

7th ICCSM

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CROATIAN SOCIETY OF MECHANICS
22 - 25 May, 2012, Zadar, Croatia

BOOK OF ABSTRACTS



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ICCSM

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Preface

Since its beginning, the Croatian Society of Mechanics is an association with the objective of promoting education, research and professional work in the fields of fluid and solid mechanics. Within the framework of this Society, scientific gatherings have been organised in order to make an environment for the exchange of new ideas in mechanical sciences. The most prominent scientific events are the International Congresses of the Croatian Society of Mechanics. The six previous congresses took place in Pula in 1994, Supetar in 1997, Cavtat, in 2000, Bizovac in 2003, Seget in 2006 and Dubrovnik in 2009. It has always been the intention to host the conference in an attractive environment. Therefore, the place of the 7th International Congress of the Croatian Society of Mechanics (7th ICCSM) is Zadar, "a city of exceptional history and rich cultural heritage, a treasury of the archaeological and monumental riches of ancient and medieval times, Renaissance and many contemporary architectural achievements". The Congress has been organised under the auspices of the Central European Association for Computational Mechanics, International Association for Wind Engineering and the University of Zagreb. The organisation has also been supported by the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb.

The scientific programme of the 7th ICCSM consists of 4 plenary lectures and 69 regular lectures divided into 16 sessions. In the plenary lectures, outstanding researchers have addressed the state-of-the-art advances in the mechanical sciences. Through the regular contributions, the scientists from all over the world have provided an international forum for the presentation and evaluation of recent advances in various fields of theoretical and applied mechanics. Thus, special attention is directed to the recent developments in solid and structural mechanics, fluid mechanics, heat and mass transfer, using numerical, analytical, and experimental methods. This scientific gathering is also an opportunity for valuable interactions between junior and senior researchers that may result in new ideas as well as new and prospective research directions.

This book contains the abstracts of the 4 plenary contributions and the abstracts of all 84 regular presentations. The abstracts have been accepted after having been carefully reviewed by the members of the Scientific Committee and the Organising Committee. However, the editors do not assume liability for possible errors or inconsistencies, as the papers have been included in the final form contributed by authors. The extended versions of the presentations are published on CD-ROM.

We gratefully acknowledge the valuable contribution of the plenary speakers who gladly accepted our invitations, as well as all the authors of the contributed papers. Furthermore, we express our gratitude to the members of the Scientific Committee who promoted our Congress worldwide. Many thanks go to the colleagues who made effort to review the abstracts. In addition, we would like to thank our sponsors, especially the Ministry of Science, Education and Sports of the Republic of Croatia. Special thanks go to the staff at Studio Hrg d.o.o. for their work in preparation activities. Finally, we would like to thank everyone who helped in any way in organising this scientific event.

*Zdravko Virag
Ivica Smojver
Hrvoje Kozmar*

AIMS AND SCOPE

It is the intention of the organizers to bring together researchers in the various fields of mechanics and present the state of the art in their respective areas. Furthermore, the congress is to exemplify the current research within the Croatian scientific environment. The scope of the congress is very extensive and covers all fields of analytical, computational, and experimental mechanics. Nevertheless, all proposed papers will be submitted to thorough reviewing process in order to ensure the high quality of presented information, with special emphasis on originality and relevance. Among other topics, of particular interest are:

- Biomechanics and Computational Biology
- Composites and Other Advanced Materials
- Concrete Structures
- Damage, Fatigue and Fracture
- Dynamics
- Environmental Fluid Dynamics
- Experimental Mechanics
- Fluid Mechanics
- Geomechanics
- Heat and Mass Transfer
- Meshless and Other Novel Computational Methods
- Multi-physics and Multiple-scale Modelling
- Nanomechanics and Nanostructured Materials
- Optimal Design of Engineering Structures
- Plasticity
- Solid and Structural Mechanics
- Wind Engineering

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ABSTRACTS

KEYNOTE SPEAKERS



COMBINED SUN-WIND ENERGY HARVESTING – IS THE SUPP TECHNOLOGY A SUSTAINABLE SOLUTION?

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1. Introduction

The today actual economic European and worldwide context, clean energy and sustainable development are amongst the top priorities. The inexhaustible solar radiation input in the deserts and the powerful updraft of warm air have enhanced, in recent years, a new idea for renewable energy sources: a combined sun-wind energy solution. The technology, namely “Solar Updraft Power Plant” (SUPP), is promoted by groups of experts around the world and the first solar updraft tower is currently under construction in China. The working principle - based on the natural updraft of warm air in a chimney and conversion into electric power by wind turbines at the chimney foot - is interesting and not yet exploited. The dimensions of the structures are a real challenge in civil engineering: towers up to 1500 m in height are envisaged, with a glass collector up to 5 or 7 km in diameter at the base.

This paper briefly reviews the state of the art on Solar Updraft Towers and addresses a fundamental question in the framework of the actual European economic context – the sustainability of the SUPP technology.

2. Solar Updraft Towers: why?

Solar Updraft Power Plants represent a new generation of renewable energy sources, converting solar radiation into electric power. While fossil energy bases are finite and thus their exploitation is limited, renewable energies are inexhaustible and will give an ever increasing contribution to the world energy supply. There are several sources of renewable energy: for example wind, water, geothermal, biomass and solar. The Solar Updraft Power Plant technology addresses a very challenging and not yet exploited idea of combining two kinds of renewable energy: wind and solar.

The strengths of this technology are a lot. The main weaknesses are, of course, the challenge of constructing an extremely high structure and the high initial cost. In fact, it is the long lifetime period (more than 100 years) and the energy production - practically for free after a depreciation period - which would made of Solar Updraft Towers the most convenient technology in the world. All of that, with absolute absence of pollutant emissions (if one incorporates the CO₂ emissions during construction, one ends up with a few grams of CO₂ per kWh of produced electricity). No water is needed at all for power generation and several studies have been conducted regarding deposition of sand on the collector (Hoeffler, 2010). The last one is in fact a considerable problem for other solar technologies in the deserts, e.g. parabolic troughs. Moreover, a thermodynamic optimization proposed by Pretorius (2007) would guarantee a control of the power output according to specific demand patterns.

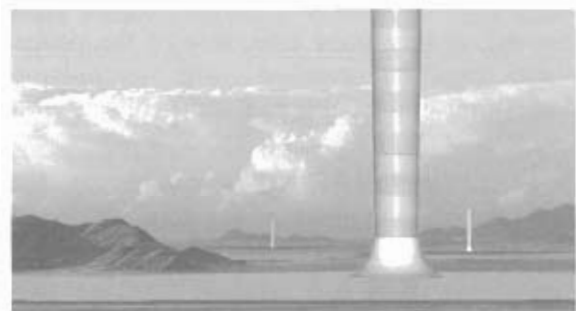


Figure 1. Solar Updraft Power Plant

3. Working principle

A SUPP consists of three elements: the collector area, the turbine(s) with coupled generator(s) as power conversion unit, and the solar tower. In the collector area, a large glass-covered area, wide-banded solar radiation heats the collector ground and consequently warms up the air inside the collector, through the

mechanism of natural convection. The buoyant air rises up into the chimney of the plant, thereby drawing in more air at the collector perimeter and thus initiating forced convection which heats the collector air more rapidly. The driving force or potential that causes air to flow through the solar tower is due to the pressure difference between a column of cold air outside and a column of hot air inside the chimney. The stream of warm air turns the turbines at the chimney foot and in the power conversion unit the kinetic energy of the flow is then transformed into electric power.

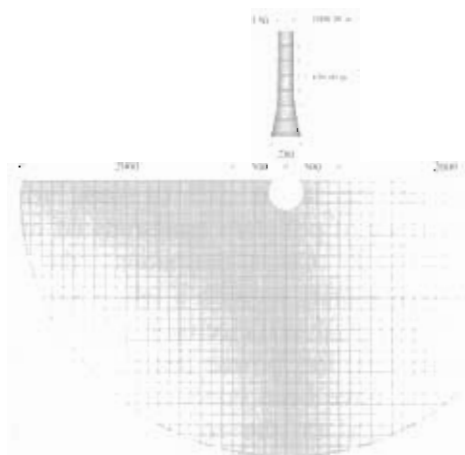


Figure 2. Solar Updraft Power Plant

The production of energy is proportional, in a non-linear manner, to the volume of the cylinder having the height of the tower and the diameter of the collector. For this reason, provided a sufficiently high solar radiation input (as it happens, for example, in the great deserts), a very good efficiency of the power plant can be reached with extra-large dimensions of the tower and/or the collector.

4. Production of energy

High solar radiation is a fundamental prerequisite for a good efficiency of the plant. Solar Updraft Power station concepts will deliver sufficient efficiency in areas with solar radiation input higher than 2.0-2.3 MWh/a. It does occur in all great deserts up to $\pm 30^\circ$ of latitude (North or South of the equator). The distribution of yearly solar radiation is shown in Figure 3.

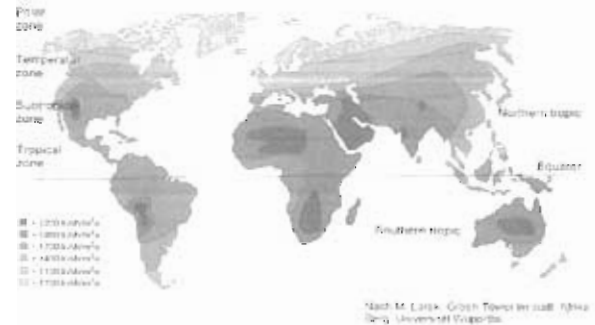


Figure 3. Distribution of yearly solar irradiation

The efficiency of the power plant is the product of the individual component efficiencies (Schlaich, 1995):

$$\eta = \eta_{coll} * \eta_c * \eta_{turb} \quad (1)$$

The efficiency of the collector (η_{coll}) describes the effectiveness with which solar radiation is converted into heat. It depends on the area of the collector and on the temperature difference between the collector inflow and outflow (its typical value is $\Delta T \approx 30$ K). The efficiency can be increased by improving the quality of the glass. For example, simulations show the superiority of coated double glazing (Krätzig, 2010).

The efficiency of the chimney (η_c) describes the effectiveness with which the quantity of heat delivered by the collector is converted into flow energy. The pressure difference between a column of cold air outside and a column of hot air inside the chimney is the driving force that causes air to flow through the tower. From Bernoulli's equation one receives the formula for the maximum air velocity at the chimney's entrance:

$$V_{c, \max} = \sqrt{2gH \frac{\Delta T}{T_0}} \quad (2)$$

The efficiency of the chimney is fundamentally dependent on its height. Of secondary importance (but influencing, anyway) is the diameter of the tower. Pretorius (2007) suggests a simplified formula for evaluation.

Last but not least, there is the efficiency of the wind turbines η_{turb} . The turbines at the base

of the chimney convert free convection flow into rotational energy and experience a certain pressure drop. It can be proved (Schlaich, 1995) that the power is maximized if such a pressure drop is about two thirds of the total pressure difference available ($k = 2/3$).

Finally, the electrical output of the Solar Updraft Power Plant is proportional to the product between height of the tower and area of the collector HA_{coll} , i.e. to the volume included within the chimney height and the collector area:

$$Pel = \frac{2}{3} \eta_{turb} \eta_{coll} \eta_c \frac{gH}{C_p T_0} A_{coll} G \quad (3)$$

A tower of 500 m in height can deliver about 50 MW of energy, but values up to 400 MW can be achieved with structures up to 1.5 km in height and 7 km in diameter of the collector.

5. From the initial idea to the recent pre-designs

The Solar Updraft Power generation was first proposed in 1903 by the Spanish Colonel Isidoro Cabanyes, followed by a publication of the German scientist Günther in 1931 (Günther, 1931). Starting in 1980, a team led by the German engineer J. Schlaich constructed a prototype in Manzanares (Spain) with 200 m high solar tower and a maximum power output of 50 kW. This plant operated successfully for more than six years and provided basic features for all future developments (Schlaich, 1995).

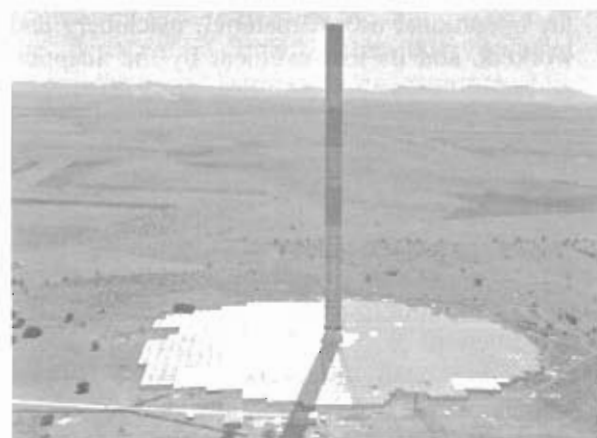


Figure 4. Prototype in Spain in 1982 (Schlaich, 1995)

In recent years, W.B. Krätzig and his team have influenced the development of SUPPs with investigations and several pre-designs, which transfer insights from designs of many natural draft cooling towers' projects to solar chimneys. The aim is to render the solar tower resistance more effective, in order to reduce investment costs of SUPPs (Krätzig et al., 2008, 2009, 2011). The smallest solar tower designed by Krätzig&Partners (Bochum, Germany) has a height of 500 m. A complete overview, demonstrating the way from the highest cooling tower in the world, built in Niederaussem (close to Cologne, Germany), to chimneys of SUPPs up to an elevation of 1500 m is shown in Figure 5 (Krätzig et al., 2009).

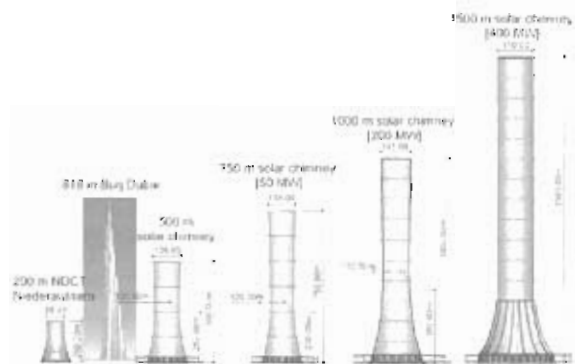


Figure 5. From the world-highest cooling tower to pre-designs of future SUPPs (Krätzig et al., 2009)

6. Structural vulnerability to natural hazards

Solar Towers are subjected to loads and other actions typical for high reinforced concrete towers: ¹⁾ dead load; ²⁾ wind loading acting both on the external and the internal surfaces of the shell, taking into account pressure fluctuations, caused by wind and body induced turbulence, and resonant vibrations of the structure caused by gustiness; ³⁾ cross wind loading caused by vortex separation; ⁴⁾ seismic action, ⁵⁾ temperature effects; ⁶⁾ shrinkage effects ⁷⁾ pre-stressing (if applicable).

Wind and seismic activity are the main natural hazards under consideration.

The sensitivity of a structure to earthquake loading depends on the natural frequencies of the tower which are excited by the ground motion. The eigenfrequency of the fundamental

oscillation of a Solar Tower - a beam-like bending mode - is around 0.17 Hz for 1-km tower. Such a value lies beyond the region of the seismic spectra which shows the most important contributions to the response. Therefore, the tower acts as a very soft cantilever. The relative displacement between the base point and the tip of such flexible structure is approximately equal to the ground displacement. As a consequence, the absolute displacement of the tip of the cantilever remains considerably small during the earthquake impact, so that a dominant increase of the base bending moment is not expected. However, the curvature of the shell associated with the ovalizing modal shapes (eigenfrequencies of 0.3 – 0.35 Hz characterize the 3rd and the 5th eigenmodes) may lead to a considerable increase of local stresses.

The wind action on solar towers comprises the mean wind load, averaged over ten minutes, the fluctuating wind load due to turbulence, the amount of resonance to turbulence, and the load induced cross-wind by vortex shedding.

A preliminary investigation of 1-km tower under an equivalent static wind load has already been performed. The aims of the investigation were: 1) assess the feasibility of Solar Updraft Towers, with regard to the wind action, 2) find a strategy of optimization of the structure to minimize wind induced tensile stresses (reduction of steel reinforcement).

In all of that, it has been proved that a key role in the structural design is played by the stiffening rings, i.e. circular ring beams (made of either reinforced concrete or steel, eventually with internal spokes). They are applied at the top and at various levels along the height, in order to stabilize the cross section and achieve a better distribution of internal forces. The stiffening rings guarantee a beam-like behaviour at the first eigenmode.

A more sophisticated wind loading model for evaluation of local effects (e.g. in the vicinity of the ring beams) is currently under investigation. A large experimental campaign was performed in the wind tunnel at Ruhr-University Bochum (Germany) during 2011. A

cross-check of results at CRIACIV wind tunnel (Figure 6, Prato, Italy) is currently running.



Figure 6. Wind tunnel model of the Solar Tower
(CRIACIV-University of Florence, Italy)

7. The construction

A crucial key for a break-through to realization of SUPP is the reduction of both the high investment costs and the erection time of the concrete chimney. This especially concerns: i) the horizontal and vertical transport facilities, especially for steel and concrete, the formwork-technologies and further the pumping and pouring of the high-performance concrete; ii) the increase of the quota of prefabrication by innovative fabrication and transportation methods; iii) the evaluation of methods to stabilize the tower during construction, like stiffening rings, guying etc., especially considering the immense height and the necessities of protection of labor safety; iv) the investigation of logistic concepts concerning the operational use of material, machinery and workers, and its improvement by the adaption of radio frequency identification RFID for a better building-site visibility.

In Germany, much experience exists in the construction of hyperbolic cooling tower shells in reinforced concrete. Proven technologies for formwork, scaffold climbing systems, material and bulk transport are available for the construction of such large hyperbolic shells with thin wall thickness. However, these techniques need to be calibrated and optimized for such a new technology. Such an issue, which still represents a challenge and is not yet

completely solved, is addressed in (Strauss, 2010).

The well-established climbing technology for cooling towers with vertical steps of 1.0 m, 1.2 m or 1.5 m has to be revised in light of the newest slip-form technology. Another optimised solution has to be found for the installation of the rings for stiffening using a special supplemental steel construction or prefabricated concrete elements for climbing purposes. Rope-stabilised tower cranes have been used up to now for the transport of materials but high-speed elevators have to be considered additionally for the new challenge, in order to put huge concrete masses into position in time. Moreover, new technologies have to be developed for surveying, so to adjust formwork equipment in certain levels and in awareness of flexible deformations in range of several meters at the top of the site.

8. Strategy for break-through and levelled electricity costs

Solar Updraft Towers are able to serve public electricity demands on high-potential level provided huge dimensions of the power plant. The initial challenge for constructing such giga-towers, never built in the world and above current experience, is thus very high. The recent construction of super tall buildings in Dubai is, in any case, a promising step.

Smaller prototypes (e.g. 500 m in height) are then recommended at first, in order to spread and test the viability of the SUPP concept. The convincing feature must be the minimization of the depreciation time. After that, levelled electricity costs decrease to an extremely low level (below 1.5 €ct/kWh) and the service-life of around 120 years is the winning move over other apparently more effective but short-terms sources of energy. Such long service-lives are realizable with modern concrete technologies. Glass roof and machinery can be renewed at some service-life intervals.

9. Acknowledgments

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MODERN TECHNIQUES FOR WIND LOADING OF LARGE ROOFS

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1. Introduction

In the last twenty years there has been a huge increase in the number of covered sports venues constructed around the world. For example, during this period in the city of Melbourne, Australia, a multi-sport 50,000 seat stadium with a fully closable roof, a football (soccer) and rugby arena (30,000 seat), and two tennis arenas with movable roofs and an aquatic centre have all been newly constructed. The historic Melbourne Cricket Ground (100,000 seat approximately) has also been re-built with fully-covered accommodation during this period. Similar developments have occurred in other parts of the world.

Wind loading for the large roofs on these facilities has generally been a challenge for engineers. Structural engineers usually do not have the background or expertise to go much beyond the provisions of wind loading codes and standards (e.g. [1], [2]) and turn to wind engineers for input on wind loads. This has usually involved the use of boundary-layer wind tunnels, now commonly used around the world.

Contrary to the perceptions of some, large roofs on sports arenas, and similar-sized roofs such as convention centres, are not generally dynamically wind-sensitive. There are exceptions such as very large roofs with low natural frequencies and roofs with unusually flexible support systems, including some roofs cantilevered from one side.

However, a special challenge in determination of wind loads results from the influence lines for many load effects (e.g. internal shear forces, axial forces and bending moments) having both positive and negative values. This will lead to upward-acting net wind pressures on one part of a roof having an opposite effect on an internal force to that of an

upward acting net pressure on a different part of the roof. As a result the simple quasi-steady approach to wind loading adopted by most codes and standards, although adequate for small structures, can produce large errors in wind load effects in large roofs.

Traditional aeroelastic structural wind-tunnel models, as commonly used in the 1970s and 1980s, had major disadvantages in that they were strongly dependent on the structural system, and thus require the structural design to be complete before a wind-tunnel model can be constructed. They were primarily used to measure deflections (a serviceability design criterion), and they were not able to give complete loading distributions for the design of the structure.

This paper will describe the bases for modern wind-tunnel techniques, developed in the last twenty years, for determination of wind loads and effective static wind load distributions, on large roofs, and discuss some anecdotal case studies, from the author's experience.

2. Components of wind loading

Wind load effects, and the wind load distributions producing them, can conveniently be divided into three components:

- The mean or static component
- The quasi-static or background fluctuating component
- Resonant components

Two methods are possible to determine the effective static distributions for the mean and background (quasi-static components):

- A direct approach in which simultaneous time histories of fluctuating pressures from the whole roof are recorded and stored. These are subsequently weighted with

structural influence coefficients to obtain time histories of load effects. The instantaneous pressure distributions coinciding with peak load effects are then identified and averaged.

- In the other approach, correlations between pressure fluctuations at different parts of the roof are obtained, and expected pressure distributions corresponding to peak load effects are obtained by calculation.

The first method is commonly used in North America and the U.K. for derivation of wind loads for actual large roof structures, whereas the second method has generally been adopted in Australia and Germany. The latter approach is discussed in detail in a later section of this paper.

Although the mean and background load distributions result directly from wind pressures, the effective loading distribution for the resonant loading is an *inertial* loading, proportional to mass, acceleration, and the mode shape coordinate for the particular mode of vibration being considered. This is quite similar to that used in design for seismic loads.

The resonant components are sensitive to the assumed level of damping (both structural and aerodynamic). Unfortunately the structural damping is not well known for actual large roofs, so that a high degree of accuracy in incorporating the resonant components is usually not warranted. The incorporation of the resonant contributions into the analysis is discussed in a subsequent section.

3. Mean and background components

The second technique described in the previous section, involves the measurement of correlation coefficients for panel pressures from rigid wind-tunnel models and the determination of structural influence coefficients by the designers, and the use of Kasperski's Load Response Correlation (LRC) equation (Kasperski, 1992; Kasperski and Niemann,

1992) to determine effective static load distributions. However there were some differences in the subsequent analysis techniques. In some cases, Proper Orthogonal Decomposition (POD) (Holmes, 1990, 1992) can be used to simplify the calculations.

The LRC method gives an expected effective static load distribution corresponding to a particular load effect or response, r , with an influence coefficient $I_r(z)$, i.e. the value of r when a unit load is applied at position, z . In practice, the rigid pressure model is divided into a number of panels, N , with the number of panels dependent on the structural system. Influence coefficients for particular responses of interest (e.g. axial or shear forces, bending moments or deflections) are derived by the structural engineer, by applying a series of patch loads, each one corresponding to a uniformly distributed load applied to the wind tunnel model panel. The influence coefficient vector for each response will have N values.

For a structure of length, L , the instantaneous value of r is:

$$r(t) = \int_0^L p(z, t) I_r(z) dz \quad (1)$$

$p(z, t)$ is the fluctuating pressure at position z , where $0 < z < L$.

The mean value of r is:

$$\bar{r} = \int_0^L \bar{p}(z) I_r(z) dz \quad (2)$$

By covariance integration (Holmes and Best, 1981), the standard deviation of r is given by:

$$\sigma_r = \left[\int_0^L \int_0^L \overline{p(z)p(z')} I_r(z) I_r(z') dz dz' \right]^{1/2} \quad (3)$$

The expected maximum value of r is given by:

$$\hat{r} = \bar{r} + g_B \sigma_r \quad (4)$$

where g_B is a peak factor, 'B' denoting background response.

The LRC formula for the distribution of $p(z)$ corresponding to $\hat{r}$ is:

$$[p(z)]_r = \bar{p}(z) + g_B \rho_{pr}(z) \sigma_p(z) \quad (5)$$

where $\rho_{pr}(z)$ is the correlation coefficient between $p(z,t)$ and $r(t)$, given by:

$$\rho_{pr}(z) = \frac{\int_0^L p(z,t) \int_0^L p(z',t) I_r(z') dz'}{\sigma_p \sigma_r} = \frac{\int_0^L \overline{p(z,t) p(z',t) I_r(z')} dz'}{\sigma_p \sigma_r} \quad (6)$$

where σ_r is given by Equation (3).

Note that the numerator in Equation (6) remains a function of position, z .

That Equation (5) is correct can be demonstrated, by computing the peak value, $\hat{r}$, from it.

$$\begin{aligned} \text{Thus, } \hat{r} &= \int_0^L [p(z)]_r I_r(z) dz \\ &= \int_0^L [\bar{p}(z) + g_B \rho_{pr}(z) \sigma_p(z)] I_r(z) dz \\ &= \int_0^L \bar{p}(z) I_r(z) dz + g_B \frac{\int_0^L \left(\int_0^L \overline{p(z,t) p(z',t) I_r(z')} dz' \right) I_r(z) dz}{\sigma_r} \\ &= \bar{r} + g_B \frac{\sigma_r^2}{\sigma_r} \quad (\text{using Equation (3)}) \\ &= \bar{r} + g_B \sigma_r \quad (\text{i.e. Equation (4)}) \end{aligned}$$

For a roof divided into N discrete panels, Equation (6) can be re-written as a finite summation:

$$\rho_{pi,r} = \frac{\sum_{i=1}^N \overline{p_i(t) p_j(t) \beta_i \beta_j}}{\sigma_{p_i} \sigma_r} \quad (7)$$

$$\text{where } \sigma_r = \sum_{i=1}^N \sum_{j=1}^N \overline{p_i(t) p_j(t) \beta_i \beta_j} \quad (8)$$

i and j represent panel numbers, N is the total number of measurement channels. β_i is the influence coefficient for panel, i , on the load effect, r , and incorporates the area of the panel - that is, β_i is the value of r when a uniformly distributed 'patch load' of unit magnitude is applied to the panel.

4. Resonant components

The effective load distribution due to the resonant part of the loading is given, to a good approximation, by the distribution of inertial forces over a roof. This is based on the assumption that the fluctuating wind forces at the resonant frequency are balanced by the damping forces, once a stable amplitude of vibration has been established.

Thus, an equivalent load distribution for the resonant response in mode 1, $p_R(z)$, is given by:

$$p_R(z) = g_R m(z) (2\pi n_1)^2 \sqrt{a_1'^2} \mu_1(z) \quad (12)$$

where g_R is the peak factor for resonant response; $m(z)$ is a mass per unit length; n_1 is the first mode natural frequency; $\sqrt{a_1'^2}$ is the r.m.s. modal coordinate (resonant contribution only), and $\mu_1(z)$ is the mode shape for the first mode of vibration.

Determination of the r.m.s. modal coordinate requires knowledge of the spectral density of the exciting forces, the correlation of those forces at the natural frequency (or aerodynamic admittance), and the modal damping and stiffness.

For low damping ratios, the mean square resonant modal coordinate for the 1st mode of vibration is to a good approximation:

$$\overline{a_1'^2} = [\pi n_1 S_{F1}(n_1)] / [4 K_1^2 \zeta_1] \quad (13)$$

where a_1 is the generalised coordinate for the 1st mode of vibration;

n_1 is the natural frequency of the 1st mode,

K_1 is the generalised stiffness equal to $(\omega_1^2 G_1)$,

$\omega_1 (= 2\pi n_1)$ is the circular frequency,

G_1 is the generalised mass,

ζ_1 is the critical damping ratio for the 1st mode,

$S_{F1}(n)$ is the power spectral density of the generalised force for the first mode.

The mean square fluctuating value of a load effect, r , resulting from the resonant response in mode 1, can be written:

$$\sigma_{r,R1}^2 = \alpha_1^2 \omega_1^4 \overline{a_1'^2} \quad (14)$$

ω_1 is the circular frequency in mode 1 ($= 2\pi n_1$)

α_1 is the integral: $\int_0^L m(z) \cdot \mu_1(z) \cdot I_r(z) dz$

$m(z)$ is the mass per unit length for a continuous structure

$\mu_1(z)$ is the mode shape for the first mode of vibration

$I_r(z)$ is the influence line for the load effect in question.

Equation (14) indicates that the resonant contribution to a load effect depends strongly on the value of α_1 – i.e. the coincidence of the influence line and the mode shape.

In the case of a roof divided into N discrete panels, α_1 can be written :

$$\alpha_1 = \sum_{i=1}^N m_i \mu_{1,i} \beta_i \quad (15)$$

The total peak response can be estimated by:

$$\hat{r}_T = \bar{r} + \sqrt{\hat{r}_B^2 + \hat{r}_R^2} = \bar{r} + \sqrt{g_B^2 \sigma_B^2 + g_R^2 \sigma_{R,1}^2} \quad (16)$$

The peak factor for the resonant response, g_R , is given by (Davenport, 1964):

$$g_R = \sqrt{(2 \log_e n_1 T) + 0.5772} / \sqrt{(2 \log_e n_1 T)} \quad (17)$$

5. Combined effective load distributions

The correct method of combining effective static load distributions for mean, background and resonant components (one mode) is as follows (Holmes, 2002):

$$p_c(z) = \bar{p}(z) + W_B p_B(z) + W_R(z) p_R(z) \quad (18)$$

The absolute values of the weighting factors W_B and W_R are given by:

$$|W_B| = \frac{g_B \sigma_{r,B}}{(g_B^2 \sigma_{r,B}^2 + g_R^2 \sigma_{r,R}^2)^{1/2}} \quad (19)$$

$$|W_R| = \frac{g_R \sigma_{r,R}}{(g_B^2 \sigma_{r,B}^2 + g_R^2 \sigma_{r,R}^2)^{1/2}}$$

Equation (18) can be validated by multiplying both sides by the influence line and integrating over the structure. The sign of W_B depends on the sign of g_B , i.e. on whether a maximum or minimum response is being considered. The sign of W_R depends on the sign of α_1 , and will differ from the sign of W_B , if α_1 is negative.

Additional terms can be added to Eq. (18) to incorporate additional resonant modes. i.e.

$$p_c(z) = \bar{p}(z) + W_B p_B(z) + W_{R1}(z) p_{R1}(z) + W_{R2}(z) p_{R2}(z) + \text{etc.} \quad (20)$$

where,

$$|W_{R1}| = \frac{g_{R1} \sigma_{r,R1}}{(g_B^2 \sigma_{r,B}^2 + g_{R1}^2 \sigma_{r,R1}^2 + g_{R,1}^2 \sigma_{r,R1}^2)^{1/2}}$$

$$|W_{R2}| = \frac{g_{R2} \sigma_{r,R2}}{(g_B^2 \sigma_{r,B}^2 + g_{R2}^2 \sigma_{r,R2}^2 + g_{R,2}^2 \sigma_{r,R2}^2)^{1/2}} \quad (21)$$

6. Some case studies

6.1 Case A

This stadium (Figure 1) actually has a number of separate roofs – four fixed and two moving roofs. The stadium has an oval planform with the fixed roofs on four sides, supported by large trusses. The two moving roofs are in the fully closed position give a totally enclosed arena. In the open position the moving roofs retract out over the fixed roofs on the east and west sides. The roof sections are nearly flat.



Figure 1. 1/400 scale model for Case A

A scale model of the stadium and roofs, at a geometric scaling ratio of 1/400, was constructed and installed in a boundary-layer wind tunnel, in which a boundary layer with characteristics of strong winds over urban terrain were simulated. The turbulent boundary-layer flows were generated by a plain barrier spanning the start of the test section, followed by about 10 metres of small roughness blocks up to the model, which was mounted on a turntable.

Basic wind pressure information was obtained from measurements on a number of panels on the top and bottom surfaces of each roof. Each panel pressure was obtained by pneumatically averaging the fluctuating pressures from up to ten individual pressure tappings. Specially designed pressure tubing systems enabling accurate measurements of the fluctuating panel pressures to a frequency of 200-300 Hertz on the model - equivalent to

about 1-2 Hertz on the full-scale structure, were used.

Five different roof configurations were studied in the wind tunnel for this stadium, to cover the most likely moving-roof configurations. Measurements were carried out for sixteen different wind directions.

The various roofs of this stadium were considered to be sufficiently stiff that the resonant dynamic response was negligible. 16 pressure measurement panels were provided on the fixed West roof, and 12 on the fixed North roof. In both cases, two panels were provided for the underside of the roofs. The loads on the fixed East and South roofs were obtained from the West and North roofs, by symmetry. One half of the moving roof was also covered with 14 upper surface panels and 2 underneath.

In order to minimise the amount of pressure tubing under the moving roof in the open position (and thus allow free flow of air between the moving roof and the fixed roof underneath), the pressure panels of the moving roof were internally manifolded through machined channels in the thickness of the model roof, in order to separate the pressure measurements on the top and bottom surfaces.

Nearly simultaneous pressure sensing was carried out for each instrumented roof using an electronic pressure scanner (ZOC-16 scanner from Scanivalve Corporation). In-house software on a PC enabled the mean, r.m.s. and peak pressure coefficients to be determined. In addition, all the correlation coefficients for every pair of panels within a single roof were calculated. This information enabled the effective static load distributions and peak load effects, to be determined using Eqs. (1) to (6), with structural influence coefficients obtained from the structural engineer.

Examples of influence coefficients for a deflection and for an axial force in the top chord member of the main truss supporting one half roof, are given in Table 1. These values represent the deflection or truss force, as a uniformly distributed panel load of unit

magnitude is applied in turn to every pressure panel on the roof.

Loaded Panel	Influence Coefficient deflection at Panel 8 (mm)	Influence Coefficient for force in top chord of truss (kN)
1	6.1	580
2	2.1	331
3	-0.1	70
4	111.9	2705
5	68.1	1760
6	28.1	770
7	1.6	37
8	281.3	1872
9	140.4	1210
10	39.5	533
11	0.9	28
12	91.6	367
13	49.9	184
14	11.1	418
15	-614.7	-4612
16	-217.7	-6253

Table 1. Influence coefficients for a deflection and the axial force in a top chord member (*Case A*)

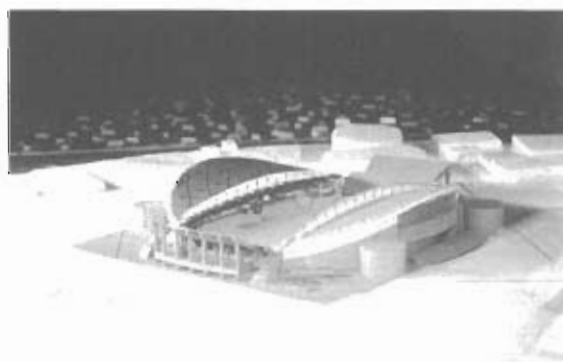


Figure 2. 1/500 scale model for *Case B*

6.2 Case B

These roofs (Figure 2) are of hyperbolic paraboloid (saddle) shape on an oval planform, but open in the middle over the playing area. The two roofs are supported along the leading edges by space-frame arches.

The model for the wind-load study was constructed at a scaling ratio of 1/500, but otherwise the experimental techniques were similar to *Case A*. Because of the symmetry of the stadium, only one half was instrumented with pressure taps. However, there were two different configurations for the end stands giving total stadium capacities of 110,000 and 80,000 persons respectively. Although these stands did not have roofs themselves, there was a possibility of aerodynamic shielding or interference effects on the roofs for which the wind loading was required, and tests were carried out for both end-stand configurations.

Twenty-four panels on the top surface of the roof were defined for pressure measurement for the structural loads. Each panel pressure was obtained by pneumatically averaging the fluctuating pressures from three or four individual pressure tappings. A specially designed tubing system (Holmes and Lewis, 1987) enabled accurate measurements of the fluctuating panel pressures to a frequency of 300 Hertz on the model (equivalent to about 3 Hertz on the full-scale structure) to be made. The panel arrangement for one half of the roof is shown in Figure 3.

The two lowest resonant modes of each roof half were calculated to have frequencies of 0.64 and 0.73 Hertz, necessitating some consideration of dynamic resonant response.

The critical wind direction for upward (negative) wind pressures on the west roof of Stadium Australia was found to be west-south-west. For this wind direction, a proper orthogonal decomposition (P.O.D.) analysis found that the first three modes accounted for 48, 12, and 11% of the pressure fluctuations on the roof. As for many other structures (e.g. Holmes 1990), the first mode (Figure 4) has a shape very similar to the mean pressure distribution. This can be explained by a quasi-steady model of fluctuating pressures on roofs (e.g. Holmes, 2007), in which slowly varying wind velocity fluctuations are transmitted as pressure fluctuations.

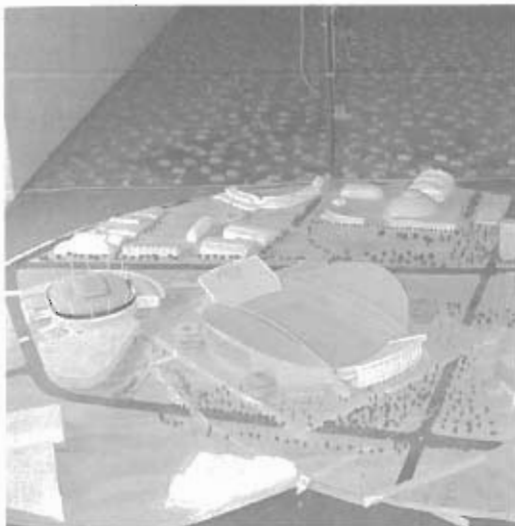


Figure 3. Surrounding buildings for Cases B and C

Critical pressure distributions (quasi-static) were determined by combining contributions from the first five P.O.D. modes, which accounted for over 80% of the pressure fluctuations. Figure 5 shows an effective static pressure distribution, determined using the methods described previously, for a critical member in the roof structure of the West roof.

Resonant contributions from the first two modes of vibration based on an assumed damping of 2% of critical were added; these amounted to less than 10% of the quasi-static contributions. The resulting distributions were grouped and enveloped before supplying to the structural designers.

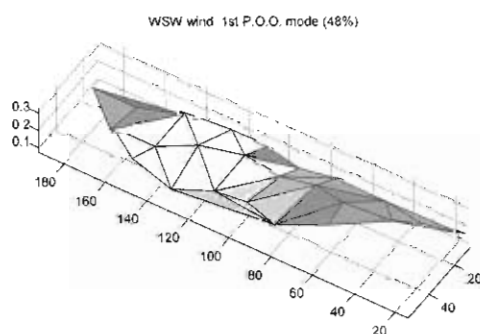


Figure 4. First P.O.D. mode for West roof of Case B (WSW wind)

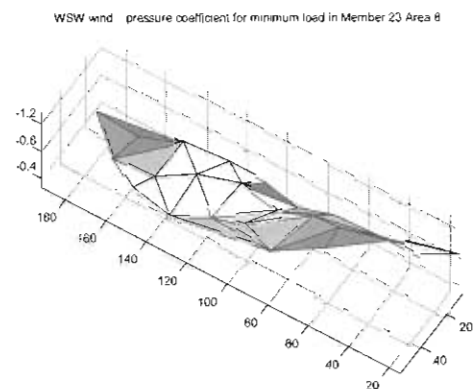


Figure 5. Critical upwards pressure distribution for a member in the West roof structure, Case B (WSW wind)

6.3 Case C

Case C is a multi-use arena with a playing area of 47 metres by 77 metres, and seating capacity of 18,000 (left-hand model in Figure 3, and Figure 6).

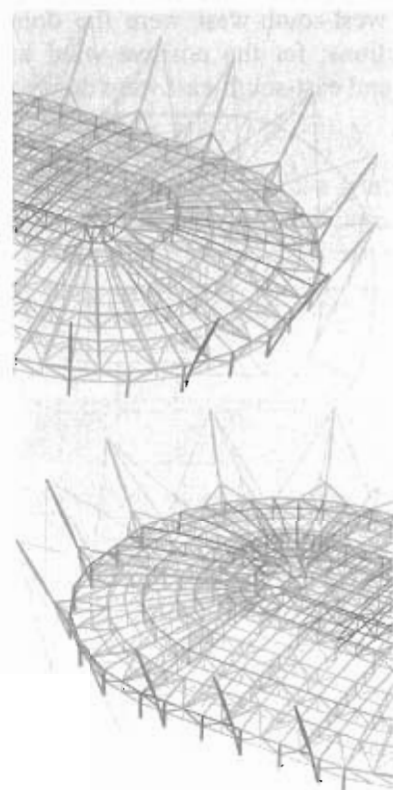


Figure 6. Structural arrangement of the roof- Case C

Wind loads were determined using a similar procedure to that used for the roofs of *Case B*, except that, in this case, resonant response to wind was regarded as negligible and not taken account of at all. A length scale of 1/500 was used for the model. In this case, 28 panels were defined on the top surface of the roof (Figure 7), and the fluctuating pressures from each panel were obtained by pneumatically averaging pressures from twelve individual pressure tappings.

The structural engineers defined 120 different load effects, and expected maximum and minimum load effects for each of these were calculated. For this roof, supported continuously around the circumference, resonant dynamic effects were regarded as negligible and not included in the calculations.

Effective static load distributions for each case were also obtained. These were enveloped and reduced to a single upward (negative), and a single downwards (positive) wind load distribution. In the former case west and west-south-west were the dominant wind directions; for the positive wind loads, south-east and east-south-east were dominant.

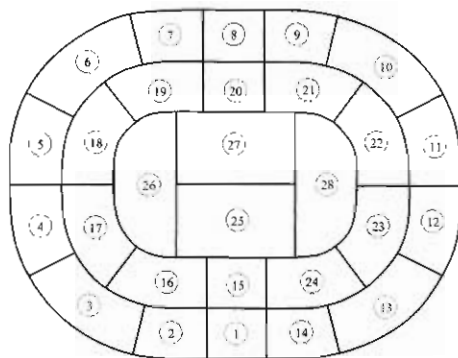


Figure 7. Panel arrangement for *Case C*

Figure 8 compares an effective static load distribution for the highest maximum (ESE wind direction), and lowest minimum (SSW direction) calculated by the LRC formula (Equation (5)), and by the direct method. Since it is based on correlations, the former method gives a more statistically stable result, than the direct or 'conditional sampling' approach, which suffers from significant sampling errors, unless some averaging process is used. Also shown in Figure 8 are the

extreme pressure limits of pressure, i.e. the maximum and minimum non-simultaneous pressures on each of the panels. Any calculated (quasi-static) pressure distribution must fall within the limits of these pressures.

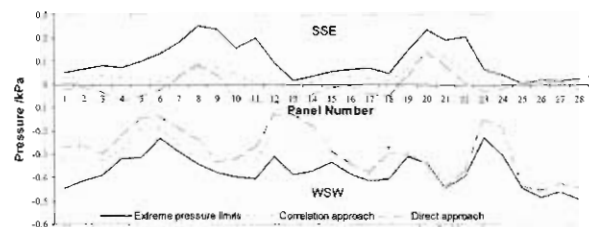


Figure 7. Comparison of effective static pressure distributions by direct calculation and by the correlation method (*Case C*)

The maximum and minimum load effects obtained by the correlation technique are typically 30% less than those obtained by assuming that the maximum (or minimum) pressures occur simultaneously all over a roof. However, there are some cases (about 30% of all load effects studied for the roof of *Case C*) for which the simultaneous maxima (or minima) underestimate the peak load effects. This is because of influence lines that change sign, so that pressures on some parts of the roof act to reduce the load effects, rather than increase them.

6.4 *Case D*

Case D (Figure 8) is a very large sports stadium with a total roof area of about 40,000 m², of which about 13,700 m² is movable. One half of the roof is supported by a large trussed arch with a rise of 133m and a span of 316m.



Figure 8. Roof and supporting arch - *Case D*

Wind-tunnel tests were carried out with a model built to a scale of 1/400 (Figure 9). The roof has a fully-closed configuration (Figure 10) and an open configuration in which part of the roof at the east and west roofs slide over the south roof.

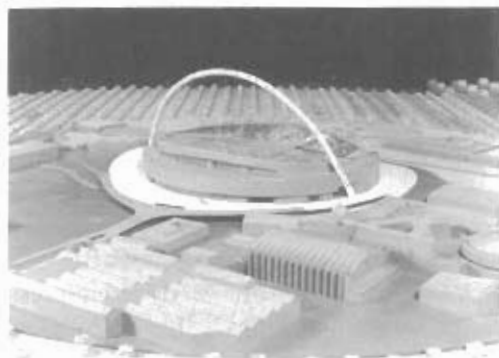


Figure 9. Wind-tunnel model - Case D

For the closed configuration, the roof area was divided into 75 panel areas, with pressures averaged from four pressure tappings within each panel on the upper surface (Figure 10).

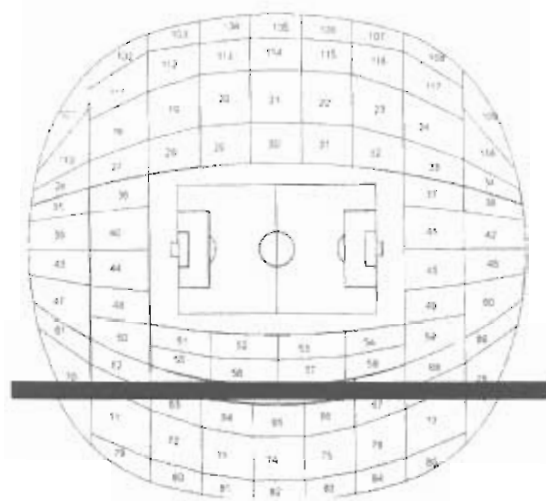


Figure 10. Panel layout closed configuration - Case D

Peak load effects for the quasi-static (mean and background) wind loading were calculated using both techniques discussed in Section 2 – i.e. the direct weighting approach, and the correlation approach. These estimates agreed to within 1.5% of each other.

An unusual feature of this very large roof was the large number of resonant dynamic modes considered to be significant contributors to the effective loads. In fact thirteen dynamic modes were included. The contribution of the resonant modes varies widely between the load effects, depending on the similarity, or otherwise, of the influence lines to the mode shapes. For ten load effects (i.e. compression and bending moments in the arch, bending moments within the roof structure and catenary reactions), the dynamic response factors varied between 1.05 and 1.69. In this case the dynamic response factor is defined as ratio of peak load effect *including* the resonant contributions, to the peak load effect *excluding* the resonant contributions.

When calculating equivalent static wind load distributions, the quasi-static distribution was simply factored by the dynamic response factor, if its value was 1.25 or less. For larger values of dynamic response factor, the more rigorous approach of Equations (20) and (21) was adopted.

6.5 Case E

The final example is the roof of a relatively small stadium, with an unusual structure consisting of a series of interconnected shells forming the walls as well as the roof (Figure 11).



Figure 11. Interconnected shell roof- Case E

Structural loads for the design of the roof were obtained from pressure measurements from a 1/150 scale wind-tunnel model, averaged within four panels of each shell of the east roof of the structure. Correlations between panel pressures were obtained from the recorded time histories, and the quasi-static wind load distributions were obtained using the techniques described in Section 3 of this paper. As for the other examples discussed, the structural engineers provided influence coefficients for various load effects, by applying uniform 'patch' loads to the areas of the structure represented by the defined panels

The predicted lowest natural frequency of the roof was about 0.9 Hertz, and hence the resonant response was not expected to be significant. However, to cover this and other uncertainties in the measurements and processing, a simple load factor of 1.35 was applied to the wind load distributions obtained as discussed in the previous paragraph.

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The author, and JDH Consulting, acknowledge the cooperation of many structural and wind engineering groups over the last fifteen years, including the Arup Group, Aurecon (formerly Connell-Wagner), BMT-Fluid Mechanics, MEL Consultants, MODUS, SKM, Wind Engineering Services (University of Sydney) and Windtech Consultants. This has led to greatly improved techniques for the definition of design wind loads on large roofs.

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Variational collision integrators in forward dynamics and optimal control

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The numerical simulation of multibody dynamics often involves constraints of various forms. First of all, we present a structure preserving integrator for mechanical systems with holonomic (bilateral) and unilateral contact constraints, the latter being in the form of a non-penetration condition. The scheme is based on a discrete variant of Hamilton's principle in which both the discrete trajectory and the unknown collision time are varied (cf. [Fete 03]). As a consequence, the collision event enters the discrete equations of motion as an unknown that has to be computed on-the-fly whenever a collision is imminent. The additional bilateral constraints are efficiently dealt with employing a discrete null space reduction (including a projection and a local reparametrisation step) which considerably reduces the number of unknowns and improves the condition number during each time-step as compared to a standard treatment with Lagrange multipliers (cf. [Leye 12]). Without collisions, this structure preserving integrator has been used in previous works on discrete mechanics and optimal control for constrained systems (DMOCC), leading to structure preserving simulation of optimal control problems for rigid multibody systems, whereby possible contacts or collisions between the bodies have been disregarded see [Leye 10]. In the formulation presented here, both collision avoidance as well as explicitly planned collisions between non-smooth bodies are included. To this end, a subdifferentiable global contact detection algorithm, the supporting separating hyperplane linear program (SSHLP), based on the signed distance between supporting hyperplanes of two convex sets, is used in the simulation of optimal control problems.

1 Introduction

In the first part of this work (Sections 2 and 3) we present an variational algorithm that handles equality and inequality constraints on the same footing, see [Leye 12]. Our approach closely follows the route described in [Fete 03] where a seamless variational integrator for collision problems is introduced; by 'seamless' we mean that both the integration scheme for the free motion and the contact conditions at the points of collision follow from a single variational principle. Here we extend the seamless variational scheme to the case where, besides the non-penetration condition for a chain of beads, additional holonomic constraints are present during the entire motion. Although our method is an ordinary time-stepping scheme, it shares some of the spirit of event-driven methodology in that it combines an implicit integrator for the free motion of the chain that allows for large time-steps with an exact treatment of the collision events. In particular, the collision times are computed on-the-fly by solving a simple quadratic equation.

For smooth systems, variational integrators with constant time-step preserve various properties of the exact dynamics such as symplecticity or momentum maps (e.g., linear or angular momentum) at the discrete level [Wend 97, Mars 01]. The key idea is simple: rather than discretising the continuous-time equations of motion, variational integrators directly discretise Hamilton's principle by appropriate quadrature rules; the numerical scheme then follows from the now discrete Euler-Lagrange equations. Variational integrators have proven useful, e.g., in understanding the long-term stability of symplectic integrators using backward error analysis [Hair 06]. Moreover, the (discrete) variational principle allows for easy generalisations so as to treat, e.g., infinite-dimensional systems [Brid 06], systems with constraints [Jay 07], or contact problems in continuum mechanics [Cira 05].

The standard molecular dynamics algorithm for systems with holonomic constraints, that is in fact a variational integrator, is the SHAKE/RATTLE algorithm [Ryck 77, Ande 83] which can be considered an augmented version of the Verlet algorithm where the constraints are enforced by suitable Lagrange multipliers. As an alternative to Lagrange multipliers that may cause stability problems when the integration time-steps are small (e.g., close to a collision point) the discrete null space method eliminates the constraint force and the Lagrange multipliers by projecting the forces in the system onto the space of admissible momenta. It does so by taking advantage of the d'Alembert principle that states that the constraint forces are always

acting perpendicular to the constraint manifold and therefore have no component in the tangential direction. The discrete null space method has been introduced in conjunction with an energy-momentum conserving time integration scheme in [Bets 05, Bets 06] and has been transferred to variational time-stepping schemes in [Leye 08]. The new idea in this work is to use the discrete null space methodology also for the treatment of the contact constraints while they are active.

When simulating the optimal control problems for three-dimensional multibody systems, the treatment of contact always imposes a challenge. One simple solution is to formulate a smooth problem where the bodies are allowed to overlap by a certain amount, which is penalised via a penalty potential. Of course, the drawbacks of inadmissible configurations and inexact contact forces that go along with this approach are obvious. However, for the forward dynamics simulation of many applications, the results might be accurate enough, in particular when high penalty parameters in combination with small time steps are tolerable from the computational effort's point of view. If the bodies are not supposed to overlap, the discrete event of contact has to be considered as part of the system's dynamics, thus making the dynamics non-smooth. See e.g. [Cira 05, Gloc 00, Lein 03, Pand 02, Pang 96] and many references on contact formulations in different contexts therein. Non-smooth formulations require the specification of a contact force that (for elastic collisions) reflects the momentum normal to a contact surface at a given time and configuration. In the context of a structure preserving time integration method (being consistent in the evolution of energy and momentum maps) that uses a predefined equidistant time grid, there is no other known procedure except for resolving the collisions in the sense that each contact time (which is likely between the nodes), configuration, and force are exactly computed, see [Fete 03, Leye 12].

The second part of this work (Sections 4 and 5) will show how the described alternatives for the treatment of collisions can be included in the context of optimal control problems. In the smooth formulation with a penalty potential, there arises some difficulty for the optimiser to distinguish between a contact and a control force, since both might point into the same direction. For the non-smooth treatment, the optimal control problem formulation gives more freedom than the forward dynamics problem, since periodic boundary conditions or leaving the exact placement of time nodes (within certain bounds) to the optimiser gives the freedom to assume that contact takes place at a certain time node without loss of generality, i.e. without fixing its physical time. A further challenge is the detection of contact for non-smooth non-convex three-dimensional bodies where a new strategy based on a supporting separating hyperplane linear programming (SSHLP) approach is used, see [John 12]. One major advantage of this strategy is that the subgradient of the SSHLP, supplying the direction of the contact force, can be readily evaluated. Reconfiguration manoeuvres with collision avoidance and with planned collisions are considered as examples.

2 Hamilton's principle for collision problems

Our formulation of the variational collision integrator follows the route taken in [Fete 03] and extends it to the case of a system that, besides a non-penetration condition, is subject to holonomic constraints. For a better understanding of the approach, it is instructive to look at the continuous formulation first. Let $Q \subseteq \mathbb{R}^n$ denote the n -dimensional configuration space of our system where n equals three times the number of particles. We suppose that the system is subject to m holonomic constraints $g(q) = 0$ with $g = (g_1, \dots, g_m)^T$ being the vector of constraints with the requirement that $G = \nabla g$ has maximum rank m on the admissible set of configurations, $C = \{q \in Q: g(q) = 0\} \subset Q$, and with $\det GG^T \geq a > 0$ being bounded away from zero.¹ The m_c non-penetration conditions for the particles can be expressed in terms of a vector of smooth unilateral constraints $g_c(q) \geq 0$ by which the set of admissible configurations turns out to be

$$C_+ = \{q \in Q: g(q) = 0, g_c(q) \geq 0\} \subset C.$$

Calling

$$TQ = \{(q, v): q \in Q, v \in T_q Q\}$$

the state space of the system consisting of the unconstrained positions and velocities, we define the Lagrangian (we use the natural identification of all tangent spaces $T_q Q$ with $\mathbb{R}^n$)

$$L: TQ \rightarrow \mathbb{R}, \quad L(q, v) = \frac{1}{2} v \cdot M v - V(q)$$

¹The latter requires that the function g has a certain regularity that we can safely assume for most cases of interest.

with $M \in \mathbb{R}^{n \times n}$ being the symmetric and positive definite mass matrix and V being a smooth potential energy. Now let $\gamma: [0, T] \rightarrow Q$ be a curve in Q that is everywhere twice continuously differentiable except at an isolated impact point $q_i = \gamma(t_i)$, $t_i \in (0, T)$ where the curve γ hits the boundary ∂C_+ of the admissible set (i.e., where exactly one of the components of g_c is zero). The classical action is then of the form

$$S[\gamma, t_i] = \int_0^{t_i} \hat{L}(\gamma(t), \dot{\gamma}(t)) dt + \int_{t_i}^T \hat{L}(\gamma(t), \dot{\gamma}(t)) dt \quad (1)$$

where

$$\hat{L}(q, v) = L(q, v) - g(q) \cdot \lambda \quad (2)$$

is the augmented Lagrangian involving the constraints g and the Lagrange multiplier $\lambda \in \mathbb{R}^m$ (cf. Lagrange multiplier theorem e.g. in [Zeid 95]). Taking variations with respect to both γ and the unknown collision time t_i , we find

$$\begin{aligned} \delta S[\gamma, t_i] &= \delta \left(\int_0^{t_i} \hat{L}(\gamma(t), \dot{\gamma}(t)) dt + \int_{t_i}^T \hat{L}(\gamma(t), \dot{\gamma}(t)) dt \right) \\ &= \int_0^T \left(\frac{\partial \hat{L}}{\partial \gamma} - \frac{d}{dt} \frac{\partial \hat{L}}{\partial \dot{\gamma}} \right) \cdot \delta \gamma dt - \left[\frac{\partial \hat{L}}{\partial \dot{\gamma}} \cdot \delta \gamma + \hat{L} \delta t_i \right]_{t_i^-}^{t_i^+}. \end{aligned} \quad (3)$$

Requiring that the integral vanishes, yields Newton's law

$$M\ddot{q} = -\nabla V(q) - G(q)^T \lambda, \quad g(q) = 0 \quad (4)$$

for the motion away from the collision. The remaining boundary terms provide the contact conditions at the point of impact. To make this precise we call g_c the scalar component of g_c that is zero at the moment of collision (cf. 3.2 for the treatment of multiple collisions at a time) and note that $g_c(\gamma(t_i)) = 0$ entails $\delta g_c(\gamma(t_i)) = 0$, i.e.,

$$G_c(\gamma(t_i)) \cdot (\delta q_i + \dot{\gamma}(t_i) \delta t_i) = 0$$

where $q_i = \gamma(t_i)$ denotes the impact point. The last equation is satisfied if either

$$\delta q_i = -\dot{\gamma}(t_i) \delta t_i$$

or

$$\delta q_i \perp G_c(q_i) \quad \text{for} \quad \delta t_i = 0$$

which determines the admissible variations of the curve at the collision point. Taking joint variations of q_i and t_i under the constraint $\delta q_i = -\dot{\gamma}(t_i) \delta t_i$ implies

$$\left[\hat{L} - \frac{\partial \hat{L}}{\partial \dot{\gamma}} \cdot \dot{\gamma} \right]_{t_i^-}^{t_i^+} = 0 \quad (5)$$

which, using that

$$E = \frac{\partial \hat{L}}{\partial \dot{\gamma}} \cdot \dot{\gamma} - \hat{L},$$

is equivalent to conservation of energy E . Conversely, if we vary q_i orthogonal to the contact surface while keeping t_i fixed (i.e., $\delta t_i = 0$), then

$$\left[\frac{\partial \hat{L}}{\partial \dot{\gamma}} \cdot \delta \gamma \right]_{t_i^-}^{t_i^+} = 0 \quad \text{for} \quad \delta \gamma(t_i) \perp G_c(q_i)$$

implies

$$\left[\frac{\partial \hat{L}}{\partial \dot{\gamma}} \right]_{t_i^-}^{t_i^+} = G_c^T \lambda_c. \quad (6)$$

Here $\lambda_c \in \mathbb{R}$ is an unknown Lagrange multiplier that must be determined by solving (5). Defining the conjugate momentum $\mathbf{p} = \partial L / \partial \dot{\mathbf{q}}$ in the usual way, equations (5) and (6) can be recast as

$$\mathbf{p}(t_i^+) = \mathbf{p}(t_i^-) + G_c^T(\mathbf{q}_i)\lambda_c, \quad \mathbf{p}(t_i^+) \cdot \mathbf{M}^{-1}\mathbf{p}(t_i^+) = \mathbf{p}(t_i^-) \cdot \mathbf{M}^{-1}\mathbf{p}(t_i^-) \quad (7)$$

But the last equation is simply the momentum reflection law for an elastic collision. That is, the change in momentum occurs normal to the contact surface where the amount by which the momentum changes is determined from the conservation of energy during the collision (note that the potential V is not needed to determine the unknown multiplier λ_c). As a consequence of (7), both total linear and angular momentum and the total energy are conserved during the collision.

We stress that the contact condition (7) follows seamlessly from the boundary terms arising in (3). In the next section we will show, again following the approach described in [Fete 03], how the discrete variant of (1) naturally gives rise to a fully variational collision integrator.

3 Variational collision integrator for constrained problems

Our derivation below follows closely the route taken in [Fete 03], but extends it to the case of systems subject to holonomic constraints. The constraints are treated fully variationally using the discrete null space method that has been introduced in [Leye 08]. The new idea here is to use the discrete null space methodology also for the treatment of the contact forces while they are active.

3.1 Discrete variational principle and discrete null space reduction

Assume that the time nodes $t_0, t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_N$ with a constant basic time-step $h = t_{n+1} - t_n$ are given, however the collision time t_i with $t_{i-1} \leq t_i \leq t_{i+1}$ is unknown. Let the discrete trajectory be denoted by $\mathbf{q}_d = \{\mathbf{q}_n\}_{n=0}^N$ with $\mathbf{q}_n \approx \mathbf{q}(t_n)$, and let $\lambda_d = \{\lambda_n\}_{n=0}^N$ with $\lambda_n \approx \lambda(t_n)$ approximate the Lagrange multipliers. As usual in the context of discrete variational principles (e.g., see [Mars 01]), the discrete Lagrangian is an approximation to the action integral of the continuous Lagrangian over one time interval. In accordance with (2), this yields away from a collision

$$\hat{L}_d(\mathbf{q}_n, \mathbf{q}_{n+1}, t_n, t_{n+1}) \approx \int_{t_n}^{t_{n+1}} L(\mathbf{q}, \mathbf{v}) - g(\mathbf{q}) \cdot \lambda dt$$

In this work, a midpoint discrete Lagrangian

$$L_d(\mathbf{q}_n, \mathbf{q}_{n+1}, t_n, t_{n+1}) = (t_{n+1} - t_n) L\left(\frac{\mathbf{q}_n + \mathbf{q}_{n+1}}{2}, \frac{\mathbf{q}_{n+1} - \mathbf{q}_n}{t_{n+1} - t_n}\right) \quad (8)$$

is used in the following discrete augmented Lagrangian

$$\hat{L}_d(\mathbf{q}_n, \mathbf{q}_{n+1}, t_n, t_{n+1}) = L_d(\mathbf{q}_n, \mathbf{q}_{n+1}, t_n, t_{n+1}) - \frac{t_{n+1} - t_n}{2} (g(\mathbf{q}_n) \cdot \lambda_n + g(\mathbf{q}_{n+1}) \cdot \lambda_{n+1}).$$

For the clarity of exposition, the dependence of the discrete Lagrangian on given time nodes is not stated explicitly in the sequel and it is assumed that only one collision $g_c(\mathbf{q}_i) = 0$ occurs at t_i during the time interval $[t_{i-1}, t_{i+1}]$; see Figure 1 for an illustration. The extension to multiple collisions is formally straightforward, however more involved from the implementation point of view (see 3.2). An approximation to (1) is given by the discrete action sum

$$\begin{aligned} S_d = & \sum_{n=0}^{i-2} L_d(\mathbf{q}_n, \mathbf{q}_{n+1}) - \frac{t_{n+1} - t_n}{2} (g(\mathbf{q}_n) \cdot \lambda_n + g(\mathbf{q}_{n+1}) \cdot \lambda_{n+1}) \\ & + L_d(\mathbf{q}_{i-1}, \mathbf{q}_i, t_{i-1}, t_i) - \frac{t_i - t_{i-1}}{2} (g(\mathbf{q}_{i-1}) \cdot \lambda_{i-1} + g(\mathbf{q}_i) \cdot \lambda_i + g_c(\mathbf{q}_i) \cdot \lambda_c) \\ & + L_d(\mathbf{q}_i, \mathbf{q}_{i+1}, t_i, t_{i+1}) - \frac{t_{i+1} - t_i}{2} (g(\mathbf{q}_i) \cdot \lambda_i + g(\mathbf{q}_{i+1}) \cdot \lambda_{i+1} + g_c(\mathbf{q}_i) \cdot \lambda_c) \\ & + \sum_{n=i+1}^{N-1} L_d(\mathbf{q}_n, \mathbf{q}_{n+1}) - \frac{t_{n+1} - t_n}{2} (g(\mathbf{q}_n) \cdot \lambda_n + g(\mathbf{q}_{n+1}) \cdot \lambda_{n+1}). \end{aligned} \quad (9)$$

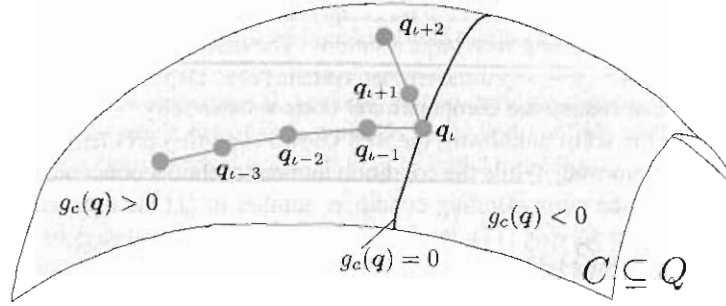


Figure 1. Discrete trajectory in constraint manifold.

As before, the discrete variational principle for the constrained motion requires that $\delta S_d = 0$ for all admissible variations $\delta q_1, \dots, \delta q_i, \dots, \delta q_{N-1}, \delta \lambda_1, \dots, \delta \lambda_N, \delta \lambda_c, \delta t_i$. This then yields discrete equations of motion for the dynamics off the contact surface with additional boundary conditions at the collision points. The equations for the collision-free motion are stated first:

Pre- and post-collision As long as no collision takes place, i.e., for $n = 1, \dots, i-2$ the discrete variational principle yields the following system

$$D_2 L_d(q_{n-1}, q_n) + D_1 L_d(q_n, q_{n+1}) - \frac{t_{n+1} - t_{n-1}}{2} G^T(q_n) \lambda_n = 0 \quad (10a)$$

$$g(q_{n+1}) = 0 \quad (10b)$$

which is solved for $q_2, \dots, q_{i-1}, \lambda_1, \dots, \lambda_{i-2}$. Here, $D_i L_d$ denotes the derivative of the discrete Lagrangian with respect to the i -th argument. Note that just as in the continuous case described in Section 2, there exists a Lagrange multiplier theorem relating stationary points of the discrete action (9) to the finite dimensional system (10), see [Mars 01]. Once the collision configuration q_i and time t_i and the first post-collision configuration q_{i+1} have been determined (as will be described below), normal time-stepping continues for $n = i+1, \dots, N-1$, i.e., (10) is solved for $q_{i+2}, \dots, q_N, \lambda_{i+1}, \dots, \lambda_{N-1}$. Obviously, (10) is a two-step method, thus special care must be given to the initialisation of the simulation. Assuming that no collision takes place during the first time-step, the following equations determine q_1 and λ_0 from given initial data $(q(0), \dot{q}(0)) \in TC$. First, one sets $q_0 = q(0)$ and $p_0^- = (\partial L / \partial \dot{q})(q(0), \dot{q}(0))$ and then solves

$$p_0^- + D_1 L_d(q_0, q_1) - \frac{t_1 - t_0}{2} G^T(q_0) \lambda_0 = 0$$

$$g(q_1) = 0$$

Collisions in the first time-step can be handled analogously to later collisions described below.

Discrete null space reduction Details on the reduction of the discrete variational equations of motion with constraints via the discrete null space method with local reparametrisation can be found in [Leye 08]. The main idea is the elimination of the constraint forces from the discrete system via the premultiplication with an appropriate null space matrix $P(\cdot) : \mathbb{R}^{n-m} \rightarrow TC$, i.e., the null space matrix fulfils range $P(q_n) = \ker G(q_n)$. Secondly, a local reparametrisation of the constraint manifold $q_{n+1} = F_d(u_{n+1}, q_n) \in C$ in terms of the discrete generalised coordinates $u_{n+1} \in \mathbb{R}^{n-m}$ representing the system's change during one time-step ensures that the constraints are fulfilled and (10b) becomes superfluous. The reduced equations read

$$P^T(q_n) [D_2 L_d(q_{n-1}, q_n) + D_1 L_d(q_n, F_d(u_{n+1}, q_n))] = 0 \quad (11)$$

Away from the collision, they are solved for $u_2, \dots, u_{i-1}$ and $u_{i+2}, \dots, u_N$ while in the very first step,

$$P^T(q_0) [p_0^- + D_1 L_d(q_0, F_d(u_1, q_0))] = 0$$

is solved for u_1 . In contrast to an absolute parametrisation in generalised coordinates with respect to the initial configuration q_0 reading $q_{n+1} = F_d(u_{n+1}, q_0)$, locality of the discrete parametrisation avoids singularities present, e.g., when dealing with large rotations. The described procedure reduces the $(d+m)$ -dimensional system (10) to the $(d-m)$ -dimensional system (11). Depending on the particular problem under consideration, this can reduce the computational costs substantially. Due to the elimination of the Lagrange multipliers from the set of unknowns, the well known condition problem associated with discretisations of index 3 DAEs is removed. While the condition number of the Jacobian matrix in the linearisation of (10) is of the order h^{-3} , the corresponding condition number in (11) is independent of the time-step (see [Leye 08]). Note that after solving (11), the Lagrange multipliers can always be determined as a post-processing step if one is interested in the constraint forces. This is particularly important when the contact forces themselves are eliminated using the discrete null space reduction (cf. equation (15) below).

Collision Before integrating forward in a new time-step, the contact inequality condition is checked. If it is violated, i.e., if $g_c(q_t) < 0$, then q_t is discarded and the collision configuration q_t and time t_t as well as λ_{t-1} are determined by solving

$$D_2 L_d(q_{t-2}, q_{t-1}) + D_1 L_d(q_{t-1}, q_t, t_{t-1}, t_t) - \frac{t_t - t_{t-2}}{2} G^T(q_{t-1}) \lambda_{t-1} = 0 \quad (12a)$$

$$g(q_t) = 0 \quad (12b)$$

$$g_c(q_t) = 0 \quad (12c)$$

which have been obtained from $\delta S_d = 0$ in (9) for variations $\delta q_{t-1}, \delta \lambda_t, \delta \lambda_c$. After that, taking admissible variations $\delta q_t, \delta \lambda_{t+1}, \delta t_t$ yields

$$D_2 L_d(q_{t-1}, q_t, t_{t-1}, t_t) + D_1 L_d(q_t, q_{t+1}, t_t, t_{t+1}) - \frac{t_{t+1} - t_{t-1}}{2} (G^T(q_t) \lambda_t + G_c^T(q_t) \lambda_c) = 0 \quad (13a)$$

$$g(q_{t+1}) = 0 \quad (13b)$$

$$D_4 L_d(q_{t-1}, q_t, t_{t-1}, t_t) + D_3 L_d(q_t, q_{t+1}, t_t, t_{t+1}) = 0 \quad (13c)$$

from which $q_{t+1}, \lambda_t, \lambda_c$ follow. Note that (13c) is resulting from the variation with respect to the time node t_t , thus it is a conservation condition for the discrete energy.

Discrete null space reduction Equivalent to (12), the reduced system yielding u_t, t_t reads

$$P^T(q_{t-1}) [D_2 L_d(q_{t-2}, q_{t-1}) + D_1 L_d(q_{t-1}, q_t, t_{t-1}, t_t)] = 0 \quad (14a)$$

$$g_c(q_t) = 0 \quad (14b)$$

and instead of solving (13), the unknown u_{t+1} can be obtained from

$$P_c^T(q_t) P^T(q_t) [D_2 L_d(q_{t-1}, q_t, t_{t-1}, t_t) + D_1 L_d(q_t, q_{t+1}, t_t, t_{t+1})] = 0 \quad (15a)$$

$$D_4 L_d(q_{t-1}, q_t, t_{t-1}, t_t) + D_3 L_d(q_t, q_{t+1}, t_t, t_{t+1}) = 0 \quad (15b)$$

As mentioned before, the Lagrange multipliers and in particular the contact forces $G_c^T(q_t) \lambda_c$ can be computed as a post-processing step.

3.2 Multiple collisions

If multiple collisions occur at the same time t_t , i.e. $g_c(q) \geq 0 \in \mathbb{R}^{m_c}$ is really vector valued with $m_c \in \mathbb{R}$, then $(\sum_{k=1}^{m_c} g_{c_k}(q_t)) \lambda_c$ is used in the discrete action (9). Note that corresponding to the single unknown collision time t_t , the Lagrange multiplier λ_c is scalar. Variation with respect to λ_c yields $(\sum_{k=1}^{m_c} g_{c_k}(q_t)) = 0$ in (12c) which, together with the positiveness of all components, is equivalent to $g_{c_k}(q_t) = 0$ for $k = 1, \dots, m_c$. Accordingly, $(\sum_{k=1}^{m_c} G_{c_k}^T(q_t)) \lambda_c$ appears in (13a) and then, as for single collisions, the discrete energy conservation condition for the complete system (13c) determines λ_c . The discrete null space reduction works analogously to the single collision case.

If collisions follow each other in quick succession, the algorithm does not return to the regular grid immediately but computes another collision time node within a basic time-step h .

3.3 Example: chain of four beads in a box

As an example, we consider a linear chain of four beads (of the same mass, length and radius) in a three-dimensional box of size $8 \times 10 \times 14$ (relative to the bead size with radius $r = 0.1$). Depending on the initial conditions, planar or real three-dimensional, periodic or non-periodic motion takes place where multiple collisions (between multiple beads or between multiple beads and multiple walls) occur. We consider an example of non-integrable three-dimensional motion. The initial conditions

$$q_0 = \left(-\frac{3}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, 0, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, 0, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, 0, \frac{3}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, 0\right)$$

$$\dot{q}_0 = \left(-\frac{1}{10}, \frac{1}{10}, -\frac{1}{10}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right)$$

lead to chaotic motion of the chain. Figure 3 illustrates different configurations, where beads in contact with other beads are coloured in blue while the red beads are in contact with a wall of the box. Note that the third row depicts a bead-bead collision happening a very short time before a bead-wall collision and the fourth row shows a bead-wall collision followed immediately by a bead-bead collision. In particular, the time between the described collisions is shorter than the basic time-step $h = 0.1$. Obviously, the algorithm is capable to handle multiple collisions (at one time node) and collisions following each other rapidly in a stable way exhibiting good energy behaviour (fluctuations are of the order of magnitude 10^{-4} , they do not show any drift, see Figure 2).

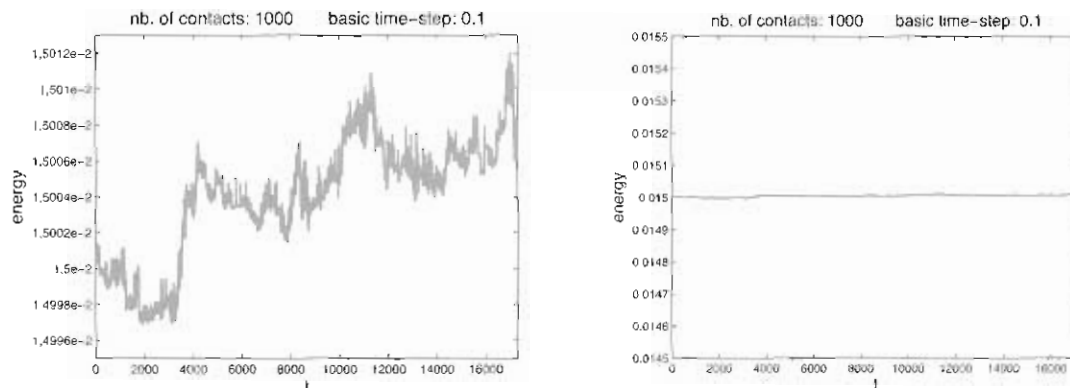


Figure 2. Chain of four beads, chaotic motion: evolution of energy during a simulation of 1000 collisions, including various collisions following each other in quick succession.

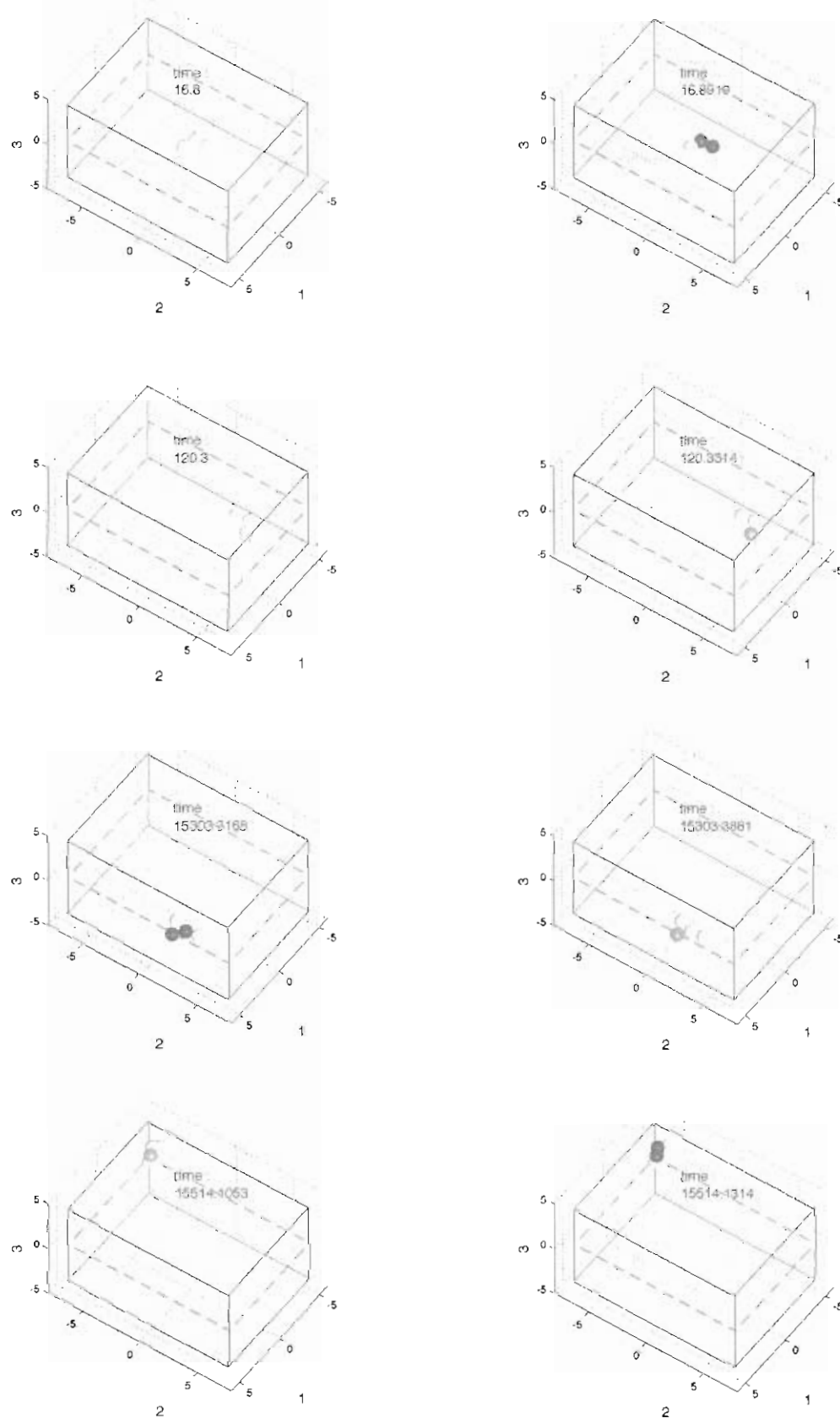


Figure 3. Chain of four beads, chaotic motion: different configurations including bead-bead and bead-wall collisions following each other rapidly.

4 Optimal control with collision avoidance

In order to avoid collisions or even any contact between bodies (including grazing collisions, i.e. the mere touching contact without a velocity component in the direction normal to the contact surface), the first step is to detect contact or overlapping. This can be quite a challenge for three-dimensional non-smooth and maybe non-convex bodies. Then, the second step is to enforce the avoidance of collisions in the simulation of optimal control problems.

4.1 Contact detection for non-smooth bodies via SSHLP

To detect contact between convex bodies, a subdifferentiable global contact detection algorithm, the so called supporting separating hyperplane (SSH) algorithm, developed in [John 12] is used. Based on theorems from convex and affine geometry, this algorithm determines the signed distance between supporting hyperplanes of two convex sets. In its first formulation, the algorithm yields a quadratically constrained linear program. However, the problems's structure is such that an equivalent linear program (LP) (with explicit expressions of the subderivative supplying the contact force) can always be formulated.

For the positive integer n , let $\alpha \neq 0 \in \mathbb{R}^n$ and $a \in \mathbb{R}$, then the affine and convex set

$$H_{\alpha,a} = \{x \in \mathbb{R}^n \mid \langle \alpha, x \rangle - a = 0\}$$

is called a hyperplane in $\mathbb{R}^n$. Here, α is the normal vector to the plane while $a = \langle \alpha, a \rangle$ for some point $a \in \mathbb{R}^n$ belonging to the plane. The signed distance between a point $y \in \mathbb{R}^n$ and a hyperplane can be computed as $\langle \alpha, y \rangle - a$. Clearly, a hyperplane has two distinct sides. For $b \in \mathbb{R}^n$ and $b = \langle \alpha, b \rangle$, the signed distance between parallel hyperplanes with normal α reads $d(H_{\alpha,a}, H_{\alpha,b}) = b - a$.

Let $K_1, K_2 \subset \mathbb{R}^n$ be compact convex sets and let $\text{ext}K_1, \text{ext}K_2$ denote the sets of their extreme points (corners for polyhedral sets) and $\text{bd}K_1, \text{bd}K_2$ their boundaries, respectively. A hyperplane $H_{\alpha,a}$ is said to support the set K_1 at $x \in \text{bd}K_1$, when $x \in H_{\alpha,a}$ and K_1 is entirely contained in one of the half-spaces associated with distinct sides of $H_{\alpha,a}$, i.e. for all $y \in K_1$ either $\langle \alpha, y \rangle - a \leq 0$ or $\langle \alpha, y \rangle - a \geq 0$ holds.

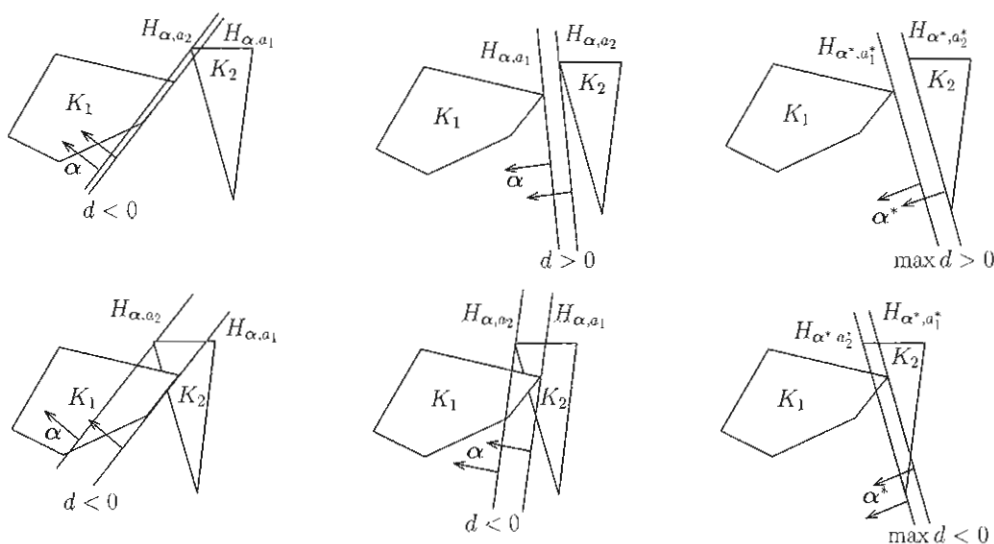


Figure 4. Strictly separable (first row) and non-separable (second row) polyhedral sets K_1, K_2 with signed distance between different supporting hyperplanes and maximum signed distance (Figure from [John 12]).

The maximum signed distance between parallel supporting hyperplanes of each set can be computed either as a solution $h(K_1, K_2)$ of the following quadratically constrained linear program (QCLP), or as the solution $g(K_1, K_2)$ of the following linearly constrained linear program (LCLP) for some unit vector $\beta \in S^{n-1}$.

QCLP	LCLP
$h(K_1, K_2) = \max_{\substack{\alpha \in \mathbb{R}^n \\ a_1, a_2 \in \mathbb{R}}} a_1 - a_2$	$g(K_1, K_2) = \max_{\substack{\alpha \in \mathbb{R}^n \\ a_1, a_2 \in \mathbb{R}}} a_1 - a_2$
subject to $\langle \alpha, x \rangle - a_1 \geq 0 \quad \forall x \in \text{ext}K_1$ $\langle \alpha, y \rangle - a_2 \leq 0 \quad \forall y \in \text{ext}K_2$ $\langle \alpha, \alpha \rangle = 1$	subject to $\langle \alpha, x \rangle - a_1 \geq 0 \quad \forall x \in \text{ext}K_1$ $\langle \alpha, y \rangle - a_2 \leq 0 \quad \forall y \in \text{ext}K_2$ $\langle \beta, \alpha \rangle = 1$

Using of the following three results shown in [John 12]

- i) **Theorem 1.** Two compact convex sets K_1 and K_2 are strictly separable, i.e. $K_1 \cap K_2 = \emptyset$ (properly separable, i.e. $K_1 \cap K_2 = \emptyset$ or $K_1 \cap K_2 \subset H_{\alpha, a_1} \equiv H_{\alpha, a_2}$) if and only if $h > 0$ (if and only if $h \geq 0$).
- ii) **Theorem 2.** giving conditions under which an optimal solution of the QCLP is equivalent to an optimal solution of the LCLP
- iii) **Remark 3** and remarks following the proof of Theorem 2 on the choice of β in praxis

we solve the LCLP to detect contact or overlapping between non-smooth convex bodies. If non-convex bodies are present, they are subdivided into convex bodies for which then contact or overlapping is tested.

4.2 Discrete mechanics and optimal control for constrained systems with collision avoidance

The goal of an optimal control problem is to determine an optimal state trajectory with the corresponding optimal control trajectory actuating the dynamical system such that an optimality criterion is reached. Thus, an objective functional is minimised with respect to the state and control trajectory while the equations of motion, initial and final conditions as well as path constraints have to be fulfilled. To simulate an optimal control problem numerically via a direct method, the problem is transformed into a finite dimensional constrained optimisation problem which can be solved e.g. by a standard SQP or interior point method. Thereby, the particular form of the discrete equations of motion (which serve as constraints for the optimisation) plays a crucial role for the resulting approximate solution. As described in the previous sections, in this discrete mechanics approach, they are derived via a discrete variational principle, leading to structure preservation (symplecticity, consistency in momentum maps in the presence of symmetry and good energy behaviour) along the discrete trajectories, see [Mars 01, Leye 08, Leye 10].

Let the degrees of freedom are actuated by the discrete control sequence $\tau_d = \{\tau_n\}_{n=0}^{N-1}$, approximating the control trajectory $\tau(t) : [t_0, t_N] \rightarrow \mathbb{R}^{n-m}$ by a constant $\tau_n \in \mathbb{R}^{n-m}$ in each time interval $[t_n, t_{n+1}]$. Using $\tau_{n-1}^+ = \frac{h}{2}\tau_{n-1}$ and $\tau_n^- = \frac{h}{2}\tau_n$, redundant n -dimensional control forces $B^T(q_n) \cdot (\tau_{n-1}^+ + \tau_n^-)$ can be computed using the configuration dependent input transformation matrix $B^T(q_n) \in \mathbb{R}^{n \times (n-m)}$. In the presence of non conservative actuation forces and constraints, a discrete constrained Lagrange-d'Alembert principle yields the variational integrator – in this case the constrained forced discrete Euler-Lagrange equations. Altogether, discrete mechanics and optimal control for constrained systems (DMOCC) yields the following finite dimensional constrained optimisation problem for the simulation of the optimal control problem with boundary values $(q(t_0), \dot{q}(t_0)) = (q^0, \dot{q}^0)$ and $(q(t_N), \dot{q}(t_N)) = (q^N, \dot{q}^N)$. Note that J_d, C_d are discrete approximations to the objective and cost functionals, respectively, while s_d, h_d, r_d are discrete versions of the initial, final and path constraints.

Let the contact constraint function be denoted by $g_c : Q \rightarrow \mathbb{R}$ and assume that it is computed as the solution of the LCLP. Then collision avoidance requires to augment DMOCC by the inequality constraint $g_c(q_n) > 0$ for $n = 1, \dots, N-1$.

DMOCC with collision avoidance
minimise discrete objective function

$$\min_{\mathbf{u}_d, \tau_d} J_d(\mathbf{u}_d, \tau_d) = \min_{\mathbf{u}_d, \tau_d} \sum_{n=0}^{N-1} C_d(\mathbf{u}_n, \mathbf{u}_{n+1}, \tau_n)$$

subject to the constraints for $n = 1, \dots, N-1$

discrete equations of motion

$$\mathbf{P}^T(\mathbf{q}_n) \cdot [D_1 L_d(\mathbf{q}_n, \mathbf{F}(\mathbf{u}_{n+1}, \mathbf{q}_n)) + D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n) + \mathbf{B}^T(\mathbf{q}_n) \cdot (\tau_{n-1}^+ + \tau_n^-)] = \mathbf{0}$$

collision avoidance

$$g_c(\mathbf{q}_n) > 0$$

initial value constraints

$$\mathbf{s}_d(\mathbf{u}_0, \mathbf{u}_1, \tau_0, \mathbf{q}^0, \dot{\mathbf{q}}^0) = \mathbf{0}$$

path constraints

$$\mathbf{h}_d(\mathbf{u}_n, \mathbf{u}_{n+1}, \tau_n) \leq \mathbf{0}$$

final point constraints

$$\mathbf{r}_d(\mathbf{u}_{N-1}, \mathbf{u}_N, \tau_{N-1}, \mathbf{q}^N, \dot{\mathbf{q}}^N) = \mathbf{0}$$

The constrained optimisation problem is solved in Matlab using the fmincon function (SQP algorithm with active set strategy or interior point method).

4.3 Example: puzzle assembly with collision avoidance

The considered puzzle consists of three non-smooth non-convex rigid bodies that are initially at rest in fully specified initial configurations. All bodies are fully actuated and supposed to reconfigure such that at the final time $t_N = 0.29$ (where $h = 0.01$ and $N = 29$), they are in a prescribed relative placement and orientation to each other. In other words, the puzzle manoeuvre ends at rest in a fully assembled configuration whose absolute placement and orientation in space is left free. During the manoeuvre, contact is detected using the SSHLP described in Section 4.1 and collisions are avoided. The initial guess for the optimisation is determined via an inverse dynamics problem, i.e. collision free reconfiguration trajectories have been guessed for the three bodies and the corresponding actuation has been determined solving the discrete equations of motion (see DMOCC with collision avoidance in Section 4.2) for the discrete control sequence τ_d . The goal of the optimisation is to minimise the control effort, i.e. $C_d = h \tau_n^T \cdot \tau_n$ while bounds on the optimisation variables $\mathbf{u}_d, \tau_d$ ensure that a local minimum with relatively small displacements is found. The control effort is reduced from a value of $J_d = 89$ at the initial guess to $J_d = 0.0630$ for the optimised solution. See Figure 5 for snapshots of different configurations during the puzzle's assembly.

5 Optimal control problems with contact

In many optimal control tasks like e.g. docking manoeuvres or walking and jumping motion, contact can not be avoided or is even necessary. However, in most cases, it is not known in advance when or where contact takes place. In particular, one can optimise the contact time and configuration with respect to specified goals.

5.1 Smooth penalty formulation

A penalty potential is an easy way to treat contact without loosing the generality of the problem formulation while retaining its smoothness, i.e. in DMOCC with collision avoidance described in Section 4.2, the collision avoidance inequality condition is left away. In case that the bodies overlap, i.e. $g_c(\mathbf{q}_n) < 0$, an n -dimensional penalty force $\mathbf{f}_n^c = \nabla(\frac{\mu}{2} g_c^2(\mathbf{q}_n))$ with the positive penalty parameter $\mu \in \mathbb{R}$ is included in the discrete equations of motion in DMOCC, which then read

$$\mathbf{P}^T(\mathbf{q}_n) \cdot [D_1 L_d(\mathbf{q}_n, \mathbf{F}(\mathbf{u}_{n+1}, \mathbf{q}_n)) + D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n) + \mathbf{B}^T(\mathbf{q}_n) \cdot (\tau_{n-1}^+ + \tau_n^-) + \mathbf{f}_n^c] = \mathbf{0}$$

Note that an explicit expression for the subderivative of the SSHLP can be readily evaluated such that ∇g_c can always be computed.

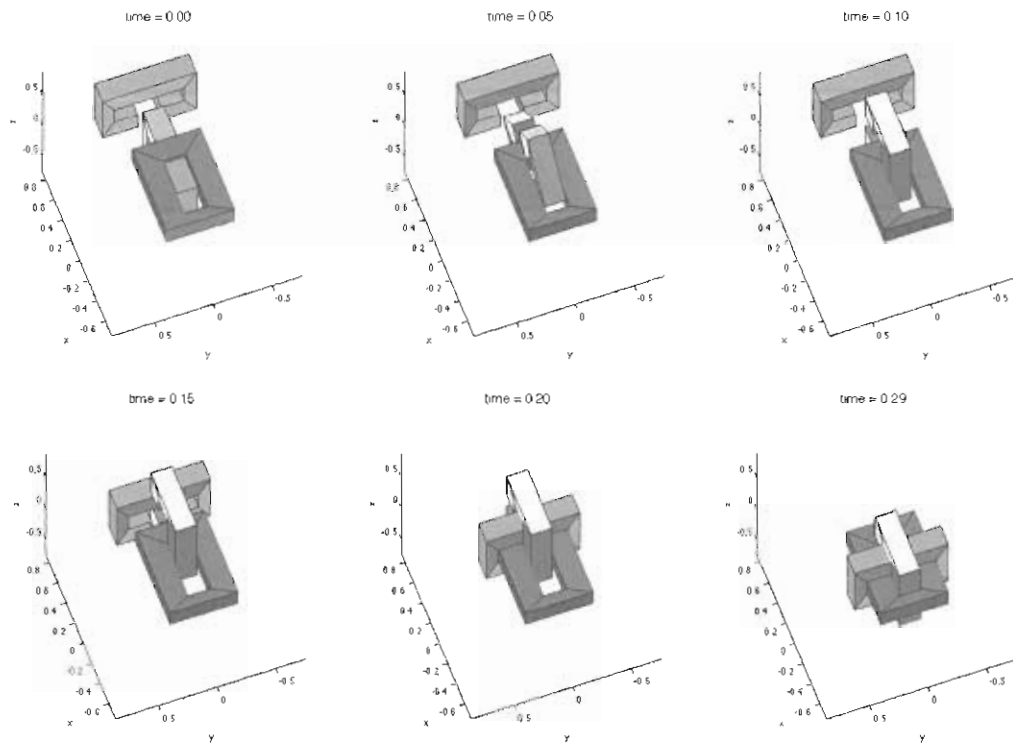


Figure 5. Puzzle assembly with collision avoidance: snapshots of different configurations.

5.2 Non-smooth contact formulation

Suppose that a mechanical system is to be optimally controlled from an initial to a final state in such a way that precisely one collision takes place during the manoeuvre at a time node, say at $t_i \in [t_0, t_N]$ with $g_c(q_i) = 0$. If the physical contact time was prescribed, the problem formulation would lose its generality. To retain generality, we let the contact time be determined as an optimisation variable. One way to realise this is to introduce positive scaling factors $\sigma_1, \sigma_2 \in \mathbb{R}$ for the time step before and after the contact time, see Figure 6. To ensure that time steps do not degenerate, the scaling factors can be bounded in the path constraints. Furthermore, the total manoeuvre time $t_N = (\iota\sigma_1 + (N - \iota)\sigma_2)ht$ can either be left free, or bounded or fixed by a scalar valued function $w(\sigma_1, \sigma_2, t_N) \leq 0$ as desired.

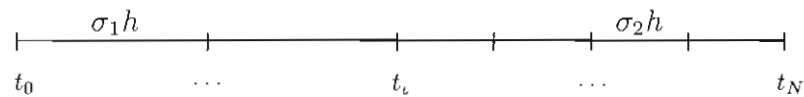


Figure 6. Time grid with differently scaled time step $\sigma_1 ht$ and $\sigma_2 ht$ before and after the contact time node t_i .

The most crucial point in treating the contact is to determine the correct change in momentum due to the collision. Let's first assume that the collision is perfectly elastic and frictionless, i.e. the kinetic energy (and therewith the total energy) does not change when computed immediately before and after the impact. When a point mass hits a surface, its linear momentum component in the direction of the surface normal at the contact point is reversed while the tangential component remains unchanged. However, the present situation is more complex since we deal with systems of rigid bodies having translational and rotational degrees of freedom on the one hand, and on the other hand, their kinematics is described using a constrained formulation reflecting their rigidity and possible interconnections by joints. These configuration constraints

give rise to hidden constraints on velocity and on momentum level. Using the following discrete reduced Legendre transforms

$$\begin{aligned} {}^P p_n^- &= P^T(q_n) \cdot [-D_1 L_d(q_n, F(u_{n+1}, q_n)) - B^T(q_n) \cdot \tau_n^-] \\ {}^P p_n^+ &= P^T(q_n) \cdot [D_2 L_d(q_{n-1}, q_n) + B^T(q_n) \cdot \tau_{n-1}^+] \end{aligned}$$

the discrete equations of motion in DMOCC can be interpreted as ${}^P p_n^+ - {}^P p_n^- = 0$ which is a balance of discrete momenta at t_n , where the discrete momenta have been computed from the past (${}^P p_n^-$) and the following time interval (${}^P p_n^+$), respectively, and both are mapped by the null space matrix into the appropriate reduced cotangent space (being consistent with the constraints on momentum level). When a contact configuration has been reached, i.e. $g_c(q_i) = 0$, the component of ${}^P p_i^+$ in the direction of the surface normal has to be reflected. Note that in the reduced formulation, this surface normal is given by $({}^P \nabla g_c(q_i))^T = P^T(q_i) \cdot (\nabla g_c(q_i))^T$. Thus, using the reduced mass matrix ${}^P M(q_i) = P^T(q_i) \cdot M \cdot P(q_i)$ the component of ${}^P p_i^+$ in the direction of the surface tangent, denoted by $({}^P p_i^+)_{\parallel}$, fulfills

$$\langle ({}^P \nabla g_c(q_i))^T, ({}^P p_i^+)_{\parallel} \rangle_{({}^P M(q_i))^{-1}} = 0 \quad (16)$$

where $\langle a, b \rangle_A = a^T \cdot A \cdot b$ is a norm for the vectors a, b and the square matrix A of appropriate dimension. Inserting the momentum decomposition into the tangential and the normal direction ${}^P p_i^+ = ({}^P p_i^+)_{\parallel} + ({}^P p_i^+)_{\perp}$ into Equation (16), the normal momentum component can be found explicitly via

$$({}^P p_i^+)_{\perp} = \frac{\langle ({}^P \nabla g_c(q_i))^T, {}^P p_i^+ \rangle_{({}^P M(q_i))^{-1}}}{\langle ({}^P \nabla g_c(q_i))^T, ({}^P \nabla g_c(q_i))^T \rangle_{({}^P M(q_i))^{-1}}} ({}^P \nabla g_c(q_i))^T$$

Finally, for frictionless collisions with a coefficient of restitution $e \in [0, 1]$, where $e = 1$ represents a perfectly elastic and $e = 0$ a perfectly plastic collision, the post collision momentum can be computed according to the following momentum reflection in normal direction at the contact surface

$${}^P p_{i,post}^+ = {}^P p_{i,pre}^+ - (1 + e) ({}^P p_{i,pre}^+)_{\perp}$$

With these preliminaries, the DMOCC with contact problem can be formulated as follows.

DMOCC with contact	
minimise discrete objective function	
$\min_{u_d, \tau_d, \sigma_1, \sigma_2}$	$J_d(u_d, \tau_d, \sigma_1, \sigma_2) = \min_{u_d, \tau_d} \sum_{n=0}^{N-1} C_d(u_n, u_{n+1}, \tau_n, \sigma_1, \sigma_2)$
subject to constraints for $n = 1, \dots, N-1$	
discrete equations of motion	${}^P p_n^+ - {}^P p_n^- = 0$
contact	$g_c(q_i) = 0$
momentum reflection	${}^P p_{i,pre}^+ - (1 + e) ({}^P p_{i,pre}^+)_{\perp} = {}^P p_{i,post}^+$
initial value constraints	$s_d(u_0, u_1, \tau_0, q^0, \dot{q}^0) = 0$
path constraints	$h_d(u_n, u_{n+1}, \tau_n, \sigma_1, \sigma_2) \leq 0$
final point constraints	$r_d(u_{N-1}, u_N, \tau_{N-1}, q^N, \dot{q}^N) = 0$
total time condition	$w(\sigma_1, \sigma_2, t_N) \leq 0$

In particular, collision avoidance can be required at all other time nodes in the path constraints vector. If multiple, say $N_c \in \mathbb{N}$, collisions are planned at the time node numbers $\iota_1, \dots, \iota_{N_c}$, then multiple contact conditions $g_c(q_{\iota_1}) = 0, \dots, g_c(q_{\iota_{N_c}}) = 0$ with the corresponding momentum reflection conditions are constraining the optimisation in DMOCC with contact. Furthermore, $N_c + 1$ scaling factors are present in the total time condition $w(\sigma_1, \dots, \sigma_{N_c+1}, t_N) \leq 0$.

ι	4	9	14
σ_1	2.1795	0.9653	0.6186
σ_2	0.8185	1.0149	1.3337
t_c	0.2180	0.2172	0.2165
J_d	$1.0244 \cdot 10^{-6}$	$1.0194 \cdot 10^{-6}$	$1.0005 \cdot 10^{-6}$

Table 1. Cube hitting a wall: values of scaling factors, contact time and control effort for different contact node numbers ι .

5.3 Example: cube hitting the wall

In the first example, a cube starts in a prescribed initial state with a translational velocity towards a close by wall. In the prescribed final state, the final velocity is reversed and bounds on the actuation are set such that the final state can not be reached without a collision of the cube with the wall. Here, $N = 30$ time nodes and a time step of $ht = 0.025$ are used and the control effort is minimised. The final time is prescribed as $t_N = 0.75$. This example serves as numerical evidence that the resulting optimal state and control trajectories and the resulting optimal contact time are independent of the chosen contact node number. Table 1 and Figure 7 shows that for $\iota \in \{4, 9, 14\}$ qualitatively the same contact time, control effort and evolution of actuating force and torque are obtained (of course the scaling factors yielding this contact time are different).

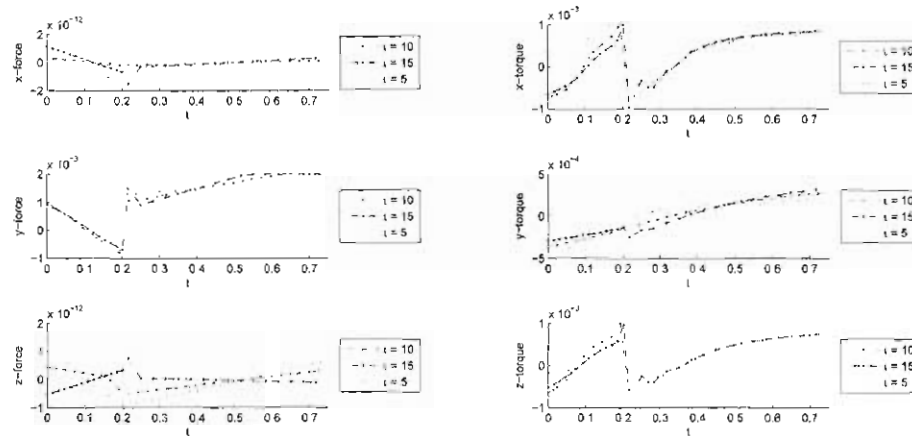


Figure 7. Cube hitting a wall: evolution of actuating force and torque for different contact node numbers ι . The contact time is marked in green.

5.4 Example: two cubes with two planned collisions

The following problem considers two cubes with fully specified initial configurations. The green cube is initially at rest. It is not actuated, thus its final state is not prescribed. The blue cube has an initial velocity and is required to be steered into a prescribed final rest position that overlaps with the green cube's initial configuration. Thus, the blue cube must push the green cube out of the way, wherefore two collisions are planned at the time node numbers $\iota_1 = 20, \iota_2 = 39$ and collisions are avoided at all other time nodes. Altogether, $N = 50$ time nodes and $ht = 0.01$ are used, while the final time is prescribed as $t_N = 0.5$, thus the total time condition $w(\sigma_1, \sigma_2, \sigma_3, t_N) = (\sigma_1 \iota_1 + \sigma_2 (\iota_2 - \iota_1) + \sigma_3 (N - \iota_2))ht - t_N = 0$ is used in DMOCC with contact described in Section 5.2. As a result of the control problem minimising the control effort to $J_d = 2.8573 \cdot 10^{-5}$, collisions happen at $t_{\iota_1} = 0.1975, t_{\iota_2} = 0.3900$, thus $\sigma_1 = 0.98725, \sigma_2 = 1.0132, \sigma_3 = 1.0004$. The initial guess for the optimisation is obtained via a forward dynamics simulation (treating contact according to the decomposition contact response formulation given in [Cira 05], which has been extended for the case of oriented bodies and constrained dynamics) yielding also a guess for possible

contact times. The left hand side plot in Figure 8 shows the evolution of the actuating force and torque for the blue cube. The corresponding net torque evolution as well as the evolution of angular momentum for the complete system are shown in the right hand side plot, where the lowest plot shows the consistency of the angular momentum evolution, i.e. the change in angular momentum exactly represents the actuation (up to the numerical tolerance) giving numerical evidence of the structure preservation properties of the discrete equations of motion. Snapshots of different configurations are depicted in Figure 9.

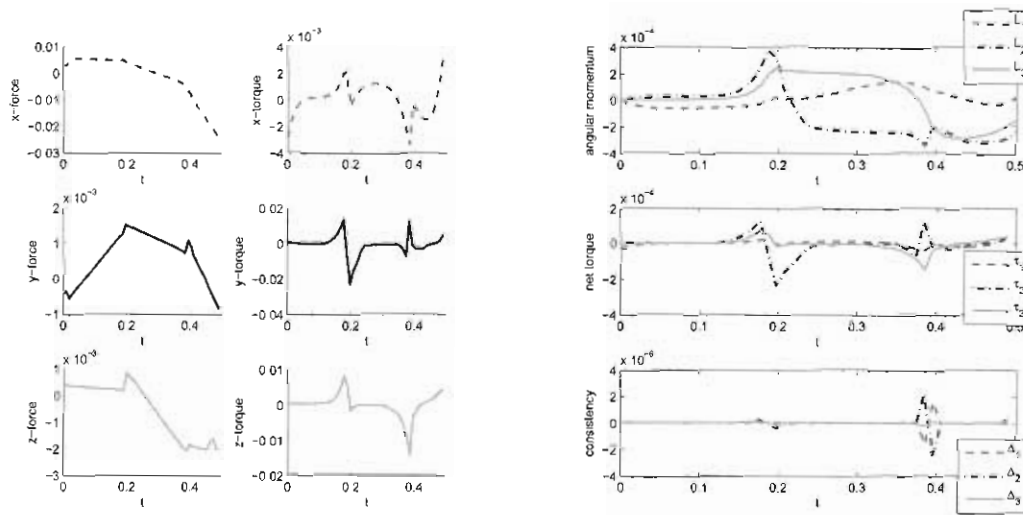


Figure 8. Two cubes with two planned collisions: evolution of actuating force and torque on blue cube (left) and evolution of angular momentum, net torque and consistency of angular momentum for the complete system (right).

6 Conclusion

Starting from a discrete version of Hamilton's principle, we have derived a structure preserving integrator for mechanical systems including holonomic (bilateral) constraints as well as unilateral contact (inequality) constraints. The basic idea of the discrete variational principle for systems involving collisions is to treat both the discrete-time trajectory and unknown collision time as unknowns; the resulting discrete Euler-Lagrange equations plus boundary conditions then lead to a time-stepping algorithm that includes the contact force and the constraint forces corresponding to the holonomic constraints. As we have shown all reaction forces can be eliminated from the discrete equations of motion using a discrete null space reduction (involving a projection and a local reparametrisation step) which considerably reduces the number of unknowns as compared to the standard treatment with Lagrange multipliers. As a consequence, the condition number during iteration stays low. This fact, together with the possibility to determine the contact time by solving an algebraic equation rather than searching it via bisection strategies, leads to much lower computational costs while, at the same time, it increases the accuracy to which the contact time is determined.

With the supporting separating hyperplane linear programming (SSHLP) approach, a very efficient method for detecting contact between non-smooth bodies has been described briefly. The resulting signed distance between the bodies is first of all used as an inequality constraint in optimal control simulations with collision avoidance. Secondly, optimal control manoeuvres with planned contacts and collisions are considered. Here it is an advantage that the subgradient of the SSHLP, supplying the direction of the contact force, can be readily evaluated. For the simulation of contact manoeuvres, the time grid is variable, such that the contact time can be determined as part of the optimisation while the contact node number can be fixed, what facilitates the implementation substantially. The presented formulation and results constitute first steps of ongoing work and many theoretical aspects and more complex examples will be considered in the future. Furthermore, a comparison of the SSHLP's efficiency with standard contact detection methods is required.

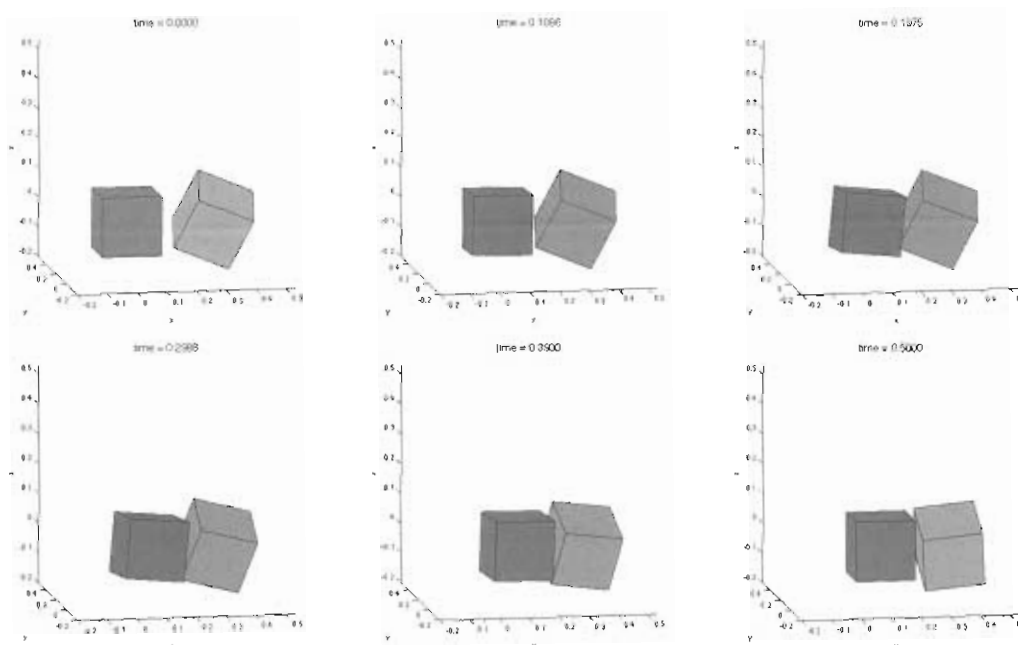


Figure 9. Two cubes with two planned collisions: snapshots of different configurations.

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BIOMECHANICS AND COMPUTATIONAL BIOLOGY

OPTIMAL SHAPE, MOTION AND ENERGETICS OF UNDULATORY SWIMMING ORGANISMS

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1. Introduction

Undulatory swimming organisms come in many different shapes and utilize different motion patterns across the scales, and they are considered as superior swimmers. It is not known, however, to what extent is that diversity due to locomotory considerations. The goal of this study is to show, using a numerical optimization of a theoretical model, what would be the shape, motion and energetics of optimal organisms across the scales, if these were obtained by optimizing initial populations of organisms with random characteristics for locomotory performance measures.

2. Model Description

The theoretical model of undulatory swimming consists of coupled hydrodynamical, structural and muscular submodels (Tokić and Yue, 2011). Each submodel is chosen to be simple enough to facilitate optimization, but general enough to accurately describe the relevant physics across the scales. Hydrodynamics of swimming is modelled using slender-body potential flow theory (Lighthill, 1960). Internal forces are described by Euler-Bernoulli beam equation with visco-elastic effects (Cheng et al., 1998). The muscle behaviour for periodic contraction is described using a novel model (Tokić and Yue, 2011).

The body shape and motion are parameterized and the parameters are used as optimization variables. A general shape is expressed in terms of Chebyshev-based shape functions and general undulatory motion is expressed by a Fourier-Chebyshev series (Tokić and Yue, 2011).

From an evolutionary standpoint, achieving maximal sustained swimming speed and minimal cost of transport has benefits in terms of survival. To study the extent to which swimming speed and cost of transport influence an organism's morphology, these conflicting performance measures are used as optimization objectives.

The multi-objective optimization is conducted using evolutionary optimization algorithm MO-CMA-ES (Igel et al., 2007). The optimization is conducted for a total of 9 optimization variables, evolving populations of 500 individuals have been evolved through 500 generations to obtain the final Pareto front.

3. Results and discussion

The optimization is conducted over nine orders of magnitude in size, for organisms weighing from 0.001 kg to 1,000,000 kg. For a large range of size scales, the results show that fish-like body shapes and motion indeed emerge from an initial population of random characteristics when aforementioned objectives are optimized. The kinematic and energetic characteristics of optimal organisms match those of fish and cetateans. This supports the notion that optimizing sustained swimming speed or cost of transport could be the driving force of evolution in many species.

Acknowledgements

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APPLICATION OF 3-D G&R MODEL TO ABDOMINAL AORTA

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1. Introduction

Adaptations of large arteries to sustained alterations in hemodynamics that cause changes in both caliber and stiffness are increasingly recognized as important initiators or indicators of cardiovascular risk to high flow, low resistance organs such as the brain, heart, and kidneys. There is, therefore, a pressing need to understand better the underlying causes of geometric and material adaptations by large arteries and the associated time courses. Although such information must ultimately come from well-designed experiments, mathematical models will continue to play a vital role in the design of these experiments and their interpretation. In this work, we present a multi-layered model of the time course of abdominal aorta growth and remodeling in response to sustained changes in blood pressure and flow. We show that multi-layered model provide additional insight into other important metrics such as the residual stress related opening angle and the axial pre-stress, both of which are fundamental to arterial homeostasis and responses to injury or insult. Herein we show results from our numerical model compared to available experiments.

2. Material and Methods

We developed model restricted to elastostatics but capable of describing 3D behaviors of arteries, Karšaj & Humphrey, 2012. The model describes behavior of intima, media and adventitia separately. Each layer could have different structure as well as growth parameters. Only structurally significant constituents are modeled; elastin, collagen (4 fiber families) and smooth muscle cells. Each constituent has its own natural configuration, generally different from others, which is taken as a possibly evolving stress free reference configuration. Deformations and G&R of a constrained mixture of all constituents are

tracked within the current mixture configuration. The model is capable of describing evolving residual stresses, revealed in a reference mixture configuration through the classical opening angle. The stress-free configuration is significantly different from configuration in which all the external loads are removed. This can be easily shown by cutting the unloaded vessel: the wall will spring open immediately into noncircular arc characterized by its opening angle. To determine stress-free configuration, more cuts should be made, until no further change of strain can be noticed. Key features of the developed model are examined using a cylindrical geometry, prior to implementation in finite element models.

3. Results and Discussion

We simulated G&R for an abdominal aorta as our final goal is numerical simulation of aneurysmal growth and rupture at that location. From available results in literature we reconstructed structure of older population aorta as aneurysmal changes are characteristic for population above age of 55. It is not possible to find all needed information so we bring them as observed (can be found in literature) and bounded (we know just their limits).

We decided to verify time changes of aortic characteristics (first example). There are no such results for humans so we have additional checks with experiments on excised human abdominal aorta (second example).

In a first example we simulated experiment done by Fung & Liu, 1989. We compare our time course of opening angle with their results, Figure 1. Fung & Liu performed experiments on rats by putting band on arterial arch causing hypertension in upper and hypotension in lower parts of aorta. In regular intervals they sacrificed rats and measured values of opening angles. Having in mind different structure of arterial layers between human and rat, our goal

is to calculate similar time course of opening angle values.

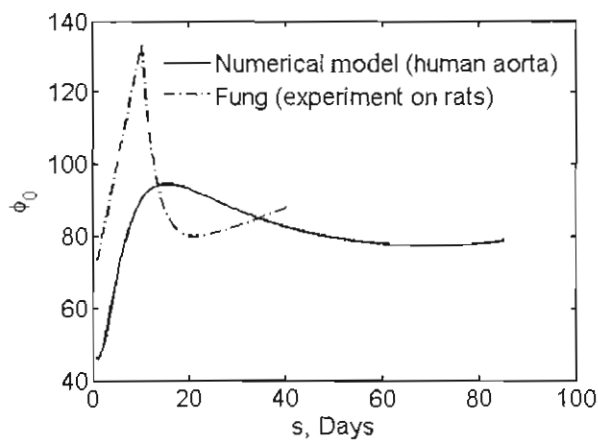


Figure 1 Time course of opening angle change – our model and Fung & Liu, 1989

As a second verification problem we use experiments from Holzapfel et al., 2007.

Abdominal Aorta (all layers together)		
	ρ	λ
Experiment	0,023±0,035	1,01±0,087
Numerical model	-0,00017	1,31
Tunica Intima		
	ρ	λ
Experiment	0,131±0,207	0,963±0,041
Numerical model	0,0864	1,08
Tunica Media		
	ρ	λ
Experiment	0,032±0,048	1,081±0,034
Numerical model	-0,0057	1,31
Tunica Adventitia		
	ρ	λ
Experiment	0,003±0,029	1,059±0,046
Numerical model	-0,0132	1,33

Table 1 Comparison with experimental results from Holzapfel et al., 2007.

Table 1 shows comparison of curvature (ρ) and axial stretch (λ) with experimental obtained for each arterial layer (tunica intima, tunica media, tunica adventitia). Layer-based comparison enable us higher accuracy of the numerical model.

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A METHOD OF CHARACTERISTICS FOR ONE-DIMENSIONAL FLUID FLOW IN VISCOELASTIC TUBES

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1. Introduction

The blood flow in the human arterial tree, may be considered as one-dimensional flow in a network of visco-elastic blood vessels. An one-dimensional model is a good compromise between lumped and three-dimensional models [1]. In case of an elastic vessel wall, the governing equations are of hyperbolic type, and can be solved by the method of characteristics. In case of a viscoelastic wall, equations are of elliptic type due to the term describing wall viscosity. Here we extend the method of characteristics to the case of a viscoelastic wall, with the wall viscosity term treated explicitly, through the source term. The developed method is stable and efficient as in a case of elastic model.

2. Problem definition

We considered a visco-elastic tube of the constant length L and with the time-dependent diameter D . At the pipe inlet the time-varying pressure p_{in} is prescribed, and the outflow from the tube is determined by the constant resistance R ($Q_{out} = p_{out}/R$).

3. Mathematical model

The problem is governed by the following equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{A}{\rho} \frac{\partial p}{\partial x} + \frac{\partial(Qv)}{\partial x} = -fQ \quad (2)$$

$$p - p_0 = \frac{1}{C}(A - A_0) + \eta \frac{\partial A}{\partial t} \quad (3)$$

where x and t are the space and time coordinate, respectively, A is the cross-sectional area ($A = D^2\pi/4$), v is the average velocity ($v = Q/A$), C is the areal compliance of the wall ($C = dA/dp$), η

is the viscous wall resistance, ρ is the fluid density, f is the friction coefficient according to the Hagen-Poiseuille law $f = 32\mu/(\rho D^2)$ and μ is the fluid viscosity.

4. Method of characteristics

The tube is divided into a number of elements of length Δx , bounded by nodes. Fig. 1 shows two typical elements (i and j) and three nodes (W, P, E). The pressure is defined at the nodes, A is defined in the middle of each element and Q is defined at each end of each element. Thus, four unknowns are stored at each node, as depicted in Fig. 1.

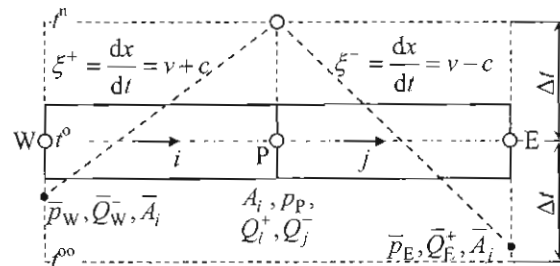


Figure 1 A part of discretized tube with unknown variables arrangement. Over bar denotes interpolated values.

Equations (1) to (3) may be transformed into ordinary differential equations that are valid along characteristics defined by $\xi^+ = dx/dt = v + c$ and $\xi^- = dx/dt = v - c$, which describe forward and backward travelling waves, respectively:

$$\frac{A}{\rho} dp + \xi^+ dQ - v^2 dA = -fQ\Delta x - \eta \frac{A}{\rho} \frac{\partial^2 Q}{\partial x \partial t} dt \quad (4)$$

$$\frac{A}{\rho} dp + \xi^- dQ - v^2 dA = -fQ\Delta x - \eta \frac{A}{\rho} \frac{\partial^2 Q}{\partial x \partial t} dt \quad (5)$$

where $c = \sqrt{A/(\rho C)}$ is the wave speed. Along the third characteristic (applied for each element), defined by $\xi^0 = dx/dt = 0$, the following relationship holds:

$$dp - \frac{1}{C} dA = -\eta \frac{\partial^2 Q}{\partial x \partial t} dt \quad (6)$$

The fourth equation related to each node is the continuity equation $Q_i^+ = Q_i^-$. Equations (4) to (6) are discretized by replacing differentials with finite differences, e.g. for pressure along the ξ^+ characteristic: $dp = p_p^n - \bar{p}_E$. To obtain a linear system of equations, all coefficients in these equations are discretized using known variable values from previous times ($t^0, t^{00}, \dots$).

If the unknowns are stored into the four element vectors X_I , where I is the node number, then the set of discretized equations takes the form:

$$[C]_I X_{I-1} + [A]_I X_I + [B]_I X_{I+1} = D_I, \quad I=1, \dots, N \quad (7)$$

where N is number of nodes, $[C]_I$, $[A]_I$, $[B]_I$ are 4×4 matrices, and D_I are 4×1 column vectors. In the case of a pure elastic tube wall, matrices $[C]_I$ and $[B]_I$ are zero matrices.

5. Results and discussion

The described method was applied to a tube of uniform cross section area $A_0 = 20 \text{ cm}^2$ (at pressure $p_0 = 0$), divided into 25 equal elements of length $\Delta x = 8 \text{ cm}$. The input pressure is defined by the equation

$$\{p_{in}\}_{\text{mmHg}} = 100 \sin(2\pi t / T) \quad (8)$$

where $T=1 \text{ s}$ is the period. In the case of inviscid flow and a pure elastic tube wall the characteristic impedance is defined by

$$Z_0 = \sqrt{\frac{\rho}{A_0 C}} = \frac{\rho}{A_0} c \quad (9)$$

If the R matches the characteristic impedance Z_0 there are no wave reflections in the tube and the pressure and flow are in phase (related by $p=R \cdot Q$) at each point of the tube. We choose the resistance $R=1 \text{ mmHg} \cdot \text{s/ml}$, $c=6 \text{ m/s}$, and ρ is selected such that $Z_0=R$. Since the wave velocity is 120 times greater than the expected maximal fluid velocity $v_{\max} = Q_{\max}/A$, the variation of the tube diameter and wave speed will be small (C is supposed to be constant). We consider two cases: a pure elastic ($\eta=0$) and a viscoelastic tube wall with η defined by the time constant $\tau = C \cdot \eta = 0.5 \text{ s}$. In both cases the integration step was $T/100$ and the total integration time was $30T$.

Fig. 2 shows the prescribed input pressure, calculated input flow and area (at the middle of the first element i.e. 4 cm from the pressure node). It is clear that the pressure and flow are in phase, and that they have the same

amplitudes according to the analytical solution. The area is also in phase with the pressure (a small difference occurs because the pressure and area are not taken from the same point). Fig. 3 shows the analogue results for the viscoelastic case, where the phase lags of flow and area are visible, as well as their amplitude reduction.

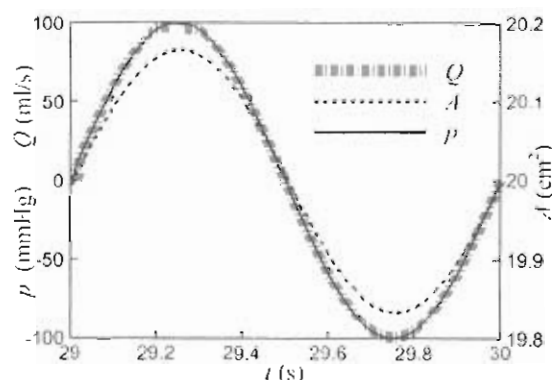


Figure 2 Input flow and pressure and first element area variations during the last integration period (elastic case)

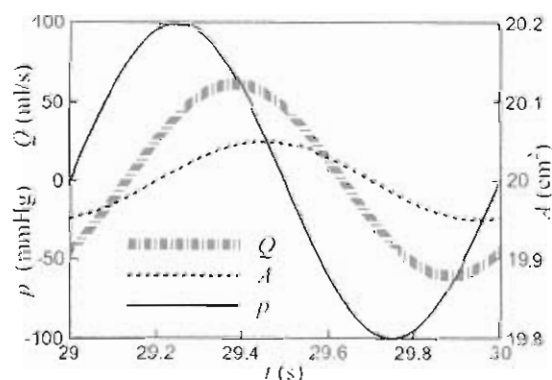


Figure 3 Input flow, pressure and area variations during the last integration period (the viscoelastic case)

6. Concluding remarks

The developed method of characteristics for a viscoelastic tube is physically clear and efficient as in the case of an elastic tube. For its application to the arterial tree it should be extended from a tube to the tree structure and it should be taken into account that the compliance depends on the pressure.

Acknowledgements

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COMPOSITES AND OTHER ADVANCED MATERIALS

HYDROELASTIC SLAMMING OF COMPOSITE PLATES

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1. Introduction

The aim of this paper is to investigate the interaction between the fluid and the structure in case of a deformable wedge entering the water. In this case, the pressure at the fluid/structure interface and the fluid motion are influenced by the structural deformation. In some cases prediction using a rigid body model are adequate while under certain conditions the deformations of the wedge greatly affect the fluid flow. Because of this mutual interaction the acceleration of the wedge or the applied hydrodynamic force can highly differ from the case of water entry of rigid bodies and, in general, the hydroelastic effects increase with the impact velocity. This topic is consequently of particular interest for the composite community since hydroelastic effects are greater when the vessels are made of composite lightweight structures, due to the higher speed that can be reached.

Hydroelastic effects introduce difficulties in the calculation of the structural stresses. Being able to correctly predict the structural deformations and stresses during the slamming phenomena is a major challenge, and suitable computer-aided design tools still have to be found. The major obstacle to get around is the very high computational cost that all the numerical models carry with them as a disadvantage. This research evaluates the capability of the Smoothed Particle Hydrodynamic Method to model hydroelasticity and to predict the impact dynamics and the structural deformation during the water entry of elastic bodies. Particular focus is given to the reduction of the computational cost. Interesting results on the parameters affecting hydroelasticity have been found, and the SPH method has been validated by comparison with the experimental results. (Panciroli 2011, Panciroli et al. 2010, Panciroli et al. 2011a, Panciroli et al. 2011b). To validate

the numerical model is the necessary first step for the practical use of SPH as design tool. The second step is the speed-up of the solution to solve full-scale 3D models of the problem. Nowadays, only two-dimensional problems can be solved with this technique (in a reasonable time), but this research presents very promising results in terms of computational speedup, showing the possibility to use SPH method to solve full-scale models.

2. Materials and Methods

The drop-weight machine was designed and built to perform hydroelastic slamming of composite plates at a laboratory scale. The testing machine is equipped with a wireless sensor system capable of recording up to four strain channels, one tri-axial accelerometer and the initial entry velocity. This machine allows different boundary conditions for the wedges and to support different geometries, e.g. cantilevered plates (already tested by the authors), fully clamped wedges, cylinders and cones. An experimental campaign on the hydroelastic behavior of fully clamped wedges will be done and the results will be compared with the previously published results to investigate the effect of different boundary conditions on the hydroelastic response.

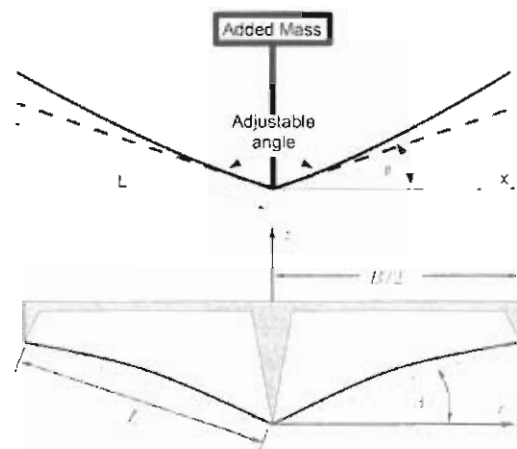


Figure 1: Cantilevered (up) and fully clamped (bottom) boundary conditions.

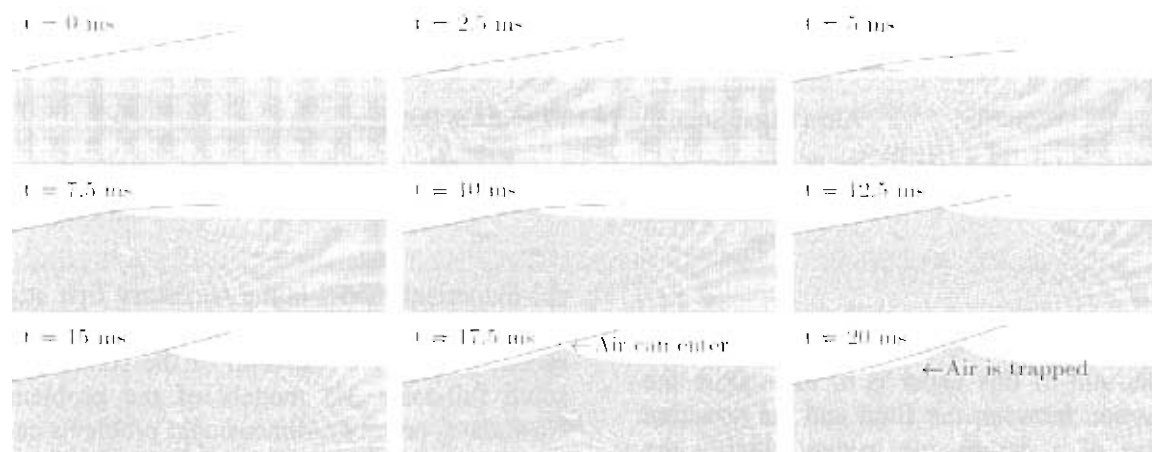


Figure 2, Mechanism of air trapping during hydroelastic impacts of a wedge (only one half of the wedge is shown due to symmetry).

Figure 2 shows a sketch of two different boundary conditions.

Alongside with the experimental campaign, numerical models are developed:

The numerical solutions are compared with the previously published ones to see if there is an influence of the boundary conditions on the hydroelastic behavior and on the air trapping phenomena.

3. Results and discussion

The experimental results show that the relative importance of hydroelasticity is strictly related to the ratio between the time necessary for the structure to get completely wet during the water entry (also called wetting time) and the natural period of the structure.

Due to this it is seen that cantilevered plates show higher hydroelastic effects than the fully clamped ones and the phenomenon of air trapping (see figure 2) in this case cannot be neglected.

Acknowledgements

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CHARACTERISATION OF MECHANICAL PROPERTIES OF LOTUS-TYPE POROUS IRON WITH PARAMETRIC COMPUTER SIMULATIONS

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1. Introduction

The use of porous materials with longitudinal pores ("Gasar" or "Lotus-type" materials) increased rapidly over the last few years in industrial applications such as biomedical implants, sports equipment, lightweight construction and dampers.

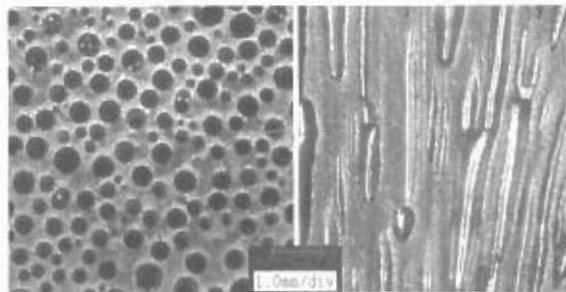


Figure 1. Microscopic pictures of cross-section of Lotus-type porous iron with longitudinal pores at 44.0 % porosity.

Transversal to pore (left) and longitudinal to pore (right) cross-section.

The advantages of lotus-type porous materials are their orthotropic mechanical and thermal properties due to unidirectional arrangement of elongated pores in the base material (Fig. 1), and already developed production processes for their serial industrial production [1]. However, the fabrication of this type of materials, with iron as the base material, is still limited to porosities lower than 50 %. Mechanical properties of lotus-type iron-based materials were so far experimentally determined for 20-50 % porosity [1, 2], since porosities less than 20 %, are not very interesting for practical applications.

New fabrication techniques for production of lotus-type materials with higher porosity are

being currently under development. Therefore, there is obvious interest in their expected mechanical properties, which can be predicted with proper computer simulations. This paper describes such a procedure and gives some estimates on mechanical behaviour of lotus-type iron materials.

2. Computational model

Parametric computer simulations were used to estimate the engineering elastic modulus and yield strength for various porosities and directions of loading in aspect to pore orientation. Four geometrical models of porous material representative volume were used for this purpose: (a) model with aligned pores, (b) with $\pi/4$ rotated aligned pores, (c) with shifted pores and (d) reconstructed pore model (Fig. 2). The porosity was varied from 0 to 0.65 by variation of pore sizes. Computer simulations were based on the finite element method, where geometrical and material nonlinearities were also considered. The finite element code Abaqus was used for computer simulations. Numerical results were compared to experimental and analytical results provided by the University of Osaka.

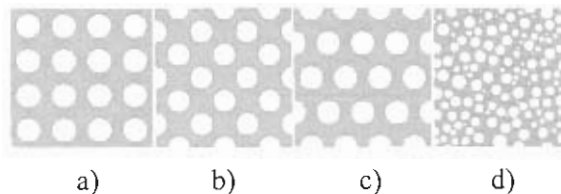


Figure 2. Four geometrical models of lotus-type porous material.

a) aligned, b) $\pi/4$ rotated aligned, c) shifted and d) reconstructed pore topology.

3. Results and discussion

Computer simulations show that the engineering elastic modulus of models with $\pi/4$ rotated aligned topology and shifted pore topology matches satisfactorily with experimental and analytical results which validate the used computational models (Fig. 3).

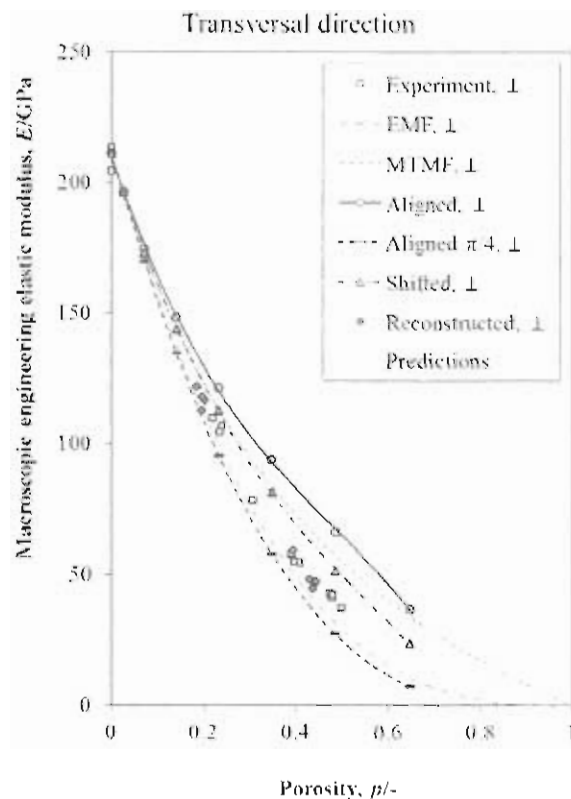


Figure 3. Comparison of engineering elastic modulus for regular and irregular computational models with experimental and analytical results in transversal ($\perp$) direction.

The same models were used to computationally estimate the engineering 0.2 % offset yield stress ($R_{p0.2}$) of lotus-type iron porous material (Fig. 4). As expected, the results for directly reconstructed models exhibit excellent correlation with experimental results.

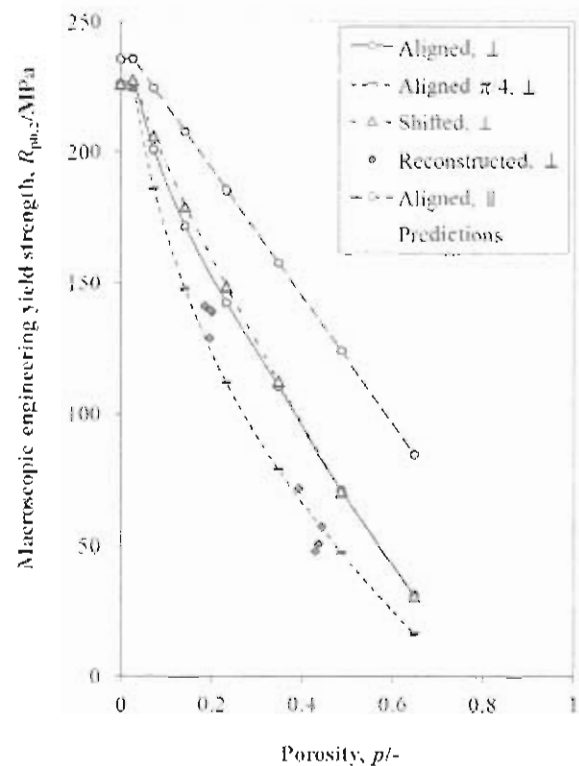


Figure 4. Macroscopic engineering yield stress dependence on porosity for regular and irregular models in transversal ($\perp$) and longitudinal ($\parallel$) direction.

4. Concluding remarks

The extensive parametric computational simulations were carried out to characterise mechanical material properties of lotus-type iron. Excellent correlation is found between numerical, experimental and analytical results.

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BEHAVIOR OF METAL FOAM UNDER IMPACT LOADING

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1. Introduction

Metal foams (Fig. 1), which have an attractive combination of physical and mechanical properties, such as low density and high specific stiffness in relation to their weight, are extremely important for many branches of modern industry [1].

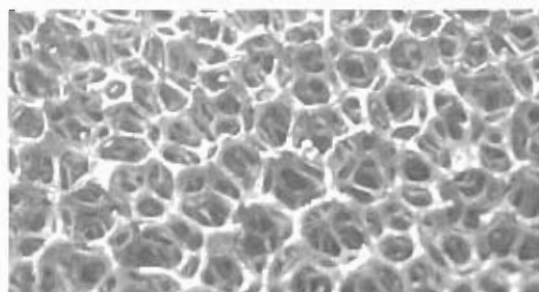


Figure 1. Open-cell metal foam

The shock wave propagation through the cellular material structure under impact loading conditions has a significant effect on its deformation mechanism and is therefore imperative to understand its effects thoroughly.

2. Underwater shock wave propagation

To observe and visualize the propagation of the shock wave the aluminium specimens were submerged in a water tank. The container with clear PMMA walls was filled with water. Open-cell aluminium foam (40x40x40 mm with 20 ppi) was placed at the bottom of the water container. The SEP explosive set in the PVC pipe was positioned above the foam specimen at the water surface (Fig. 2).

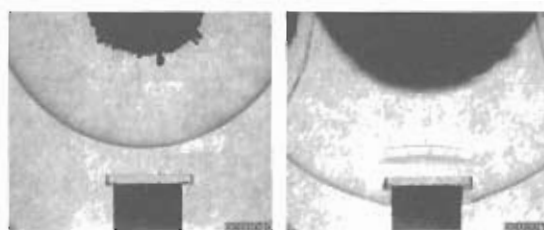


Figure 2. Underwater shock wave propagation

The behaviour of open-cell aluminium foams under underwater shock wave loading conditions was further studied with computational simulations using the LS-DYNA. The aluminium foam was modelled with a lattice model to represent the irregularity of the structure. The foam cell edges were modelled with beam finite elements and their topology was created using a developed algorithm based on Voronoi 3D regions. The PVC plate that transmitted the loading was modelled using an Eulerian mesh with a piecewise-linear plasticity constitutive model. A fluid structure interaction interface was used to transfer the pressures between the plate and water. Three different relative densities of the aluminium foam were considered. From the computational results it can be observed, that due to the high velocity loading of the PVC plate, the foam material primarily deforms at the upper part where the blast wave impacts the PVC plate.

3. Taylor impact test

The Taylor impact test was used to further study the behaviour of open-cell aluminium foam projectile at high strain rates. During the test the specimen is accelerated to the desired

velocity and then impacted into a rigid wall. Dynamic properties of the used material can be determined upon plastic deformations of the specimen after the impact in the rigid wall and using the theory proposed by Taylor. The projectiles were made from the open-cell aluminium foam and shaped in a cylindrical form with a diameter of 35 mm and height of 35 mm [2]. The combination of explosives accelerated the projectile through the 3 m long barrel to the final muzzle velocity of 400 m/s.

Optical recording of projectile behaviour before and upon impact into a rigid wall was used to analyse the deformation behaviour of open-cell aluminium foam sample (Fig. 3). Because of the high impact velocity the inertia effects dominate the deformation behaviour and the open-cell material deforms only in contact with the rigid wall. These experimental results confirm the predictions based on previous computational simulations of open-cell material behaviour under compressive loading conditions [3]. A more in depth analysis of the samples geometry before the impact in the rigid wall showed, that open-cell foam deforms already while accelerating through the barrel. This deformation was estimated to be approximately 11.4 %. The reason for this deformation is a very high acceleration.

The acceleration effect on the deformation of the open-cell foam in computer simulations was studied by parametric simulations varying the value of the acceleration. Computer simulations were done by means of a finite element method using the LS-DYNA software.

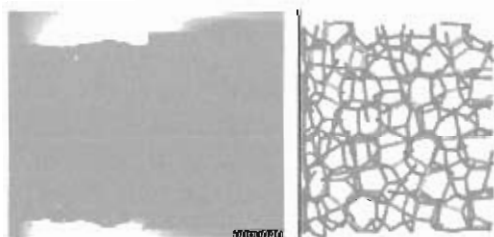


Figure 3. Experimental (left) and computational (right) impact sequence of the open-cell foam projectile into the rigid wall

Numerical model of the irregular open-cell foam was positioned on a rigid support plate, modelled by shell finite elements. The load was

prescribed as an acceleration of the support plate, acting normal to the plane of the plate in the direction of the foam model. Different accelerations were used for the conducted parametric study. Twenty simulations with differently generated models of irregular open-cell foams were carried out for each of the selected accelerations, in order to exclude from the results the impact of randomness, which was used in the preparation of the geometry.

Analysis of the acceleration effects on the average strain of the open-cell foam shows significant differences between the strains at accelerations between 10^5 and $3 \cdot 10^5$ m/s², while the differences in strain become much smaller at higher accelerations. The cause for these large differences is the behaviour of the open-cell foam under compressive loading conditions.

4. Conclusions

Experimental tests and computational simulations of aluminium foam specimens inside a water tank subjected to explosive charge have been performed. Comparison of the results shows a good correlation between the experimental and simulation results. Performed Taylor impact tests showed that the open-cell foam projectile deforms even before the impact in a rigid wall. The simulations also confirmed the assumption that the material collapse only in the vicinity of the rigid plate is the result of the uneven distribution of the mass inertia.

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MICROMECHANICAL MODELLING OF DAMAGE IN COMPOSITE STRUCTURES USING HFGMC

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1. Introduction

The High Fidelity Generalized Method of Cells (HFGMC) has been implemented into Abaqus/Explicit in order to model damage processes in composite materials at the micromechanical level. The HFGMC model discretizes the representative volume element of the heterogeneous material using $N_x \times N_y$ subcells, which can be either fibre or matrix subcells. Modelling on the micro-level enables insight into the underlying physical processes which lead to the homogenized properties of the macro-scale material. Figure 1 depicts different unit cells models which have been evaluated throughout this research.

2. Micromechanical model

The HFGMC model is based on the basic principles of the Method of Cells micromechanical model (Aboudi, 1987). HFGMC uses a Legendre type polynomial to approximate the displacement field within the subcell, thereby solving the lack of "normal-shear coupling", the main drawback of preceding micromechanical models based on Aboudi's Method of Cells, as stated in (Bansal and Pindera, 2002; Bansal and Pindera, 2006).

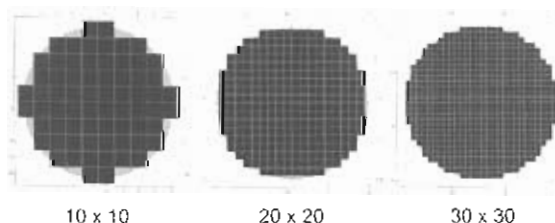


Figure 1. Composite material unit cells analyzed in this work.

The aim of the micromechanical analysis is to determine the strain concentration tensor, which relates the strain tensor of each subcell to the macroscopic strain field. The input variables for the HFGMC model are the current state of

macroscopic strain, number of subcells (N_x and N_y), as well as parameters which define properties of the composite material – fibre and matrix mechanical properties and fibre volume fraction (V_f). Each subcell is selected to be a fibre or matrix constituent, based on the fibre volume fraction, number of fibres within the unit cell and position of fibre centres. At the current phase of the methodology development, dimensions of all subcells are equal.

Abaqus user material subroutine VUMAT has been used to implement the micromechanical model into the finite element analysis. The work described in this paper is in the initial phase of implementation of the multiscale computational model, and therefore material properties have not been degraded due to damage effects. At a later stage of this research, the HFGMC model will be used to calculate equivalent composite material properties after damage progression. As the HFGMC model is computationally demanding, it should not be called at all elements and increments in the finite element model, as to achieve acceptable computational times. Puck's failure criterion has been selected as a measure which initiates calculation on the micro-scale.

Number of subcells:	10 × 10	20 × 20	30 × 30
$\bar{\sigma}_{33}$ [MPa]	-1.1931	0.3647	-0.0025
V_f	68.0%	69.0 %	70.2%
E_1 [GPa]	176.96	179.49	182.61
$E_2 = E_3$ [GPa]	11.34	11.03	11.29
G_{12} [GPa]	9.02	5.56	6.10
ν_{12}	0.236	0.239	0.237
ν_{23}	0.394	0.388	0.391

Table 1. Effect of discretization on predicted effective ply properties

In addition to the calculation of the stress field within the unit cell, the HFGMC model is used to calculate composite failure criteria on the micromechanical level. The micromechanical failure criteria implemented in the model are the Micromechanics of Failure model (MMF, used in Sun, Tay and Tan, 2011), and 3D Tsai-Hill model (for matrix failure) along with the maximum strain criterion for the fibre (used in Arnold, Bednarczyk and Aboudi, 2004).

3. Results and discussion

The effect of the unit cell discretization on effective CFRP properties is shown in Table 1. The HFGMC model has been used in combination with plane stress elements in the macro-level analysis. The micromechanically calculated average unit cell stress component in the 3 direction is used as an estimation of error of the micromechanical model, since this stress component should be zero in the plane stress case. As illustrated in Table 1, the value of $\bar{\sigma}_{33}$ converges to zero with unit cell discretization refinement.



Figure 2. Micromechanics of Failure criterion for a static tensile load case.

The HFGMC model has been tested on a relatively simple finite element model, loaded with only tensile static nodal forces. The finite element model represents a composite panel with stringer reinforcements, which is a typical structural element in aeronautical structures. In order to keep the computational time on an acceptable level, the micromechanical analysis is performed using the 20 x 20 HFGMC model. Figure 2 shows distribution of the MMF criterion value for the fibre. The criterion reaches maximal values between 0.28 and 0.31, whereas the macroscopically calculated Puck's

fibre failure criterion has reached values above 0.8 for the same loading condition (failure is initiated when the criterion reaches the value of 1.0).

4. Concluding remarks

This work presents results of the initial phase of implementation of the HFGMC model in an explicit finite element code. The HFGMC model has been used to calculate equivalent composite properties and micromechanical failure criteria. In the ongoing research, the HFGMC will be employed to predict equivalent composite properties after damage initiation caused by impact loading, blast loads and similar high velocity / high strain phenomena. Despite the differences in the prediction of failure criteria on the micro and macro-level, the HFGMC presents a promising model for further development.

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CONCRETE STRUCTURES

A COMBINED FINITE-DISCRETE ELEMENT MODEL FOR REINFORCED CONCRETE STRUCTURES UNDER STATIC AND CYCLIC LOAD

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1. Introduction

The development of the numerical models for simulation of the response of reinforced concrete structures, taking into consideration non-linear behaviour of the concrete and reinforcement, makes possible the results of high accuracy almost equivalent to the results obtained by expensive laboratory experiments. One of the main causes of concrete non-linear behaviour is cracking. A reliable model for simulation of the opening and closing the cracks is especially important in the structures under cyclic load because a significant part of energy is lost in cracking. The most of the models described in literature is based on finite element method where the cracking is described with smeared crack models or with embedded models where the cracks are modelled as discontinuity within the elements by enriching interpolation function. In contrast to the finite elements, in discrete models cracks are simulated as discontinuities of displacement between two elements.

A numerical model developed in this work is based on combined finite-discrete element method (Munjiza, 2004). The model combines standard finite element formulation for the hardening part of the constitutive law with the single-crack model for the softening part of stress-strain curve. Finite elements are used to model behaviour of the material up to the ultimate tensile strength while a discrete crack model is used for modelling of the crack opening and separation along edges of finite elements (Munjiza *et al.*, 1995). An embedded model of reinforcing bars is implemented in Y2D combined finite-discrete element code. The concrete structure is discretized on triangular finite elements, while the reinforcing bars are modelled with one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements. The

main assumption of the model is that the concrete tensile strength will be exceeded before the yield stress will be achieved in steel. The behaviour of the structure is linear-elastic up to the opening of the crack. After that joint elements in concrete as well as in reinforcing bar are occurred. The concrete and reinforcing bars are analyzed separately, but they are connected by the relation between the size of the concrete crack and strain of the reinforcing bar (Soltani and Maekawa 2008). Cyclic behaviour of the steel during the seismic load is modelled with improved Kato's model (Maekawa *et al.* 2003).

2. Modelling of the reinforced concrete structure

The model of the reinforced concrete structure with the embedded reinforcing bar is shown in Figure 1.

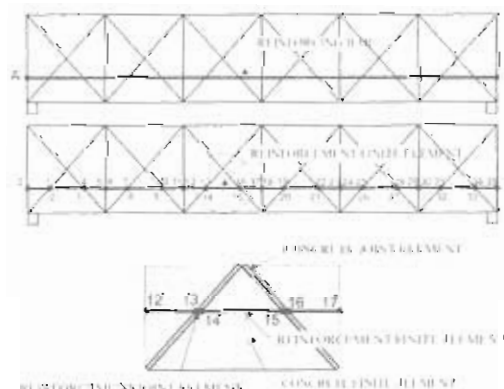


Figure 1. Discretization of reinforced concrete structure

In the first step the reinforcement is modelled as bar AB. Intersection between the sides of concrete finite elements and reinforcing bars gives the reinforcement finite elements. The behaviour of the structure is linear-elastic up to the opening of the crack. In that phase the triangular concrete element and line element of reinforcing bar behave as one body. The deformation of the triangular element influence

to the deformation of the reinforcing bars. When the crack in concrete appears, joint element in concrete as well as joint element in reinforcing bars is occurred.

A stress-strain curve for concrete in direct tension is divided into two sections (Figure 2): strain-hardening prior to reaching the peak stress (f_t) and strain-softening (Hordijk, 1992) for which the stress decreases with an increasing the strain.

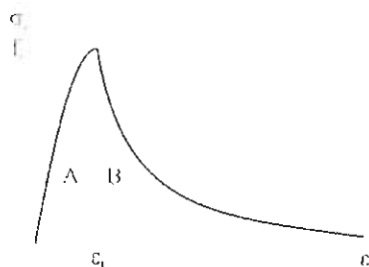


Figure 2 Typical strain softening curve defined in terms of strains

A model of the relationship between the concrete crack size and strain of the reinforcing bar developed by and Shin (Shin, 1988) is applied. The model is based on experimental strain-slip curves and represents well approximation of the behaviour of reinforcing bar with the expressed plastic strain caused by cyclic loading. Non-linear material model for steel is based on experimental stress-strain curve. Cyclic behaviour of the steel is modelled with improved Kato's model (Kato, 1976).

3. Results

Validity of the model is tested on two examples. The first one presents validation of the model for the static loading on reinforced concrete beam with numerical and experimental results taken from literature. The second one is an example of the reinforced concrete beam under the cyclic loading, which is also taken from literature. In both cases the results obtained by the numerical model based on combined FEM/DEM methods show very good agreement with the numerical and experimental results obtained in literature.

4. Concluding remarks

This paper presents a new numerical model for analysis of reinforced concrete structures

under static and cyclic load based on combined finite-discrete element method. An embedded model of reinforcing bars is implemented. The concrete structure is discretized on triangular finite elements, while the reinforcing bars are modelled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements. The behaviour of the structure is linear-elastic up to the opening of the crack. After that joint elements in concrete as well as in reinforcing bar are occurred. Interaction solutions between bars and concrete have also been developed and implemented into the open source Y2D combined finite-discrete element code. The results obtained for the reinforced beams under the static and cyclic loadings show very good agreement with the numerical and experimental results obtained in literature.

Acknowledgements

The partial financial support, provided by the Ministry of Science, Education and Sports of the Republic of Croatia under the project Non-linear dynamic analysis of three-dimensional reinforced concrete structures, is gratefully acknowledged.

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A SIMPLE REINFORCED-CONCRETE BEAM MODEL ACCOUNTING FOR THE EFFECT OF TENSION STIFFENING

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1. Introduction

Reinforced concrete is the most widely used composite material in civil engineering. It combines the advantageous mechanical properties of concrete in compression and steel in tension to provide the staple ingredient for a variety of structures including the widely used framework-type structures designed to carry the loading primarily by resisting the effects of bending. In the process, the reinforcement bars normally become stretched while the adjoining concrete experiences fully developed cracking or sizeable micro-cracked regions with limited resistance to tension (see e.g. Bažant and Cedolin, 2003). Such a mechanical behaviour is instrumental in producing the so-called tension-stiffening effect as the principal load-bearing mechanism in reinforced-concrete structures once cracking of concrete in tension and subsequent slippage of the reinforcement bars with respect to the surrounding concrete take place (see e.g. Stramandinoli and La Rovere, 2008). In this way the tensile stresses in the reinforcement at the crack positions gradually become transferred to the concrete between the cracks.

The tension-stiffening effect may be mathematically modelled in many different ways, from highly complex models involving fully spatial concrete behaviour (see e.g. Ožbolt and Bažant, 1992), non-linear damage properties of concrete (see e.g. Marfia et al., 2004), isotropic and kinematic hardening of reinforcement steel (see e.g. Marfia et al., 2004) and non-linear bond-slip relationship (see e.g. Ožbolt et al., 2002) including the detailed effects of the rib resistance (see e.g. Wang and Liu, 2006) to fairly simple yet sufficiently accurate models in which these effects are lumped together (see e.g. Stramandinoli and La Rovere, 2008).

2. Problem Description

The mechanism of crack initiation and growth is described by principles of damage mechanics in conjunction with the energy considerations originating from fracture mechanics (see e.g. Hillerborg et al, 1976). The methods to model crack initiation and growth using the finite-element method fall into two major categories: (i) the smeared-crack methods (see e.g. Ožbolt and Bažant, 1996), in which a finite band around the actual crack is considered to be either fully or partly damaged and (ii) the discrete-crack methods (see e.g. Jirasek, 2000), in which a strong discontinuity in the material takes place at the fully developed crack position. A discrete crack may be either predicted to occur at an interface element between two continuum finite elements (see e.g. Alfano and Crisfield, 2001) or it may develop within an existing element (see e.g. Jirasek, 2000).

Upon cracking, the reinforcement slips with respect to the surrounding concrete, which determines the amount of tangential stress (bond) transferred from the reinforcement to the concrete as a result of Coulomb friction and the actual shape of the bond-slip diagram as described by different authors and design codes (see e.g. Ožbolt et al., 2002). The behaviour of concrete in compression and steel in tension is defined by the standard non-linear elastic-plastic models with steel hardening.

3. Results and discussion

In this work the tension-stiffening effect is modelled on the basis of the experimental results reported in literature, which provide the force-extension relationships for a uniaxially loaded reinforced-concrete specimen. From these results, an average strain of the specimen may be easily deduced and used to define the constitutive relationship needed for the

numerical analysis. In this way, the complex issues of crack initiation and growth as well as the stress transfer from the reinforcement to the surrounding concrete are effectively by-passed.

This model is applied to beam-like structures in which, owing to the standard kinematic hypothesis of cross sections remaining planar, a relatively fast computational procedure is expected. The strain measures in the beam element are related to the displacement and rotation functions using the geometrically exact Reissner's beam model (Reissner, 1972), which in the case of geometrically linear analysis reduces to the standard Timoshenko beam model. These strain measures are then considered as the average strains along the beam element for which the resulting axial force is deduced from the known experimental results for a suitably chosen effective area of the reinforced-concrete beam in bending.

4. Concluding remarks

The approach proposed here results in a very simple model for the analysis of reinforced-concrete beams in which the analysis of the complex issues of crack initiation and growth as well as the stress transfer from the reinforcement to the surrounding concrete is effectively avoided. As the next step, a more refined model will be proposed in which a suitable non-linear bond-slip relationship will be taken to govern the stress transfer upon crack occurrence as predicted by the model codes. Finally, an embedded discrete-crack layered element of variable layer thicknesses will be proposed, in which both the pre- and post-cracking behaviour will be modelled using the fictitious crack model (Hillerborg et al, 1976).

Acknowledgements

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CONCRETE FRAMES

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1. Introduction

Reinforced concrete frame buildings with masonry infill walls have been widely constructed for commercial, industrial and multi-family residential uses in seismic prone regions in Southern Europe. A masonry infill wall panel typically consist of clay brick or concrete block walls and is constructed after the reinforced concrete frame has been finished.

These panels are generally not considered in the design process and are treated as architectural (non-structural) components. Their presence increases structural strength and stiffness, but at the same time diminishes ductility and introduces brittle failure mechanisms associated with the wall failure and wall-frame interaction.

The study presented in this paper compares seismic vulnerability of reinforced-concrete frame structures with- and without infill. An incremental nonlinear dynamic analysis of planar reinforced-concrete frames with- and without infill exposed to a set of recorded ground motions has been performed in order to evaluate their seismic vulnerability and to compare their seismic responses.

2. Model Setup

Model structures used for the analysis are based on a 7-storey reinforced concrete middle frame of the regular low-rise "Tsukuba" building with different infill configurations as presented in Figure 1.

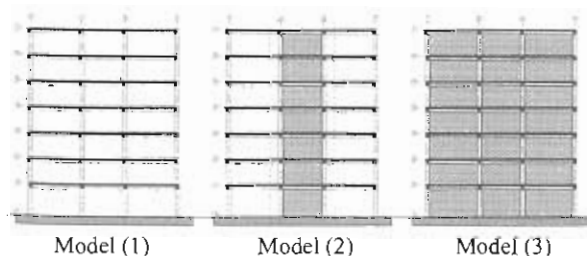


Figure 1. Middle frame models of the "Tsukuba" building with different infill configurations. (Grubišić, M., Sigmund V., 2011).

Reinforced concrete frame elements are designed according to the EC8 and are the same in all three models (1) bare frame; (2) middle bay is full-height infilled with masonry and (3) all three bays are full-height infilled with masonry.

Seismic risk of various models has been evaluated by means of probabilistic analysis that takes into account the randomness in seismic excitations and evaluates the probability of crossing a certain critical state (Performance State). An Incremental Dynamic Analysis (IDA) of the nonlinear analytical models was performed by use of the SeismoStruct ver. 5.2.2. software. Reinforced-concrete frame elements were modelled as Inelastic displacement-based fiber elements and infill walls were modelled as inelastic infill panel element (Kalman Šipoš, T., Sigmund, V., 2010).

A set of normalised earthquake records was individually applied to the structural models. The records were scaled in an increasing intensity (Incremental Dynamic Analysis) until the structural failure occurred. Structural behaviour was evaluated based on the recorded Inter-storey Drift Ratio (IDR or θ_{max}).

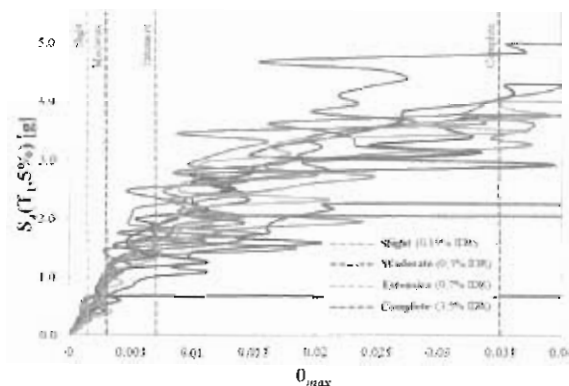


Figure 2. IDA results for 7-storey partially-infilled frame model (2) subjected to 10 earthquake records. (Grubišić, M., Sigmund V., 2011).

The results of IDA is a function that shows relationship between the ground motion

intensity as 5% damped first-mode spectral acceleration, $S_a(T_1, 5\%)$, and peak IDR, θ_{max} , for a set of ground motions, as shown in Figure 2.

3. Results and discussion

Variability of earthquake records leads to the need for statistical treatment of multi-record IDA output in order to summarize the results and use them effectively in a predictive mode, as for example in a Performance Based Earthquake Engineering (PBEE) context.

The results showed an increase in initial stiffness, strength and energy dissipation of the infilled frames in models (2) and (3) compared to the bare frame model (1). Fragility curves, created from the IDA, provide useful tools for evaluation of the structural behaviour under seismic action.

These curves, as shown in Figure 3, predict the probability of reaching a certain performance level as a function of the ground motion intensity for a particular structure.

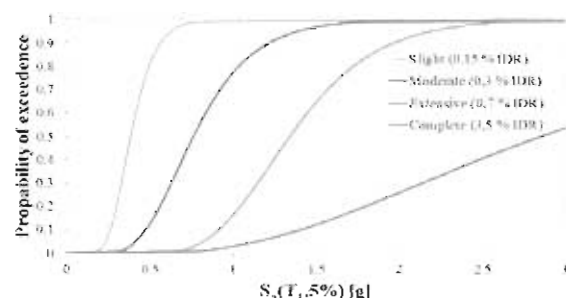


Figure 3. Fragility curves for 7-storey partially-infilled frame model (2) as assessment of the probability of reaching each performance level. (Grubišić, M., Sigmund V., 2011).

Performance levels (damage) are defined with respect to the parameters of IDR (Inter-story Drift Ratio %). Serviceability limit state (SLS) unites slight (0,15% IDR), moderate (0,3% IDR) and extensive (0,7% IDR) damage levels. Ultimate limit state (ULS) is associated with heavy structural damage close to collapse and with IDR from 0,7% to IDR $\leq$ 3,5%.

From the obtained fragility curves for all three models we concluded that partially- and fully infilled frame models could sustain higher intensities of spectral accelerations.

4. Conclusion

The analysis results indicated that infill completely changed distribution and mechanism of damage throughout the structure. Estimated probability of exceeding each performance level (i.e. collapse risk or

vulnerability in terms of Damage Measure, θ_{max}) is undoubtedly the lowest for the fully-infilled frame and highest for the bare frame where global dynamical instability is caused by second order effects (P- Δ effects). For the infilled models we had considerable dispersion of IDA results in the areas of high spectral accelerations due to randomness in seismic excitations.

Since masonry infill walls significantly affects seismic behaviour of the RC frame, they should be included in the analytical model. Infill could favourably influence the seismic response by increasing strength, stiffness, damping and lowering the IDR thus lowering damage probability, provided that they are placed regularly throughout the structure and that they do not cause shear failures of columns.

The research presented in this paper is a part of the research project "Seismic design of infilled frames", No. 149-1492966-1536, supported by the Ministry of Science, Education and Sports of the Republic of Croatia and its support is gratefully acknowledged.

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DAMAGE, FATIGUE AND FRACTURE

ROUGHNESS SURFACE OF ROBOT LASER HARDENED PATTERNS GGG 70

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1. Introduction

Characterization of surface topography is important in applications involving friction, lubrication, and wear (Thomas, 1999). In general, it has been found that friction increases with average roughness. Roughness parameters are, therefore, important in applications such as automobile brake linings, floor surfaces, and tires. In this work we investigated roughness of robot laser hardened patterns of the DIN standard GGG 70 at different temperatures. Robotic laser surface hardening heat treatment is complementary to the conventional flame or inductive hardening. The energy source for laser hardening, the laser beam which heats up very quickly and the metal surface area of ponds up to 1.5 mm and a hardness of 65 HRC. Laser hardening is a process of projecting features such as non Method told them, controlled energy intake, high performance constancy and accurate positioning process. Hard martensitic microstructure provides improved surface properties such as wear resistance and high strength. In this work we have used scanning electronic microscope (SEM), search and analyze the fractal structure of the robotic laser-hardened material. The aim of research is to find how robotic laser cell parameters for optimal tempering effect on fractal dimension of the hardened material.

2. Experimental Setup

We made patterns on of the DIN standard GGG 70. GGG 70 was hardened with the laser at different temperature. So we changed parameter of temperature $T \in [1000, 1400]^\circ\text{C}$ to 50°C steps In all these attempts we have made recordings microstructure. We made recordings hardened surface area as well as clips deep hardened zone. Interest to us is whether the robotic laser hardening hardening parameters for different roughness found microparticles. Figure 1 shows the longitudinal

and transverse cross-section of hardened patterns GGG 70.

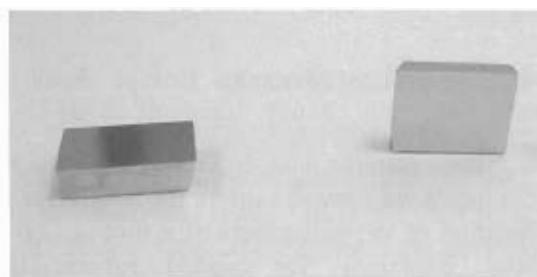
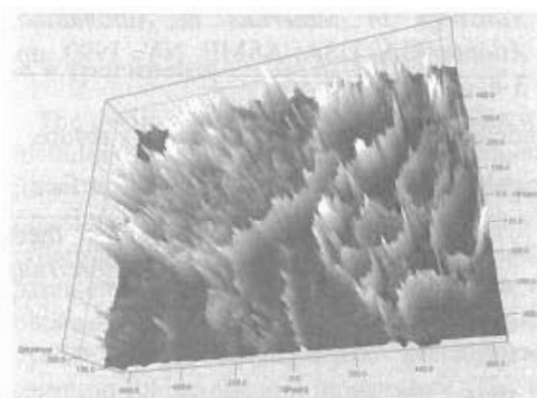


Figure 1. Longitudinal and transverse cross-section of hardened patterns GGG 70.

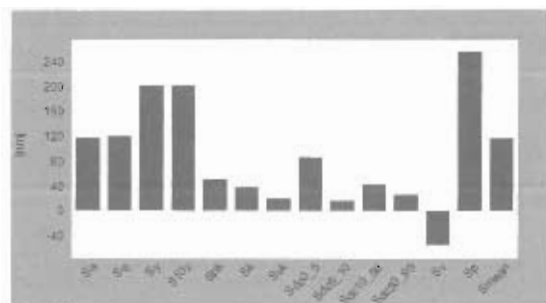
3. Results and discussion

Definitions and indications for surface roughness parameters (for industrial products) are specified. They are arithmetical mean roughness (R_a), maximum height (R_y), ten-point mean roughness (R_z), mean spacing of profile irregularities (S_m), mean spacing of local peaks of the profile (S) and profile bearing length ratio (t_p). Surface roughness is given as the arithmetical mean value for a randomly sampled area.



Graf 1: 3D graph of robot laser hardened pattern GGG 70 with 1100 °C and 3 mm/s

In Graf 2 we can see roughness with different methods.



Graf 2: Roughness of robot laser hardened pattern GGG 70

4. Concluding remarks

We calculated a roughness of robot laser hardened patterns with different temperature. In the future we want to explore the roughness as a function of the parameters of a robotic cell for laser hardening for pinned robotic laser hardening. Laser parameters as power, energy density, focal distance, temperature and speed of hardening.

Acknowledgements

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LOW-CYCLE FATIGUE INVESTIGATIONS AND NUMERICAL SIMULATIONS ON NODULAR CAST IRON FOR WIND TURBINE COMPONENTS

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1. Introduction

Most of the components of modern wind turbines are made of ductile cast iron. Ferritic nodular cast iron grade EN-GJS-400-18-LT, in which graphite spheroids or nodules in a ferritic matrix provide large ductility and fatigue strength, is commonly used. In the last decade numerous experimental studies have been presented showing influence of microstructure on the mechanical behaviour of the nodular cast iron. However, there are only limited experimental data available on fatigue crack growth behaviour of EN-GJS-400-18-LT (e.g. Mottischka et al., 2010) and still the effect of the graphite morphology (size, shape and distribution) as well as microstructure phase (ferritic and pearlitic) on fatigue properties of this material is not well documented (Čanžar et al., 2011).

This paper presents an experimental and numerical study of the low-cycle fatigue behaviour of nodular cast iron EN-GJS-400-18-LT depending on the material microstructure.

2. Materials and specimens

Four types of cast iron EN-GJS-400-18-LT produced by different technologies are considered. For producing nodular cast iron, one non standard technique (type 100) and three standard techniques (flotret – type 200, tundish – type 300, and in mould – type 400) are applied. The graphite morphology of the casting types is shown in Figure 1. As it can be seen, material type 200 produced by the flotret process has significantly larger nodules with low density distribution than the other three types of nodular cast iron. Besides, material type 200 has graphite nodules with the lowest circularity (irregularly shaped nodules)

and lower content of pearlitic grain. On the other hand, material types 300 and 400 produced by the tundish and in mould process have smaller ferrite grains and smaller nodules, more spherical and regular in shape than those in the material type 200. Furthermore, material type 100 has the highest content of pearlite.

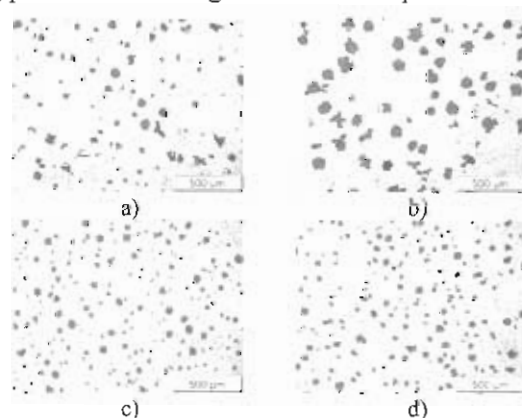


Figure 1. Microstructure of the nodular cast iron for: a) type 100; b) type 200; c) type 300 and d) type 400

3. Experiments

The cyclic plasticity model used in this work includes highly nonlinear multi-component forms of kinematic and isotropic hardening functions in conjunction with von Mises yield criterion (Tonković et al., 2008). The material parameters are identified by means of the data obtained in constant and variable amplitude cyclic tests by Walter Bai servohydraulic testing machine at room temperature. The tests are carried out on the smooth cylindrical specimens. The representative symmetrical test for the type 400 of nodular cast iron is presented in Figure 2. Furthermore, the low-cycle fatigue crack growth and the fracture toughness tests are performed. The tests are conducted on the compact tension (CT) and single-edge notched (SEN) specimens, loaded

in three-point-bending under several R -ratios. The crack length is measured using an optical measuring system Aramis. Figure 3 presents the crack growth results for $R=0.1$.

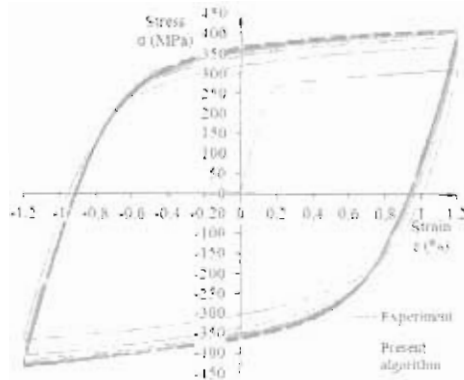


Figure 2. Comparison of the test data and the simulated model for representative stress-strain hysteresis loops

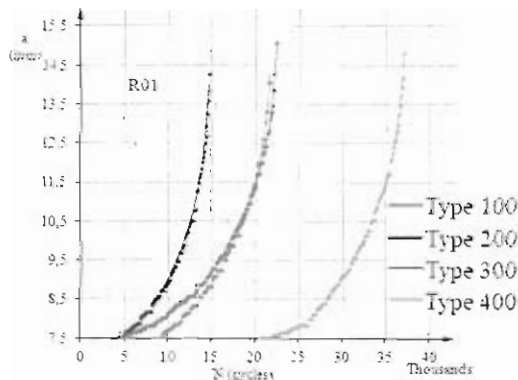


Figure 3. Number of cycles vs. crack length for $R=0.1$

4. Numerical formulation

The algorithm which enables numerical modelling of cyclic elastoplastic deformation of shell structures presented in (Tonković et al., 2008) is modified to simulate elastoplastic behaviour of a nodular cast iron. The derived computational algorithm is implemented in the software ABAQUS/Standard by using the user-defined material subroutine UMAT. Figure 2 shows a comparison of the numerical predictions and the test data obtained from the symmetric strain-controlled cyclic experiment. Good agreement of the solutions is exhibited except for the first cycle, since the kinematic hardening component is calibrated using the stabilized stress-strain curve. The cyclic plasticity model is afterwards applied to study the fatigue damage in the SEN and CT type

specimens near the notch and the crack tip. Herein, Abaqus/Standard damage initiation and evolution criteria are adopted to determine the low-cycle fatigue damage. Figure 4 shows a comparison between the experimental data and numerical predictions.

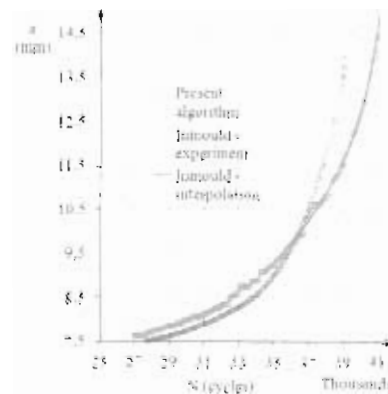


Figure 4. Comparison between the experimental data and numerical predictions

5. Concluding remarks

Experimental results show that size, shape and distribution of the graphite nodules has no significant influence on cyclic hardening of the material but they play a great role in the crack initiation and propagation process. It is shown that the larger irregularly shaped nodules reduce fracture toughness and fatigue strength. Furthermore, the results demonstrated that pearlitic phase does not strongly affect fatigue life if its proportion does not exceeds 10%.

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EXPERIMENTAL AND NUMERICAL INVESTIGATION OF CYCLIC CREEP AND RECOVERY BEHAVIOUR OF SHORT-FIBER-FILLED EPOXY

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1. Introduction

One of artificial material of human cortical bone tissue is short-fiber-filled epoxy produced by SAWBONES (Sawbones Europe AB in Malmö, Sweden). This fourth-generation epoxy is a mixture of short glass fibers and pressure injected epoxy resin around a foam core. The objective of this study is to develop a new constitutive model for this composite that predicts the experimental viscoelastic, viscoplastic and damage behavior in creep-recovery tests. The derived model is based on the ideas and approaches from Abdel-Tawab and Weitsman (2001), developed for the swirl-mat composites. The composite behavior is modeled by using the theory of viscoelasticity, viscoplasticity and continuum damage mechanics based on the thermodynamics of irreversible processes (e.g. Parsamian 2001, Dassapa et al. 2010, Marklund et al. 2006). The derived algorithm for the integration of the proposed one-dimensional constitutive model at the material point level is implemented into the finite element code ABAQUS/Standard. The accuracy of the computational procedure is tested by comparing the computed results with the real experimental data. Thereby, the creep-recovery deformation processes in short-fiber-filled epoxy at different loading levels are considered.

2. Constitutive model

In the constitutive model presented here, assuming small strains, the total creep strain $\varepsilon(t)$ is additively decomposed into viscoelastic part coupled with damage (ε_{VE}^D) and irreversible viscoplastic part (ε_{VP}), as follows

$$\varepsilon(t) = \varepsilon_{VE}^D + \varepsilon_{VP} \quad (1)$$

A schematic representation of the stress history and strain response of composite material during the creep and recovery period is presented in Figure 1. As can be seen, upon the removal of the applied load, the viscoelastic strain is recovered, while the viscoplastic strain is permanent.

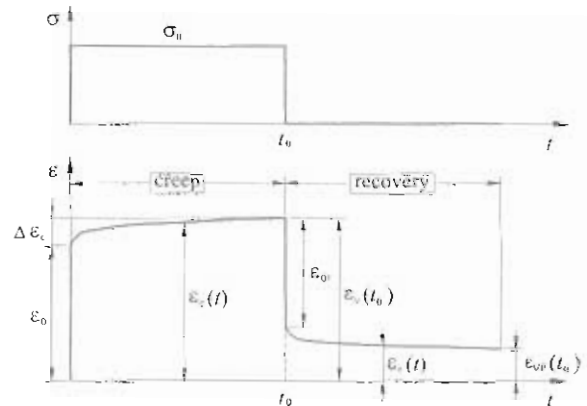


Figure 1. Schematic representation of the stress history and strain response during the creep-recovery period

According to Abdel-Tawab and Weitsman (2001) and Dassapa et al. (2010) the following evolutionary equations for strain components are adopted:

a) for creep ($0 < t < t_0$):

$$\begin{aligned} (\varepsilon_{VE}^D)_c = J_0 \sigma \left[1 - (r+1) \left(\frac{\sigma - \sigma_{th}^D}{C} \right)^r \cdot t \right]^{-1/(r+1)} \\ + J_1 \sigma t^r \left[1 + \frac{1}{1+\gamma} \left(\frac{\sigma - \sigma_{th}^D}{C} \right)^r \cdot t \right] \end{aligned} \quad (2)$$

$$(\varepsilon_{VP})_c = A \cdot \left(\sigma - \sigma_{th}^{VP} \right)^{m-n} \cdot t^n, \quad (3)$$

b) for recovery ($t > t_0$):

$$(\varepsilon_{VE}^D)_r = J_1 \sigma \left[t^r - (t - t_0)^r \right] \left[1 + \frac{t}{1+\gamma} \left(\frac{\sigma - \sigma_{th}^D}{C} \right)^r \right] \quad (4)$$

$$(\varepsilon_{VP})_r = A \cdot \left(\sigma - \sigma_{th}^{VP} \right)^{m-n} \cdot t_0^n. \quad (5)$$

In Eqs. (2-5) σ_{th}^D and σ_{th}^{VP} are the threshold stress levels for accumulation of damage and viscoplastic deformation, respectively. J_0 , J_1 and γ are the viscoelastic material parameters, A , m and n denote viscoplastic constants, while C and r stand for material parameters of damage.

3. Experiments

For the proposed constitutive model, the material parameters are determined by fitting experimental results of the creep-recovery tests. Specimens have been cut from the plates with dimensions of 130×180×2 mm using a commercial water jet cutter. Shape and dimensions of specimens are shown in Figure 2.

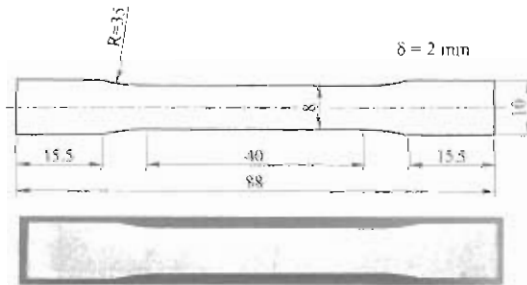


Figure 2. Specimen shape and dimensions

Tensile cyclic creep and recovery tests are carried out at room temperature on a Messphysik Beta 50-5 testing machine at ten different stress levels between 20 and 65 MPa. Each test consisted of 60 s of tensile creep at constant stress level, followed by 300 s of recovery. The results presented in Figure 3 are average values from six tests at each applied stress.

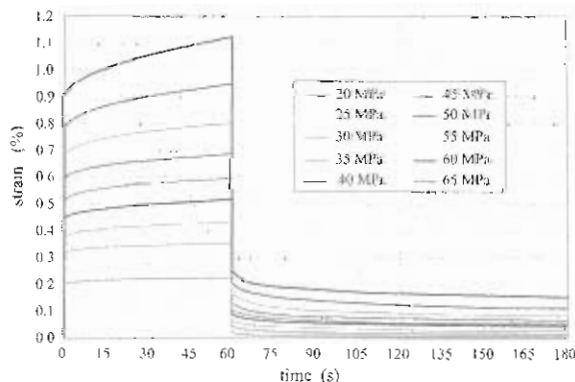


Figure 3. Experimental results from tensile cyclic creep and recovery tests

4. Numerical formulation

In order to implement the viscoelastic-viscoplastic-damage model into the finite element algorithm, the constitutive equations are transformed into an incremental form by using finite differences. The derived integration algorithm and the corresponding tangent stiffness are implemented at the material point level of the three-dimensional solid continuum finite elements in the software ABAQUS/Standard by using the user-defined material subroutine UMAT.

5. Concluding remarks

A new one-dimensional constitutive model for short-fiber-filled epoxy is proposed to simulate the viscoelastic-viscoplastic-damage behavior occurring during creep-recovery tests. The material parameters are determined by fitting experimental results. An efficient computational algorithm for the integration of the proposed constitutive model at the material point level is derived. The computational algorithm shows an excellent capability to describe the tensile behavior of short-fiber-filled epoxy for the specific mechanical condition analyzed.

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DYNAMICS

TO INVESTIGATE STRESS EFFECTS IN THE DYNAMICS OF OPTICAL LENSES

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1 Introduction

In high precision optics, very high accuracies are required to ensure resolutions down to the nanometer scale. However, due to the accuracy demands, these systems are very error-prone. Regarding dynamics, even small excitations yield to erroneous projections due to so-called dynamically induced aberrations which decrease the image quality.

The problems caused by the dynamical behavior of a lens system are aberrations due to rigid body motion of the lenses, surface deformations and stress-induced changes of the optical properties of the lens material.

In contrast to this, these effects can be created on purpose to compensate interfering aberrations by opposite aberrations. For example, lenses can be actively deformed [1]. To control this process, the influence of undesired effects like dynamic stress must be predictable. For this, dynamic stress recovery methods in elastic multibody systems have to be combined with optical simulation methods.

2 Basics of the optical part

To determine the aberrations, point light sources are used in the raytracing simulation, i.e. a set of rays starts from the same point in the object plane and the path of the rays to the image plane is calculated. By means of several point light sources distributed over the object plane the aberrations for the whole projection range can be determined. In the special case of coupling dynamics and optics, it is advantageous to separate the overall aberration into chief ray deviation and wavefront aberration.

Chief ray deviation is a measure for the movement or distortion of an image. Wave-

front aberration describes its blurring. By means of expanding the wavefront aberration in Zernike polynomials [2, 3], the blurring can be quantified.

3 Stress effects in lenses

Stress in lenses produces a locally variable refraction index. Furthermore, the refraction index varies with respect to the direction of the polarization [4]. However, this effect called birefringence will be neglected in the following. In Figure 1, the behavior of light rays in a lens with variable refraction index is demonstrated.

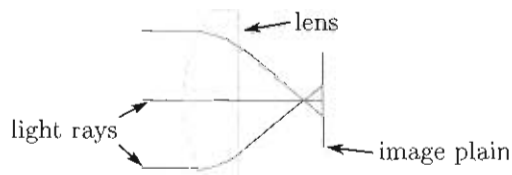


Figure 1: Curved path of light rays in a lens with locally variable refraction index.

For a specific wavelength the refraction index n depends on the principal stresses $\sigma_1(\mathbf{p})$, $\sigma_2(\mathbf{p})$ and $\sigma_3(\mathbf{p})$ at a point $\mathbf{p}$ within the lens,

$$n(x, y, z) = n_0 + (\sigma_1 + \sigma_2 + \sigma_3)K \quad (1)$$

where K is the stress-optical coefficient. Raytracing simulations with a locally variable refraction index require a numerical solution for the non-straight path of a ray through a lens. For this, the fourth order Runge-Kutta method is a good choice. Unfortunately, the step-wise raytracing leads to a correspondingly long computation time.

4 Modeling the mechanical part

The mechanical part of a lens system consists of the lenses itself, their holding devices and the housing. It is modeled as an elastic

sidered as rigid bodies and some parts are modeled as elastic bodies, see [5, 6].

For investigating stress effects, at least the lenses have to be elastic. Most commonly, elastic bodies are modeled by means of the Finite Element Method. Due to performance, the system matrices of the elastic bodies are reduced by an appropriate model order reduction method, see [7], which requires a reduction matrix V containing information about the deformation of the chosen shape functions. Additionally, the stress produced by these deformations can be computed and stored in the stress mode matrix V_σ , see [8].

When all bodies are created, they get connected to each other by means of the floating frame of reference approach. The equations of motion for the whole system are set up and finally linearized. A modal analysis yields the mode shapes of the EMBS. Each mode shape has to be applied to the optical model to determine the chief ray deviations and wavefront aberrations of the mode shapes. These aberrations are stored in the modal sensitivity matrix $\tilde{C}_s$.

Applying a modal transformation to the equations of motion, the results of a time integration are obtained in form of modal coordinates $\tilde{q}$. By multiplying the modal coordinates with the modal sensitivity matrix,

$$a(t) = \tilde{C}_s \cdot \tilde{q}(t) \quad (2)$$

the aberrations a for each time step can be calculated. In a second calculation, the stress-induced variation of the refractive index has to be neglected while applying the mode shapes to the optical model. The differences of the aberrations show the influence of the stress effects.

Acknowledgements

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ENERGY BALANCE METHOD WITH USING EIGENFUNCTIONS IN THE ESTIMATION OF OVERHEAD TRANSMISSION LINE AEOLIAN VIBRATIONS

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1. Introduction

Wind-induced overhead transmission line Aeolian vibrations can cause the damage and breaking of the cable due to the material fatigue and can significantly shorten its lifetime. Aeolian vibrations are in practice commonly estimated by using the Energy Balance Method (Hagedorn, 1987) and the Modified Energy Balance Method with Using Eigenfunctions (Hagedorn et al., 2002), which enables determination of eigenvalues of a cable with line fittings, and therefore, the calculation of bending strains at any point in the span. Warning sphere is usually modelled as a particle having only a vertical rectilinear motion. This is a crude assumption because in this way any rotation of warning sphere cannot be taken into account. In this paper the warning sphere is modelled as a rigid body which perform plane motion.

2. Energy Balance Method with Using Eigenfunctions

An overhead transmission line cable of span length L with N arbitrary mounted line fittings ($N+1$ subspans) is shown in Figure 1. Free vibration of the considered cable, modelled as an ideal taut string, inside each of the $N+1$ subspans is described by

$$-T w_i''(x_i, t) + m \ddot{w}_i(x_i, t) = 0, \quad (1)$$

where w_i is the transverse displacement of the cable at location x_i at time t , T is the conductor tension force, m is the conductor mass per unit length. The ' signs denote differentiation with respect to coordinate x , while dots denote differentiation with respect to time t .



Figure 1. A cable of length L with N arbitrary mounted line fittings ($N+1$ subspans).

The general solution of the equation (1) is

$$w_i(x_i, t) = \text{Re}((A_i e^{\frac{s x_i}{c}} + B_i e^{-\frac{s x_i}{c}}) e^{st}), \quad (2)$$

where $c = \sqrt{T/m}$ is wave velocity, s is eigenvalue, A_i and B_i are complex integration constants. From the boundary conditions at both ends of the each subspan and from the force equilibrium at the clamps of the line fittings, one obtains a set of equations that leads to the eigenvalue problem

$$\mathbf{J}(s) \mathbf{a} = \mathbf{0}, \quad (3)$$

where $\mathbf{a} = [A_1, B_1, A_2, B_2, \dots, A_{N+1}, B_{N+1}]^T$.

In forming the system matrix $\mathbf{J}(s)$, the mechanical transfer impedance matrix of rigid body (plane motion) is used instead of the mechanical driving point impedance of a particle (rectilinear motion).

Equation (3) is not a standard matrix eigenvalue problem in the complex frequency parameter s , since the elements of the matrix $\mathbf{J}(s)$ depend explicitly on s . The spectrum of this eigenvalue problem is very dense and its numerical solution presents considerable numerical problems (Verma and Hagedorn, 2007).

After the complex eigenmodes are determined, one can formulate the energy balance by equating the wind power input over the whole span to the power dissipated in all Stockbridge

dampers and the power dissipated in the cable due to cable's self-damping

$$\sum_{i=1}^{N+1} P_{Wi}(A_{\max}) = \sum_{i=1}^N P_{Di}(A_{\max}) + \sum_{i=1}^{N+1} P_{Ci}(A_{\max}). \quad (4)$$

$A_{\max}$ is here the variable to be determined by scaling the vibration amplitudes. P_{Wi} , P_{Di} and P_{Ci} are determined according to data from (Wolf et al., 2008). The general expression for bending strain is:

$$\varepsilon(x, t) = \frac{d}{2} w''(x, t). \quad (5)$$

The local curvature of a cable w'' is determined according to equations from (Hagedorn et al., 2002). d is here the bending diameter.

3. Results

The presented results are related to ACSR "Bersfort" 35.6 mm (48/7) with two Stockbridge dampers and two warning spheres, as it is shown in Figure 2 ($d = 22.2$ mm, $m = 2.37$ kg/m, $EI = 2105$ Nm², $T/P = 0.22$, where EI is the cable bending stiffness and $P = 180.1$ kN is the cable tensile strength).

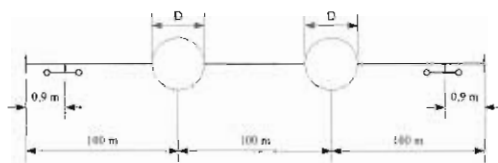


Figure 2. A cable with two Stockbridge dampers (St) and two warning spheres (WS).

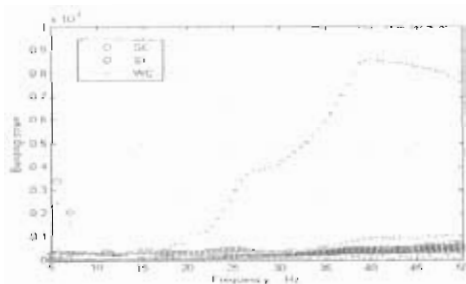


Figure 3. Bending strains at the critical points of the span (warning sphere as a particle).

Bending strains at the critical points of the span, i. e. at the suspension clamps (SC), at the points of damper attachments (St) and at the points of

warning sphere attachments (WS) are shown in Figures 3 and 4 for considered two models of warning spheres.

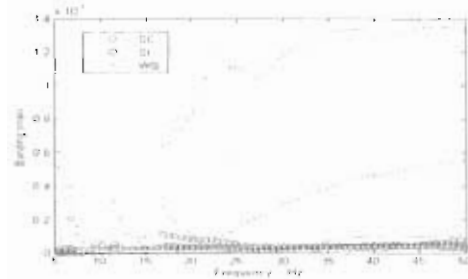


Figure 4. Bending strains at the critical points of the span (warning sphere as a rigid body).

4. Concluding remarks

The presented results show significant differences of bending strains obtained with the model of a warning sphere as a particle (rectilinear motion of a particle) and with the model of a warning sphere as a rigid body (plane motion of a rigid body). In the considered example these results differ up to 57 %.

Acknowledgements

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ON EC8-COMPATIBLE TIME-DOMAIN SIMULATION OF STRUCTURES SUBJECTED NON-UNIFORM SEISMIC GROUND MOTIONS

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1. Introduction

Seismic analysis and design in Europe are regulated by Eurocode 8. Part 1 is of overall interest since it treats seismic actions per se and general rules, while the other parts address specific types of structures. EC 8 heavily leans on the response spectrum method. It is the reference analysis method – all other alternative methods must be compatible with it. The response spectrum method tacitly assumes a uniform ground motion that envelops the entire structure. If that is not the case, then “Allowance for the variation of ground motion in space as well as time may be required for specific types of structures” (EC8:Part1:3.2.2.1 (8)). This rather plastic formulation is further detailed in Part 2 which addresses bridges. A spatial variation must be taken into account if the length of the bridge exceeds a certain limit or/and the soil properties change noticeably between the supports. In such cases there exist three analysis options: a static approach with different support displacements in combination with a classic response spectrum method, a multi-correlated response spectrum method, or a time-domain approach with time histories of the ground motion.

The generation of time histories compatible with a given response-spectrum is treated in Part 2, Annex D. The method of choice is the by now classic weighted-amplitude wave superposition method by Shinozuka (Shinozuka, 1972). It has been applied to the generation of ground accelerations by Gasparini and Vanmarcke (Gasparini and Vanmarcke, 1976a) who published a widely used computer program (Gasparini and Vanmarcke, 1976b). An extension to a field of multi-correlated time histories offers no problems. There are many further contributions to the generation of acceleration

histories, but the following discourse does not depend on the generation method, so this topic is not pursued here further.

2. The problem

The problem lies not in the generation of EC8-compatible acceleration histories, but in the fact that they are acceleration histories. When imposing a given acceleration history to a degree of freedom at a support and performing a time-history analysis, a strong linear trend can be observed in the solution. This defect has no noticeable influence on the inner forces in the case of a uniform ground motion since the trend is an almost static process (no dynamic amplification) which leads to a stress-free rigid body motion. It would be disastrous for non-uniform ground motions since two supports with unequal trends would introduce huge static inner forces.

This trend is a product of the time integration and has no physical relevance. A similar phenomenon can be observed when trying to integrate ground accelerations to ground displacements. A full seismic record of accelerations, velocities and accelerations contains incompatible information in that it is not possible to exploit the mathematical relationship between the three variables and gain one history from another by integration/differentiation (Malhotra, 2001). This phenomenon is well-known in earthquake engineering and has led to a variety of base-line correction techniques. Different techniques influence the maximum amplitude of the ground motion considerably. Such an uncertainty is unacceptable for design purposes. Here the engineer needs a trend-removal algorithm which yields a maximum ground displacement which is compatible with the provisions of Eurocode 8. Then histories of the ground displacements instead of accelera-

tions could be imposed and the numerical defect would disappear.

3. Numerical Procedure

A generated history is in two respects different from a recorded acceleration history. It is, as a sum of harmonic parts, per definition trend-free – no trends are introduced by measurement limitations. Also it is a wholly fictitious record. A response spectrum represents a response envelope to all possible earthquakes to be expected at a specific site. So the generated history can also be seen as a kind of envelope history.

The generation procedure produces first an autospectrum G_a from the given response acceleration spectrum S_a (Gasparini and Vanmarcke, 1976a) and generates an acceleration history for this autospectrum (Shinozuka, 1972). The double integration of the sum of harmonics can be done analytically. Then each harmonic gets divided by the square of the circular frequency ω . The autospectrum G_d of the resulting displacement history is identical to the autospectrum of the acceleration divided by ω^4 . Even though G_a approaches zero for $\omega \rightarrow 0$, G_d approaches infinity. This results in theoretically infinitely large quasi-static displacement components. In practice the displacement amplitudes are limited by the smallest frequency present in the history which in turn is related to the inverse of the sample length T_s . The longer the acceleration histories are, the larger the resulting displacement amplitudes, a clear indication of the randomness of the maximum displacement.

This defect in the displacement autospectrum has been recognized by Der Kiureghian and Neuenhofer in (Der Kiureghian and Neuenhofer, 1992), where they needed both G_a and G_d for their multi-correlated response spectrum method. They overcame this difficulty by multiplying G_a with a frequency-dependent filter function which reduced the spectral ordinates for low frequencies. The filter parameters were identified such that the area below G_d , i.e. the variance of the displacement history, corre-

sponds to a chosen peak factor, i.e. a given maximum displacement amplitude.

In the present paper this concept has been employed for the generation of time histories. A study of EC 8 revealed that, even though the code defines no displacement response spectra for frequencies below 0.25 Hz, it gives the maximum ground displacement for all soil conditions. Based on this target value, it is possible to scale G_a (which resulted from the target acceleration response spectrum) and generate acceleration histories for this modified autospectrum. The resulting acceleration histories are practically identical to the original histories so that the response spectrum from the acceleration histories still conforms to the target response spectrum. The displacement histories, however, are greatly affected in such a way that the maximum amplitudes are reduced to the desired level given by EC 8.

The generation tool thus produces both acceleration and displacement histories. The former are used to check their compliance with the compatibility requirements of EC 8 and scale them accordingly, the latter are scaled the same and then used for the time-domain simulation. Extensive numerical studies will be given in the full paper.

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EXPERIMENTAL ANALYSIS OF STABILITY OF VISCOUSLY DAMPED SPHERICAL PENDULUM

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1. Introduction

Engineering structures subjected to dynamic loading are often equipped with special devices proposed to suppress adverse motion. Various pendulum-based systems are very popular for their reliability and simple maintenance, see e.g. [Haxton]. However, these installations are frequently designed with certain stereotyped approach, considering only simplified model of a planar pendulum. This practice is often not sufficient, as it neglects some important features, for example the ability of the system to break the reflective symmetry and to move in non-planar direction. The article describes an experimental treatment of such system represented by the spherical, auto-parametric pendulum. It is a strongly non-linear system, with the excitation being applied in the suspension point and interpreted for instance as an excitation by periodic wind buffeting or vortex shedding. The experimental results are compared to the theoretical (semi-trivial) solution. The effect of stability loss and possible stability regain is shown in the experimental as well as in the non-linear theoretical model. An analytical, numerical and experimental study results in several recommendations for designers of these devices.

2. Theory

The spherical pendulum is considered as strongly non-linear dynamic system with kinematic external excitation in the suspension point, see Fig. 1. The double-degree of freedom mathematical model follows from the mechanical energy balance. Properties of the mathematical model are derived using the harmonic balance approach. The theoretical work is partly based on a classical paper of [Miles] and is described in [Naprstek].

3. Experimental Setup

Special experimental frame was developed. It contains a pendulum supported by the Cardan joint, see in Fig. 2. The moving support is made by means of trolley moving on two parallel rails. An electro-dynamic shaker excites kinematically the trolley. There are two magnetic units attached to the frame and to the supporting axes of rotation. They are able to reproduce linear viscous damping for both degrees of freedom. Movement of the pendulum was recorded in the both directions by a pair of identical high-speed rotary magnetic encoders. The stability of the system is analyzed experimentally and compared with theoretical results.

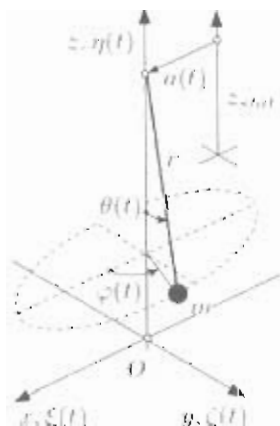


Figure 1: Theoretical model of the pendulum



Figure 2: Experimental model setup with two damping units

4. Results and discussion

Regarding the resonance curves, see Fig 3, there are several noticeable results:

1. The agreement of the numerical, theoretical and experimental values is rather good, especially for the frequencies below the critical region. It is interesting especially when taking into account, that the mathematical model has been derived using a "small amplitude" assumption.
2. The in-plane motion (ξ variable) shows different stationary amplitudes for "sweep up" and "sweep down" experiments. This fact corresponds well with the both stable branches of the theoretical resonance curve for the short pendulum. The experimental pendulum was able to follow the "unstable" part of the resonance curve under certain circumstances.
3. The transversal motion (ζ variable) of the pendulum should be usually initiated using a small initial impulse, though an "ambient" disturbance sometimes occurs. However, in the resonance region this initial impulse is amplified and the pendulum demonstrates the full spatial movement.

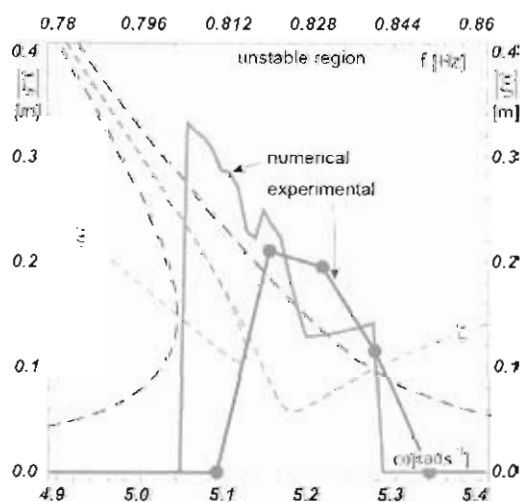


Figure 3: Experimental (solid w. bullets) and numerical (solid) resonance curves for the non-stationary transversal motion of the pendulum, together with the semi-trivial resonance curve (dashed) and ξ and ζ stability limit curves (dashed, labeled by ξ and ζ).

For the detailed theoretical analysis of the several types of the unstable resonance response of the pendulum was published before, see [Naprstek]. It is worth to emphasize, that the predicted types of response has been validated by the experiments.

5. Concluding remarks

Analytical, experimental and numerical investigation has shown that widely used linear model of the damping pendulum is acceptable only in a very limited extent of parameters. Two degrees of freedom non-linear model must be introduced. The harmonic kinematic external excitation in the suspension point has been applied both analytically and experimentally.

If the stability limits intersect the resonance curve, then four different response regimes can be consecutively encountered in the resonance domain. For large excitation amplitude and/or small damping can the instability interval occupy a relevant frequency domain. It implies a design recommendation that the damping should not drop below a certain limit to avoid the instability, which implies the decrease of the pendulum efficiency especially for broadband excitations.

Acknowledgements

The kind support of the Czech Scientific Foundation No. I03/09/0094, Grant Agency of the ASCR No. IAA200710902, IAA200710805 and AV 0720710524 research plan are gratefully acknowledged.

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TIME DOMAIN METHODS AND OBSERVER APPROACH FOR MODAL PARAMETER IDENTIFICATION

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1. Introduction

Linear time invariant (LTI) systems like linear elastic structures are characterized by modal parameters i.e. natural frequencies, natural modes and modal damping. For the given mathematical models there are number of well established procedures that calculate modal parameters (Ljung, 2006, Ewins, 2000). When the structure is given as real system, experiment is employed to detect modal parameters. The experiment comprises excitation of the structure, data acquisition, data processing and finally modal parameters extraction. During last decades a number of very sophisticated methods for modal parameters detection have been and still are being developed (Zhou, 2008). All these methods can be divided in two groups: frequency domain and time domain methods. The frequency domain methods are typically based on modal parameters detection from the frequency response functions (FRF) while time domain methods typically rely on impulse response. So we can conclude that the most of the methods detect the modal parameters in two steps: FRF or impulse response has to be calculated first and then modal parameter detection techniques applied.

2. Calculation of the Impulse response

The impulse response as free response is characterized by the modal parameters. It is related with FRF by the Fourier transform. However, besides Discrete Fourier Transform impulse response can be calculated from time series models using least squares (LS) approach or random decrement method. LS and random decrement do not require necessarily Fourier transform at any step. However, LS approach typically comprises input output time series while random decrement method is specially designed for impulse detection from ambient excitation when outputs are available only.

These methods have to introduce some assumptions in order to perform model detection. The most common assumption is zero order hold.

3. Time series and time domain methods

The common time domain methods are Ibrahim Time Domain (ITD), Least Square Complex Exponential (LSCE) and Eigensystem Realization Algorithm (ERA). All these methods detect modal parameters from the free response of the system:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0 \quad (1)$$

where $\mathbf{x}$ is displacement vector, $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are mass, damping and stiffness $N \times N$ matrices, respectively. Introducing new variable $\mathbf{w}$:

$$\mathbf{w} = \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} \quad (2)$$

the governing equation (1) can be transformed in new eigenvalue problem:

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} \quad (3)$$

where:

$$\mathbf{A} = \begin{bmatrix} [0] & [I] \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \quad (4)$$

The solution of eigenvalue problem results in $2N$ complex conjugate eigenvalues λ_r and corresponding eigenvectors ϕ_r .

The free structure response in discrete matrix form reads:

$$\mathbf{X}_0 = \begin{bmatrix} x_1(t_0) & x_1(t_1) & \dots & x_1(t_{2N-1}) \\ x_2(t_0) & x_2(t_1) & \dots & x_2(t_{2N-1}) \\ \dots & \dots & \dots & \dots \\ x_N(t_0) & x_N(t_1) & \dots & x_N(t_{2N-1}) \end{bmatrix}_{N \times 2N} \quad (5)$$

where $x_i(t_j)$ is output (acceleration/ velocity/ displacement) at measured dof i at time instant j . The ITM and LSCE methods accept these

outputs as starting point. The ERA method reduced to the single input multi output technique (SIMO) and providing that system output matrix collects all the outputs, can operate on the same data.

4. Experimental setup and modal parameters identification

The experiment time history outputs are post-processed and impulse/pulse response is obtained. The modal parameter identification using ITD, LSCE and ERA method based on given impulse are performed and compared, Figure 1 and Table 1.

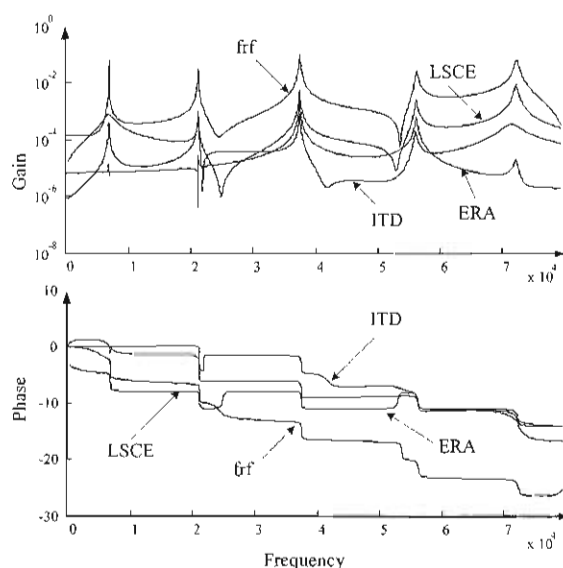


Figure 1. Fourier transform of the impulse and frfs of the identified system by the time domain methods involved

	1 st mode		2 nd mode		3 rd mode	
	ω_1	ξ_1	ω_2	ξ_2	ω_3	ξ_3
	s^{-1}	%	s^{-1}	%	s^{-1}	%
ITD	6.83	11.3	21.15	0.522	37.4	0.204
LSCE	6.82	0.819	21.14	0.243	37.4	0.205
ERA	68.2	0.034	21.14	0.105	37.4	0.187

Table 1. Identified modal parameters

The modal parameters are detected successfully by all applied identification methods. The good agreement between frequencies can be observed from Table 1.

However, discrepancy between damping ratios is significant.

5. Correlation of the experiment and numerical

The experiment and model response reconstruction discovers good correlation in frequency domain and bad correlation in time domain except for ERA. The ERA method has better correlation in time than in frequency domain.

6. Concluding remarks

The time domain methods: ITD, LSCE and ERA are implemented and compared. The comparative analysis in order to find the most suitable method for model validation and model update is performed. The special attention is paid to the identification tuning and its influence to the identified modal data.

Acknowledgements

The authors acknowledge the support to the Ministry of Science, Education and Sport of the Republic of Croatia, grant 0231744-1747.

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STRUCTURAL VIBRATIONS DUE TO HUMAN LOCOMOTION

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1. Introduction

The basic tendency in modern architecture to light-weight and slender design of pedestrian structures leads to an increased susceptibility to vibrations. Correspondingly, in recent years loads induced by human locomotion have gained more and more interest in the research community. However, many studies are based on inappropriate assumptions (e.g. loads induced by the right foot are identical to the left foot) or make use of inappropriate experimental setups as e.g. single force plates or treadmill devices (Kerr, 2001; Brownjohn, 2004) which leads to a loss of important information. Usually, the random features of the load process can not be identified or are simply ignored leading to biased load models. However, it has been shown in 2004 [Sahnaci & Kasperski, 2004] that the randomness and imperfections in the locomotion parameters lead to sub- and inter-mediate harmonic load contributions which even may become decisive in the structural design. Consequently, for the evaluation of safety and serviceability of pedestrian structures against vibrations due to human excitation, refined load models are required.

Basic aim of this study is to develop appropriate load models for both locomotion forms walking and running. Therefore, a sophisticated longer walkway has been constructed which allows to monitor a longer sequence of natural steps and thus to determine intra- and inter-individual differences. The paper analyses in detail the load traces obtained from extensive tests giving a special focus on the randomness of the basic locomotion parameters and the load contributions.

2. Experimental setup

To monitor the parameters of the locomotion processes natural walking and running the

Research Team EKIB has constructed a 28 m walkway, as shown in Figure 1. The core section is the active measuring section which is completely mounted on 16 vertical force cells. The data acquisition system uses a resolution of 16 Bit, the sampling frequency is set to 2400 Hz. These settings lead to a high information density of the loads which is required to identify the random imperfections in the locomotion parameters. To allow for monitoring only stationary locomotion conditions, i.e. to avoid biasing effects due to acceleration or deceleration processes, at both ends of the active section a 4 m lead-in and a 8 m lead-out section has been added. The width of the walkway has been chosen to 1.50 m.

Altogether 227 subjects (38 female and 189 male subjects) have participated in the tests performed at the halls of the department of mechanical engineering at the Ruhr-University Bochum. The basic test procedure for each subject is to cross the walkway two times by walking and two times by running with a deliberately chosen speed. Depending on the subjects individual step length, the number of monitored consecutive steps per crossing was 17 to 19 for walking and 13 to 16 steps for running.



Figure 1. 28m walkway with lead-in and lead-out section.

3. Analysis of the load patterns

The analysis of the monitored load patterns requires the identification of the individual phases. The locomotion form running can be described as alternating process between load phases occurring as single pulses and flight

phases where no loads are monitored. Compared to running, during the locomotion form walking the person is in permanent ground contact. The respective load pattern is made up of double-stance and single-stance phases which have to be identified separately. A typical load time history obtained from the walking tests is shown in Figure 2.

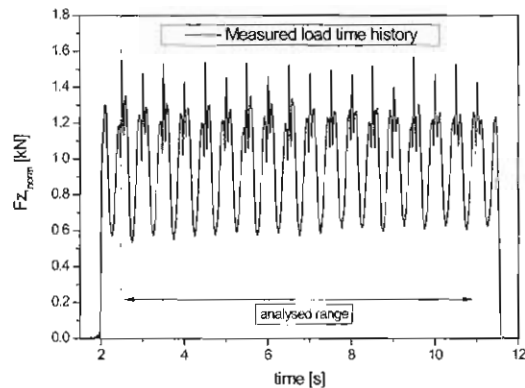


Figure 2. Typical load trace for walking (forces normalized by persons bodyweight)

The parameters obtained for each individual step are step frequency, step length, step width, double-stance and single stance phase (for walking), contact time and flight phase (for running) and the normalized harmonic load amplitudes. Since these parameters are usually different for female and male persons separate statistics are performed.

However, an appropriate load model has also to consider the differences in the parameters for the left and the right foot which lead to inter-mediate harmonic load contributions. The study, therefore, gives a special focus on the interdependencies between consecutive locomotion and load parameters. The findings later can be used to generate an artificial realistic locomotion pattern of unlimited duration for simulation purposes.

4. Results and discussion

The paper aims to develop probabilistic load models and gives recommendations on how to deal with the random scatter of the individual parameters and the interdependencies in consecutive steps. Basically, the quality of the models can be evaluated with structural

response calculations applying Newmark's step-by-step integration method. The responses obtained from the model should lead to similar maximum acceleration levels as obtained from the measured loads. However, an appropriate model also has to feature inter-mediate harmonic load content. This can be demonstrated on the example of the monitored walking patterns by performing structural response calculations based on a simple beam with variation of the natural frequency between 0.5 Hz and 10 Hz. In Figure 3 the maximum accelerations in terms of the non-exceedance probability are plotted revealing that considerable resonant excitation does not only occur for the integer harmonics, i.e. at 2 and 4 Hz and so on, but also for sub- and inter-mediate harmonics at 1, 3, 5 and 7 Hz.

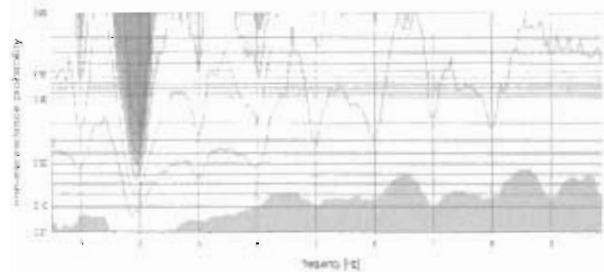


Figure 3. Maximum accelerations obtained from walking (forces normalized by persons bodyweight)

Acknowledgements

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FEM MODELS ON DYNAMIC STABILITY INVESTIGATION

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1. Introduction

Dynamic loads are extremely dangerous if they affect global or local stability of buildings or their structural elements. Small external excitations cannot produce any significant response unless the frequency of the excitation is close to one of the natural frequencies of the system. In the case of small parametric excitation large response can be produced even if the frequency of the excitation is far from the linear natural frequencies of the system. The fundamental dynamic instability problems were investigated in analytical manner by (Bolotin, 1964). The scope of this study was to investigate ability of applying the finite element method (FEM) for the numerical investigation of the dynamic instability problems. The main purpose for numerical analysis for the behavior of parametrically excited system is its application to more complex structural elements for which can't be obtained analytical solution. Due to comparison with analytical solution the simple beam structure was selected for model.

2. System considered

The absolutely elastic beam, having the length L and constant bending stiffness EI , was loaded with an axial compressive force $P(t) = P_0 + P_1 \cdot \cos 2\Omega t$ (Figure 1). The influence of this force is introduced into the model related to first critical buckling load P_c by the coefficient μ , replacing $P_1 = 2\mu \cdot (P_c - P_0)$. For the purpose of numerical analysis of a beam element's cross section the steel pipe profile with an outside diameter of 70 mm, the thickness of 4.5 mm and the beam length of 5 m is chosen. The values of stresses in the beam for the basic critical force P_c remain far below the yield point, so analysis was performed for the elastic oscillations of system. Dynamic response of this system is described with nonlinearly coupled second order differential equations

having periodical time varying coefficients, in which the excitation appears as a matrix coefficient of the displacement vector $\mathbf{u}(t)$ (Mathieu's type of equations).

To trigger the lateral vibrations of this undamped system the initial displacement of 1mm is given in the middle of the beam span. The numerical criterion of instability is the amplitude growth for triple amount of initial value in first 30 seconds of response.

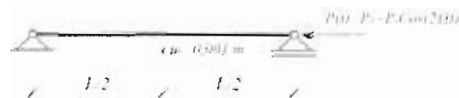


Figure 1. The model of beam element loaded with variable compressive force $P(t)$.

The numerical integration is executed by the special routine Runge-Kutta method carried out in the program language *Wolfram Mathematica 7.0*, to control the various parameters values.

The unstable response regions associated with the parametric resonance are determined considering the parameter of the compressive forces μ and the frequencies ratio Ω/ω_1 . Regions of dynamic instability determined by FEM models are shown Figure 2.

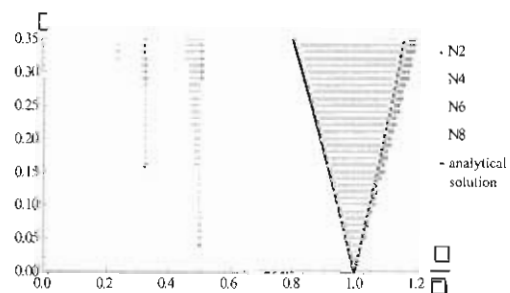


Figure 2. Overlap of the regions of dynamic instability different numbers of finite elements.

The contributions of higher modes shapes in system response are involved and that affects the regions of instability.

Plotting the response for different values of parameters we can see what is really happening

with the structure during harmonic parametric excitation, and how the chosen number of finite elements affects it.

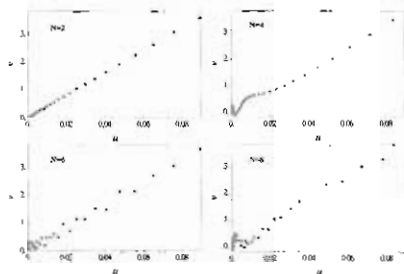


Figure 3. Poincaré maps for value of parameters $\Omega/\omega=1.00$ and $\mu=0.05$.

If $\Omega < \omega_1$ the number of finite elements doesn't have any influence on the instability boundary. In this part of region models have pure resonant response. All of them have continuous increase of amplitude, but if the system is modeled by increased number of finite elements it will need more time to reach the same limit amplitude. Resonant response of models considering more than one degree of freedom is disturbed by higher order shapes, as can be seen on the Poincaré maps (Figure 3) and phase portraits (Figure 4).

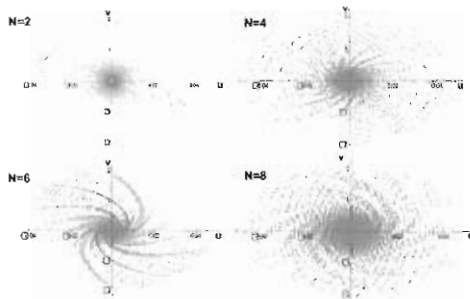


Figure 4. Phase portraits for value of parameters $\Omega/\omega=0.95$ and $\mu=0.10$.

On the opposite side, if $\Omega > \omega_1$, the unstable response boundary differs notably depending on number of finite elements in model, as we can see on Figure 2. Beam undergoes beating phenomena for parameters chosen from this section. Whether the response selection is going to be stable or unstable depends on numerical criterion. Plotted phase portraits for models that have more than one degree of freedom would indicate that system undergoes some kind of irregular behavior. Their Poincaré maps show that their responses are still of periodic nature (Figure 5).

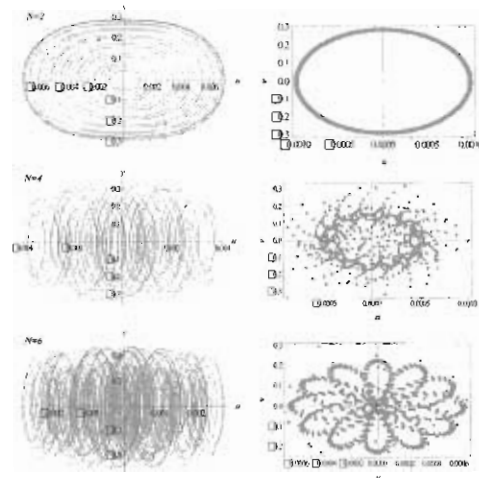


Figure 5. Phase portraits and Poincaré maps for value of parameters $\Omega/\omega=1.13$ and $\mu=0.26$.

3. Concluding remarks

The numerical investigation results indicate that the finite element models are convenient for dynamic instability problems investigation. The principal region of instability whose boundaries are determined analytically (black line on Figure 2.) agree very well with numerical results. Furthermore, even only two beam elements FEM model are quite enough to determine the unstable regions. Attention should be paid to the selection of numerical criterion for instability. The numerical criterion should not be selected arbitrarily because it influences both the duration of numerical calculation and stresses in the cross section.

Acknowledgements

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SEISMIC BEHAVIOR OF REINFORCED-CONCRETE FRAMES WITH INFILL DESIGNED FOR VERTICAL LOADS- FEM ANALYSIS

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1. Introduction

Reinforced concrete frames with masonry infill (infilled frames) are composite bearing system whose behaviour under seismic excitation is still not quantified and valorised. Influence of infill to the seismic response of such composite system in Eurocodes is not clearly defined and is usually disregarded for design of the new buildings and for evaluation of existing ones.

The presence of masonry infill significantly changes structural characteristics of frame buildings and create the new failure mechanisms. In this study we have numerically investigated the effects of different parameters on the seismic behaviour of not seismically designed masonry infilled frames by use of the nonlinear FEM software ATENA 2D ver. 4.3. Masonry infill constrains deformations of frame and increases its stiffness and strength especially in small deformations range. Reinforced concrete frames are flexible and could sustain large inelastic deformations that imply large damages. Insertion of an infill brings stiffness and strength thus improving the system behaviour and lowering potential damages provided that they are placed regularly throughout the structure and do not cause shear failures of columns. Not-seismically designed frames represent a special case that need particular attention as they represent the most of the present building stock in Southern Europe.

2. Numerical investigation

Numerical investigation of the influence of different parameters on the seismic behaviour of infilled not-seismically designed frames is presented. The parameters are chosen with aim of examining their influence on the bearing capacity and the ultimate failure mode.

Reinforced-concrete frames and masonry infill are usually divided into strong and weak

each. Masonry infill is classified into strong and weak according to its mechanical properties and reinforced-concrete frames according to the ratio of beam to column stiffness and strength. Influence of the shear capacity of columns is analysed by parameter of transverse reinforcement.

masonry infill parameters	
masonry units	
hollow clay tile (1)	
mortar	
lime (2)	lime-cement (3)
frame parameters	
dimension of the cross section of column	
25/30 cm (4)	25/50 cm (5)
amount of transverse reinforcement in column	
φ6/10 cm (6)	φ6/20 cm (7)
combination of parameters	
(4)+(6) - bare frame = S1	
(4)+(7) - bare frame = S2	
(5)+(6) - bare frame = S3	
(5)+(7) - bare frame = S4	
(1)+(3)+(4)+(6) = S5	
(1)+(3)+(4)+(7) = S6	
(1)+(2)+(4)+(6) = S7	
(1)+(2)+(4)+(7) = S8	
(1)+(3)+(5)+(6) = S9	
(1)+(3)+(5)+(7) = S10	
(1)+(2)+(5)+(6) = S11	
(1)+(2)+(5)+(7) = S12	

Table 1. Observed parameters and their combination.

The nonlinear FE micro modelling of one bay, one storey infilled reinforced-concrete frame was performed using the ATENA 2D version

4.3. Constitutive materials were described by discrete material models and the plane nine node isoparametric finite elements were used for modelling of concrete and masonry units. Reinforcement was modelled by bar elements taking into account reinforcement slip that was defined according to the CEB-FIP recommendation. Mortar and brick-mortar contact were modelled by interface elements. Values of material input parameters were defined according to the experimental results performed on samples and tested in our laboratory. In-plane pushover analysis was performed in order to obtain the failure modes (Figure 1.) and resistance envelope curves of the models.

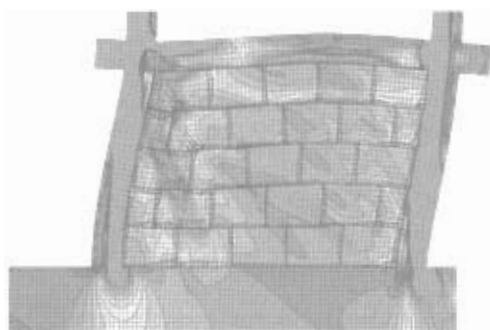


Figure 1. Stress distribution and deformed shape of infill frame FE model.

3. Results and discussion

Quantitative contribution of the parameters on the failure modes, bearing capacity, stiffness and energy dissipation of infilled reinforced-concrete frames could be observed from the results. Masonry infilled frames have higher stiffness, strength (S5 & S9 in Fig. 2) and energy dissipation, but their failure could be brittle (S5 in Fig. 2).

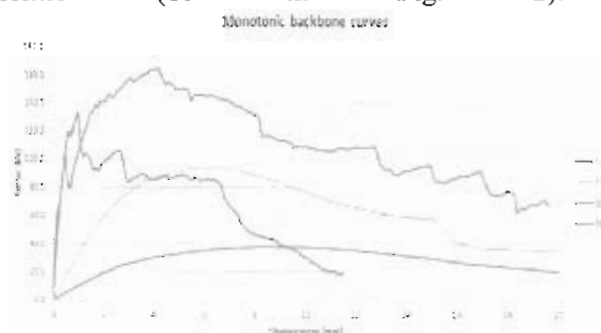


Figure 2. Response envelope curves

4. Concluding remarks

Reinforced-concrete frames with masonry infill could be modeled by FEM if the input material data and overall behavior were calibrated on the experimental results. Relative strength and stiffness of the elements influences failure mechanism and overall system behavior. For the failure type critical is the unbraced length of the column and if we could estimate the column's unbraced length we could make confident estimates of the brittleness and strength of the system. Within this work that has its experimental and analytical part we are trying to come to a plausible answer to this question.

Acknowledgements

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DESIGN OF GRANDSTANDS SUBJECT TO CROWD ACTION

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1. Introduction

The key issue to the evaluation of safety and serviceability of stand structures is crowd dynamics. Basically, the following limit states have to be considered for the design:

- the load amplitude of the active crowd exceeds the design value of the static load
- the crowd activities excite the structure to larger vibrations, the additional inertia forces exceed the design value of the assumed actions
- the vibrations lead to discomfort of people
- the vibrations lead to alarm and fear amongst the audience, which may result in a panic

At least in Germany, since many years well synchronized crowd activities form a normal pattern of the behaviour of football fans, and for many clubs rhythmic jumping today is almost the rule and not exceptional fan behaviour. However, the German Standard on imposed loads [1] gives no guidelines at all on how to design a stand structure in regard to the dynamic crowd behaviour. Quiet surprisingly, the committee members took the pragmatic view that stands are extraordinary structures which lay outside the scope of required codification.

The missing information in the Standard leads to the unsatisfactory situation, that each stand is designed differently depending on the source which has been used for the design. Unfortunately, there are guidelines, recommendations and Standards which present a fairly biased view on the inherent danger of an active crowd. The Danish Standard [2] simply suggests that stands with permanent seats leave not sufficient space for jumping. In the UK, the IstructE guidelines [3] simply state, that jumping can be related more to aerobics and vigorous dancing than to crowd behaviour in stadia.

Beside the missing load model, the Euro-code and the German Standard fail to specify limiting values of the vibration amplitudes in regard to comfort and safety of the audience. Appropriate serviceability criteria have to consider the respective bio-psychological aspects in human behaviour. Basically, fear stimuli can be sub-divided into two groups: learnt fears and innate fears. Fears can be reduced by learning, as can be seen on the example of using vehicles or aeroplanes. To some extent, vibrations are accepted as normal experience; however, above a specific threshold, the sudden movement of an aeroplane will cause fear for almost everyone. By no means, the danger of a possible panic can be removed by printing on the back of the entry ticket the advice '*the stand is designed to be safe*', as is recommended in the IstructE-document, which claims that '*Experience suggests that advance information ... reduces a tendency to panic*'.

2. Basic variables

Crowd behaviour is a random process, and perfect synchronisation is an obvious over-conservative assumption. The randomness can be separated into two groups: inter-individual and intra-individual randomness, i.e. the load time histories of a single jump will differ for different persons and will differ from jump to jump for an individual. Based on extensive laboratory tests, today sufficient information is available to simulate the loads induced by a dynamic crowd in the computer. Thereby, it is vital to distinguish four different groups of active persons in regard to their basic rhythmic abilities of meeting and keeping a triggering beat:

- best - the person meets the triggering beat on average and shows only small deviations from cycle to cycle

4.3. Constitutive materials were described by discrete material models and the plane nine node isoparametric finite elements were used for modelling of concrete and masonry units. Reinforcement was modelled by bar elements taking into account reinforcement slip that was defined according to the CEB-FIP recommendation. Mortar and brick-mortar contact were modelled by interface elements. Values of material input parameters were defined according to the experimental results performed on samples and tested in our laboratory. In-plane pushover analysis was performed in order to obtain the failure modes (Figure 1.) and resistance envelope curves of the models.

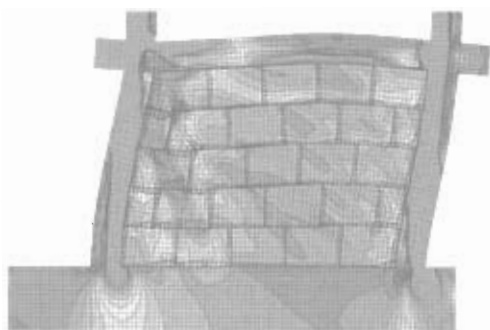


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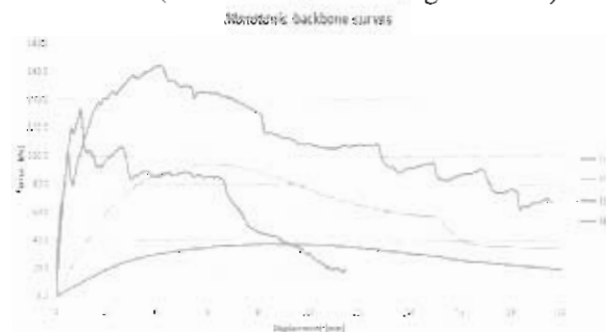


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- best - the person meets the triggering beat on average and shows only small deviations from cycle to cycle

- medium - the person almost meets the triggering beat on average, however, the deviations from cycle to cycle are larger
- bad-slow - the person performs the activity with a too low frequency
- bad-fast - the person performs the activity with a too high frequency

Since rhythmic abilities can be trained to some extent, it makes perfect sense to assume different distributions of the rhythmic abilities amongst the audience for different events. An international match with a more or less randomly mixed crowd will have more individuals with medium and bad rhythmic abilities than the crowd which permanently trains the group-exercise of jumping during the complete season.

A further basic variable is the body weight which is triggered by the gender. A conservative approach assumes only males in the crowd, since males have on average the larger body weight. The load time history of each individual jump can be re-modelled based on a sufficiently large number of harmonics and their corresponding phases.

3. Crowd factor

Based on the numerically generated load time histories, the structural response can be obtained in terms of accelerations by a simple step-by-step integration. The governing parameters are the ratio of the natural frequency to the jumping frequency and the size of the crowd. To obtain more general results it is reasonable to normalize the maximum acceleration for a specific scenario, e.g. to the average maximum acceleration induced by a single male person from the group 'best'.

Basically, the crowd factor decreases with increasing number of persons and increasing harmonic. However, there is a large scatter in the crowd-factor; and the question arises which value has to be used for the design.

A typical example of the randomness of the crowd-factor is shown in figure 1 for a group of 50 persons. All individuals are male and belong to the group 'best'. The median and the 95%-fractile value differ by a factor of 1.1, 1.2 and

1.3 for the first, second and third harmonic, respectively.

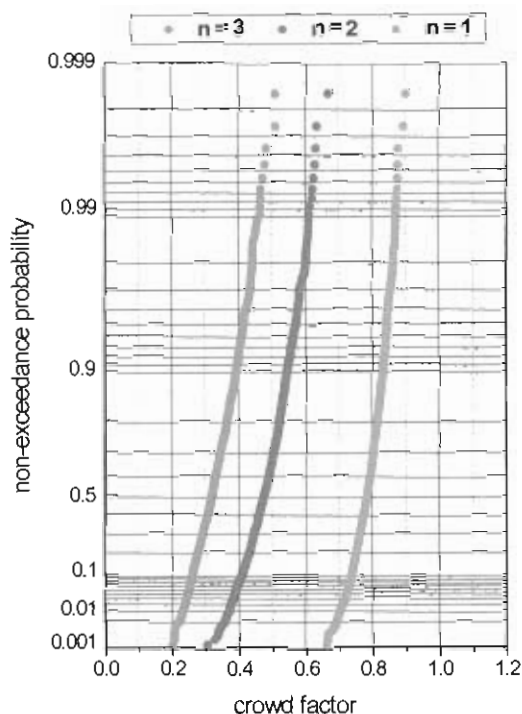


Figure 1. Randomness in the crowd factor for a group of 50 male persons with best basic rhythmic abilities.

For the specification of the appropriate target fractile value the number of independent rhythmic activities in the projected or design working life has to be considered. The full paper will discuss this item and will give a reasonable target value.

Acknowledgements

Part of this work has been sponsored by the German Research Foundation under the contract number KA675/10-1 and KA675/10-2. This support is gratefully acknowledged.

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ASSESSMENT OF STRESS CONCENTRATIONS IN LARGE CONTAINER SHIPS USING BEAM HYDROELASTIC MODEL

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1. Introduction

The increase in world trade has largely contributed to the expansion of sea traffic. As a result, the market demand is leading to Ultra Large Container Ships (ULCS), with expected capacity up to 18000 TEU and length about 400 m. The particular structural design of the container ships, leads to open midship sections, resulting in increased sensitivity to torsional and horizontal bending loads. At the same time, due to their large dimensions, the structural natural frequencies of ULCS become significantly lower so that the global hydroelastic structural responses (springing & whipping) can become a critical issue in the ship design and should be properly modelled by the available simulation tools since the present Classification Rules don't cover described operating stages completely.

The methodology of hydroelastic analysis includes the definition of the structural model, ship and cargo mass distributions, and geometrical model of ship surface. First, dry natural vibrations are calculated, and then modal hydrostatic stiffness, added mass, damping and wave load are determined. Finally, wet natural vibrations, as well as the transfer functions (RAO – response amplitude operator) for determining ship structural response to wave excitation, are obtained (Senjanovic et al., 2008a).

The hydroelastic problem can be solved at different levels of complexity and accuracy. The best, but highly time-consuming way is to consider 3D FEM structural model and 3D hydrodynamic model based on the radiation-diffraction theory. Such an approach is recommended only for the final strength analysis since it is time consuming. However, in this paper the beam model is coupled with 3D hydrodynamic model that is especially appropriate for the preliminary strength

assessment. After the global hydroelastic response is calculated in the frequency domain, the modal displacements have been imposed to the 3D FEM fine mesh substructure model and stress concentration RAOs in the considered structural detail is obtained. Both, the global and local hydroelastic responses are compared to those obtained by fully coupled 3D FEM + 3D BEM hydroelastic model.

2. Hydroelastic numerical model

Governing modal matrix differential equation for coupled ship motions and vibrations yields:

$$[\mathbf{k} + \mathbf{C} - i\omega(\mathbf{d} + \mathbf{FB}(\omega)) - \omega^2(\mathbf{m} + \mathbf{A}(\omega))] \boldsymbol{\xi} = \mathbf{F}$$

where $\mathbf{k}$, $\mathbf{d}$, and $\mathbf{m}$ are structural stiffness, damping and mass matrices, respectively, $\mathbf{C}$ is restoring stiffness, $\mathbf{B}(\omega)$ is hydrodynamic damping, $\mathbf{A}(\omega)$ is added mass, $\boldsymbol{\xi}$ is modal amplitudes, $\mathbf{F}$ is wave excitation and ω is encounter frequency. All quantities, except ω and $\boldsymbol{\xi}$, are related to the dry modes.

The structural part includes sophisticated beam model based on the advanced beam theory (Senjanovic et al., 2009). It takes into account shear influence on bending and torsion, contribution of transverse bulkheads to hull stiffness (Senjanovic et al. 2008b) as well as effective stiffness of relatively short engine room structure of ULCS (Senjanovic et al., 2010). The hydrodynamic part is based on the potential theory assumptions and it is described for zero advancing speed case as the simplest one in (Malenica et al., 2003). The hydrostatic part of the hydroelastic model includes consistent restoring stiffness formulation (Senjanovic et al., 2011).

3. Results

Transfer functions of horizontal bending moment and torsional moment obtained by beam and 3D hydroelastic model are shown in Figures 1 and 2, where very good agreement can be noticed. The fine mesh substructure

model is shown in Figure 3. Figure 4 shows the stress concentration transfer functions in the

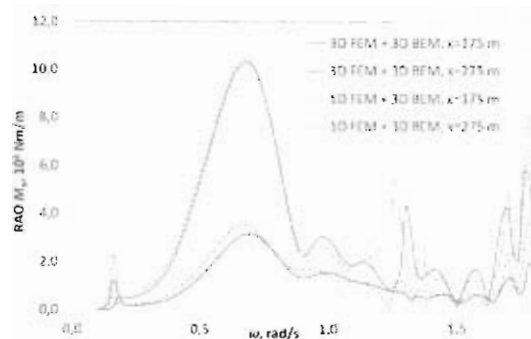


Figure 1. Transfer functions of horizontal bending moment, $\chi=120^\circ$, $U=15.75$ kn.

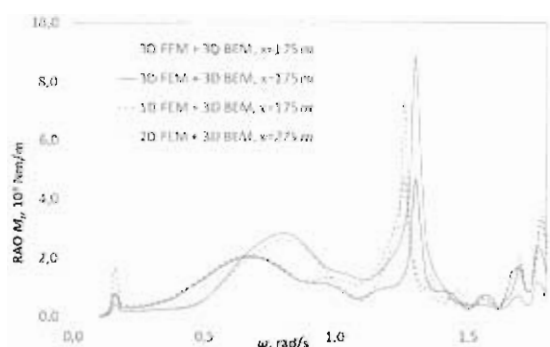


Figure 2. Transfer functions of torsional moment, $\chi=120^\circ$, $U=15.75$ kn.

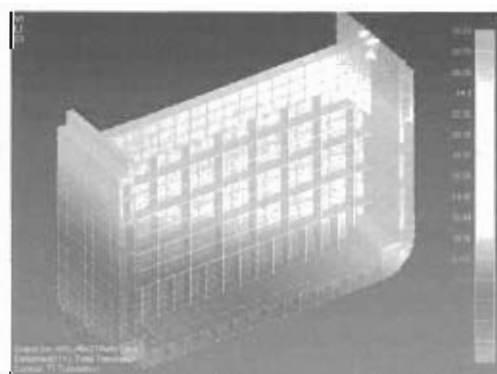


Figure 3. Deformed substructure model, $\omega=0.90$ rad/s, real component of the response.

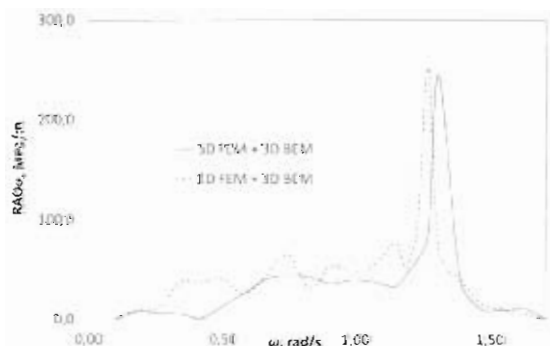


Figure 4. Transfer functions of stress concentrations.

hatch corner at the upper deck level in the middle part of the 11400 TEU container ship, with particulars presented in (Boutillier et al., 2010).

4. Conclusion

The hydroelastic model based on the sophisticated beam model, which is proven to be reliable for the global hydroelastic response calculations can be extended for the assessment of stress concentrations in combination with 3D FEM substructure model. Although, very good agreement in the high frequency range is obtained, some minor improvements in the low frequency domain are still necessary to increase the fatigue damage assessment accuracy.

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NUMERICAL INTEGRATION ALGORITHM IN LIE-GROUP SETTING FOR DYNAMICS OF MECHANICAL SYSTEMS

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1. Introduction

Design of the numerical integration methods that operate on manifolds and Lie-groups, instead of linear vector spaces like 'standard' ODE and DAE integrators, offer some attractive features such as numerical robustness and avoidance of the kinematical singularities as well as numerical efficiency of the code.

The possibility of using global rotational coordinates for kinematic definition of the system certainly appeals for numerous applications. The additional motivation for taking this approach can be based on the fact that computational geometric formulation provides integration schemes that can preserve global characteristics of motion (such as conservation of the first integrals) in 'more naturally' manner than 'classical' methods that operate in the vector spaces.

2. Geometric DAE integration procedure

The configuration space of mechanical multibody system (MBS) is modeled as $G = \mathcal{R}^3 \times \dots \times \mathcal{R}^3 \times SO(3) \times \dots \times SO(3)$, which is a n -dimensional manifold with Lie-group properties, consisting of translational and rotational kinematical domains of each rigid body in MBS.

The Lie-group composition operation $G \times G \rightarrow G$ is introduced by $p_{com} = p_1 \circ p_2$, where $p_1, p_2, p_{com} \in G$ and the identity element e of the group is defined as $p \circ e = e \circ p = p, \forall p \in G$. The Lie-algebra $\mathfrak{g} = T_e G$ (vector space that is isomorphic to $\mathcal{R}^n$) is defined as the tangent space $T_p G$ at the identity $p=e$. The tangent vector in $T_p G$ (at any

point $p \in G$) can be represented in Lie-algebra $\mathfrak{g}$ via derivation L'_p of the left translation map $L_p : G \rightarrow G, y \mapsto p \circ y$.

Thus, for $y=e$, we can define bijection $L'_p(e) : \mathfrak{g} \rightarrow T_p G, \tilde{\Omega} \mapsto L'_p(e) \cdot \tilde{\Omega}$, where $L'_p(e) \cdot \tilde{\Omega}$ is directional derivative of L_p at the point $y=e$ in direction of $\tilde{\Omega} \in \mathfrak{g}$ (since G is Lie-group, the element of Lie-algebra $\tilde{\Omega}$ defines left invariant vector field on G , similarly as it is the case with $SO(3)$).

To incorporate kinematical constraints of MBS, the function $\Phi : G \rightarrow \mathcal{R}^m$ are imposed on G , meaning that system is constrained to evolve on the $n-m$ dimensional sub-manifold $S = \{p \in G : \Phi(p) = 0\}$. Consequently, dynamic equations of MBS are shaped in the form

$$\begin{aligned} \mathbf{M}(p)\dot{\mathbf{v}} + \mathbf{Q}(p, \mathbf{v}, t) + \mathbf{C}^T(p)\boldsymbol{\lambda} &= 0 \\ \Phi(p) &= 0 \\ \dot{p} &= L'_p(e) \cdot \tilde{\mathbf{v}}, \end{aligned} \quad (1)$$

where $\mathbf{M}$ is $n \times n$ dimensional inertia matrix, $\mathbf{v} \in \mathcal{R}^n$ are system velocities, $\mathbf{Q}$ represents external and non-linear velocity forces, $\boldsymbol{\lambda} \in \mathcal{R}^m$ is the vector of Lagrangian multipliers and $\mathbf{C}$ is $m \times n$ dimensional constraint gradient matrix, such that $\Phi'(p) \cdot \tilde{\Omega} = \mathbf{C}(p)\boldsymbol{\Omega}, \forall \boldsymbol{\Omega} \in \mathcal{R}^n$ is valid.

The equation (1) represents DAE system of index 3. Within the framework of the proposed integration procedure, the equation (1) will be re-shaped into the DAE of index 1 form by

including kinematical constraints at the acceleration level $\ddot{\Phi}(p, \mathbf{v}, \dot{\mathbf{v}}) = 0$ (instead of $\Phi(p) = 0$) and integrated by the routine based on the Munthe-Kaas algorithm.

The constraint violation stabilization algorithm at the generalized position and velocity level will be introduced by using two different algorithms: first one that operates directly on the 'state-space' manifold and one that uses Cartesian rotation vectors as local coordinates for the generalized positions.

3. Results and discussion

Described Lie-group numerical integration algorithm for dynamics of constrained multibody system will be applied for numerical simulation of the satellite rotational dynamics.

The system constraints are introduced in the mathematical model *via* DAE of index 1 formulation, which is modelled and adjusted in accordance to the method's Lie-group setting. Proposed algorithm operates directly on the Lie-group of the system state space, avoiding thus kinematical singularities of the local rotation coordinates at the generalized position level.

In this way, integration algorithm 'comprehend' non-linear 'curved' manifold-structure of the system configuration space, immanently satisfying certain kinematic constraints and avoiding singularities that are always present within the vector space formulations of the large domain 3D rotation coordinates.

Furthermore, by following this route, more robust and numerically efficient computational model that do not need domain re-parameterization during numerical integration (that is common procedure within the framework of the 'standard' vector spaces numerical integrators) is synthesized.

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The first and second author acknowledge the support of the Croatian Science Foundation under the contract of the project 'Geometric

Numerical Integrators on Manifolds for Dynamic Analysis and Simulation of Structural Systems' that is conducted at Chair of Flight Vehicle Dynamics, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb.

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APPLICATION OF THE STEADY STATE MODEL FOR FLIGHT PATH PLANNING OF THE FIGHTER AIRPLANE INCLUDING TAKE-OFF AND LANDING

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1. Introduction

Navigational flight planning is an essential element for safe and successful flight, and it is presented by several data: flight route, flight velocity, altitude, heading and duration between each point on the route. Beside these, very important data is the amount of the fuel consumed, especially for the fighter airplane. Diversity of the flight missions, possibly significant restrictions, like available amount of fuel or flight restricted zones, along with the possibility of route modifications during the flight makes a flight planning for the fighter airplane interesting application. An access to the optimal flight regimes and flight limitations, like maximum available range or endurance, would present valuable information during the flight.

Methods applied for the flight path planning as a part of navigational flight planning basically could be defined as one based on the flight manual with given fuel consumption tables and one based on mathematical models. The later could range from the application of simple analytical performance calculations, total energy analysis, to the application of complex non-linear six degrees of freedom models of aircraft motion as for example used in the flight simulators. The later would give most detailed insight into the planned flight but it would also require access to the flight simulator and significant amount of time required for the analysis of the planned flight. On the other hand simple performance calculations can hardly comprehend diversity of the flight conditions in different flight phases, in particular take-off and landing, climb and descent. Both mentioned methods have limitations regarding the variation of the flight route and analysis of the available optimal

regimes along the planned route. For the application of flight path planning a mathematical model of aircraft's motion in steady state flight is applied in this paper. The steady state model assumes steady, trimmed flight along the flight trajectory. This model is capable to accurately define aircraft's trajectory describing the motion of its centre of gravity. It presents modified point mass model of the aircraft capable to describe all possible phases of flight and here it is extended to include take-off and landing.

Flight model presented in this paper is applied for flight path planning of the MiG-21, an old generation fighter airplane for which no data from the manufacturer apart from the flight and technical manuals were available. In practice at Croatian Air Force, navigational flight planning for this aircraft is preformed by its flight manual and results of this operational approach are compared to the results of steady state model.

2. Mathematical model of aircraft's motion

Steady, trimmed flight assumes equilibrium of the moments acting on the aircraft during the flight. Essential equations of the steady state model are differential equations of the velocity applied in the velocity vector axis system (Vinh, 1972, 1993). Whole system of differential equations defines change of velocity, flight path angle, heading angle, coordinates, mass and range. Due to the fact that during the take-off and landing there is important phase of rotation around the main landing gear, additional equations for pitch rate and pitch angle are introduced (Janković, 2001). For this phase of flight in the velocity equation an additional term due to the ground reaction force has to be included.

Since flight routes of interest are within small area, rotation of the Earth is ignored and flat Earth is considered. Wind is ignored due to the practical reasons of inability to model or predict wind velocity along the trajectory. Furthermore, operational utilization of the fighter airplane considered in this paper demonstrates small influence of the wind on the navigational flight planning, except for take-off and in particular for landing.

Data required for this model are aerodynamic gradients for drag, lift and pitch moment, engine characteristics for available thrust and specific fuel consumption, and inertia about the pitch axis. For the case of considered MiG-21 fighter airplane aerodynamic model is dependent on Mach number and considered airplane configuration, while engine model is depended on flight altitude, Mach number and applied regime. These data were estimated and then verified with respect to technical manuals. Its original successful application was for the flight simulator in operation at the Croatian Air Force and Defense.

3. Results and discussion

Described flight model is applied for the analysis of flight route for MiG-21 fighter airplane (Zovak, 2011). For the typical mission of MiG-21 fighter airplane (Fig.1), following specific flight procedures, results of the presented steady state model (Fig.2) are satisfactory in comparison to the flight manual data and results of analytical performance calculation.

4. Concluding remarks

Described steady state model presents modified point mass model extended in this paper for take-off and landing. Following the abilities of the presented model and its results, it can be concluded that its application for operational flight path planning is possible and it would present efficiency improvement in comparison to current navigational flight planning procedures at the Croatian Air Force based on flight manual.

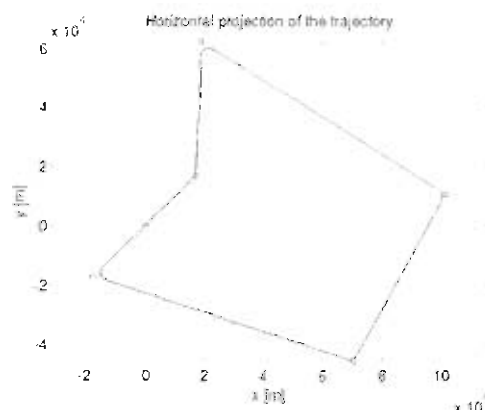


Figure 1. Horizontal projection of the trajectory with circles as navigational points of given route

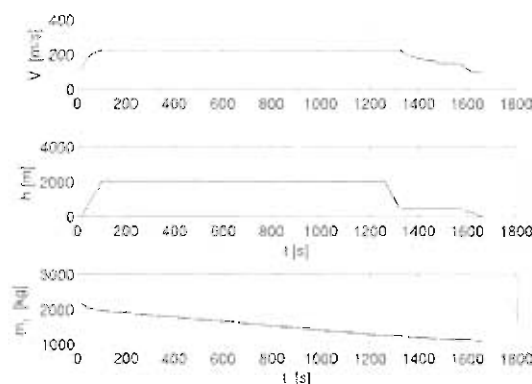


Figure 2. Results for planned flight (presented here without take-off and landing): flight speed, altitude and amount of the fuel consumed.

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ENVIRONMENTAL FLUID DYNAMICS

COMPUTATIONAL SIMULATION OF WIND FLOW BEHIND A RECTANGULAR BUILDING

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1. Introduction

A computational model has been developed to determine wind characteristics in the wake of a rectangular building. A width of the recirculation zone behind the building has been determined using the Reynolds-Averaged-Navier-Stokes (RANS) equations. Tests were performed for rural, suburban, and urban type terrains.

2. Numerical Setup

Numerical simulations were carried out using the commercial CFD code Fluent 6.2. Investigated building geometry corresponds to Mavroidis and Griffiths (2009), who exposed a rectangular building with length $a = 10.96$ m, width $b = 4.24$ m and height $c = 1.56$ m to the atmospheric boundary layer flow. In this study, tests were performed for wind direction normal to the longer, as well as to the shorter building side.

Turbulence intensity and integral length scale profiles at the inlet of the computational domain were set according to specifications given in ESDU 85020 (1985) for the rural, suburban, and urban type terrain, i.e. for the aerodynamic surface roughness length $z_0 = 0.001$ m, $z_0 = 0.01$ m, $z_0 = 0.1$ m, respectively.

In each configuration, wake dimension behind the building was determined as the largest closed vorticity isoline.

3. Results

Mean velocity profiles corresponding to rural, suburban, and urban type terrains according to ESDU 85020 (1985), which were set at the inlet of the computational domain, modified along the domain, as depicted in mean velocity profiles given at the building position,

Figure 1. At this position, mean velocity profiles, normalized using the corresponding reference velocity recorded at the building height, show little difference to each other. However, differences in turbulence kinetic energy profiles k , reported for various terrains in Figure 2, are expected to predominantly determine wake dimensions behind the building, as e.g. Castro and Robins (1977) demonstrated that the size of the cavity region is reduced by the presence of upstream turbulence.

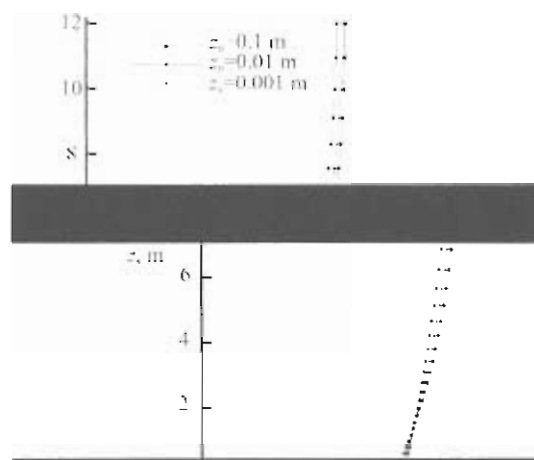


Figure 1. Mean velocity profiles at the building position observed for various terrains

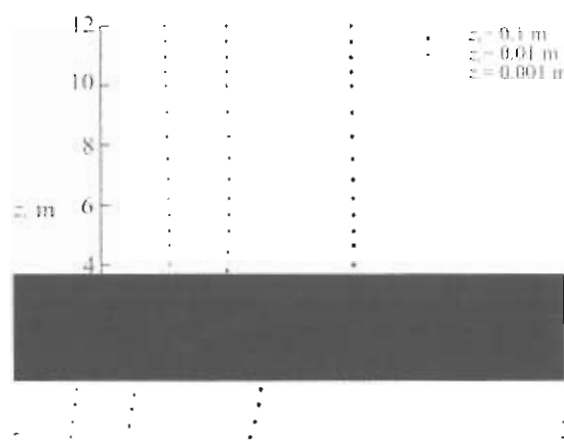


Figure 2. Turbulence kinetic energy profiles at the building position observed for various terrains

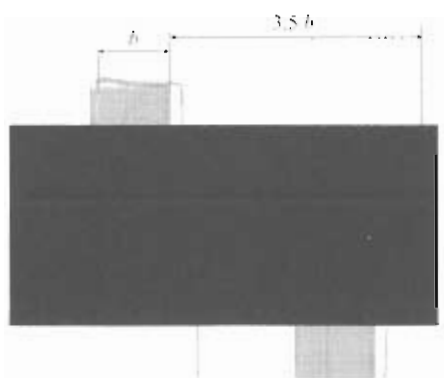


Figure 3. Wake dimensions for aerodynamic surface roughness length $z_0=0.001$ m

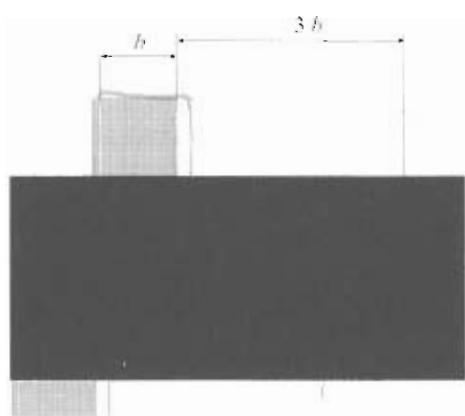


Figure 4. Wake dimensions for aerodynamic surface roughness length $z_0=0.01$ m

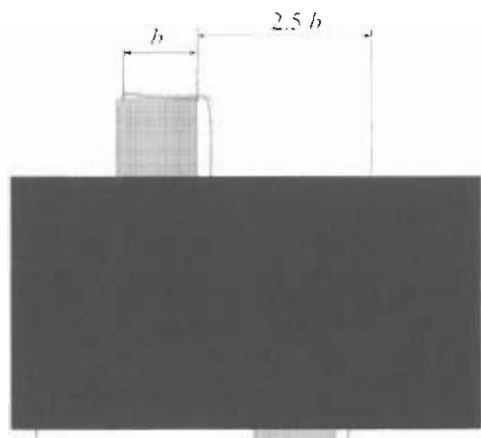


Figure 5. Wake dimensions for aerodynamic surface roughness length $z_0=0.1$ m

Computational results reported in Figures 3, 4, and 5 for wind normal to the longer building side indicate a reduced wake length behind the building with an increase in terrain roughness, i.e. for stronger wind turbulence, which is in agreement with trends presented by Castro and Robins (1977). Similar trends were observed for wind normal to the shorter building side.

4. Concluding remarks

A computational model has been developed to determine wind characteristics in the wake of a rectangular building for rural, suburban, and urban terrain exposure. A width of the recirculation zone behind the building has been determined. The computational results compared to wind-tunnel experiments indicate a reduced wake length behind the building for rougher terrains, i.e. for stronger wind turbulence in the oncoming flow.

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EVALUATION OF TURBULENCE FLUXES IN THE NIGHTTIME STABLE BOUNDARY LAYER IN KUTINA, CROATIA

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1. Introduction

Evaluation of turbulent fluxes in the surface layer is important for characterization of the atmospheric boundary layer (ABL). The turbulent heat flux in the surface layer is significant for the surface energy balance, while momentum turbulent flux is responsible for production of turbulence in stably stratified boundary layer. Additionally, turbulence, and specifically, turbulent fluxes strongly affect the fate of pollutants in the boundary layer. In order to investigate turbulent characteristics, it is necessary to separate turbulent perturbations from the mean flow. Turbulent and synoptic scales are typically separated by the spectral energy gap, which is often found at the mesoscale.

The aim of this study is to investigate turbulent momentum and heat fluxes in the vicinity of the petrochemical industry plant. Here we focus on nighttime conditions, which are generally favorable for the establishment of pollution episodes.

In order to determine turbulent quantities from fine-resolution wind measurements, it is necessary to assess averaging time appropriate for the turbulent timescales. Therefore, here we also address to the problem of proper selection of averaging time interval.

2. Experimental Setup

Wind data were measured by five ultrasonic anemometers with a sampling frequency of 20 Hz. Anemometers were placed at five heights, namely, 20, 32, 40, 55 and 62 m above the ground. The meteorological mast is located in a suburban, industrial area of Kutina, Croatia. The mast is above the grassy surface, and it is surrounded by approximately 20 m high trees, where the closest trees are about 20 – 30 meters distant.

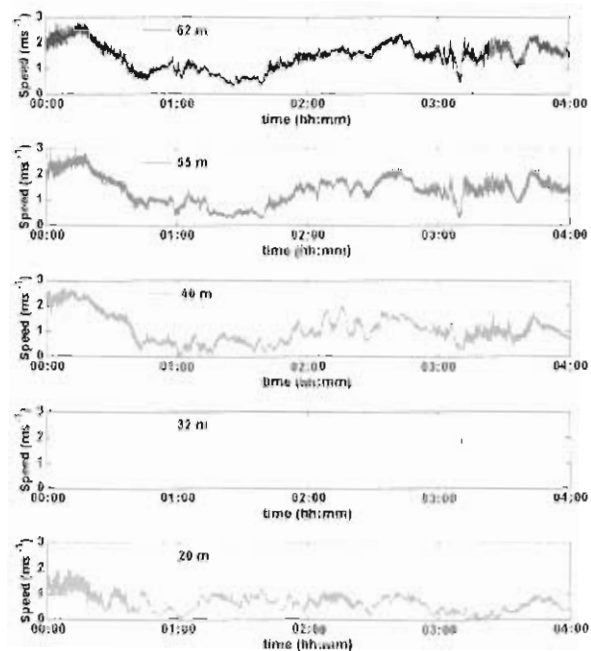


Figure 1. Mean wind at 5 levels on 22 December 2008 from 00 to 04 LST

3. Methods

Lengths of averaging interval are determined by two methods based on Fourier analysis. The first method (Metzger and Holmes, 2008) takes the length of interval as an inverse of frequency at which a minimum in the velocity spectrum is found. The second method (ogive) uses cumulative integral of cospectrum of perturbances of any two turbulent quantities (Oncley et al., 1996). Here we employ perturbations of wind components and/or sonic temperatures. Afterwards, the averaging interval is taken as a value at which ogive converges to a constant.

4. Results and discussion

Figure 1 shows mean wind variations at five levels for one case of the nighttime, stable

boundary layer, while Fig. 2 illustrates mean power spectrums of u -components for 20 and 62 m AGL.

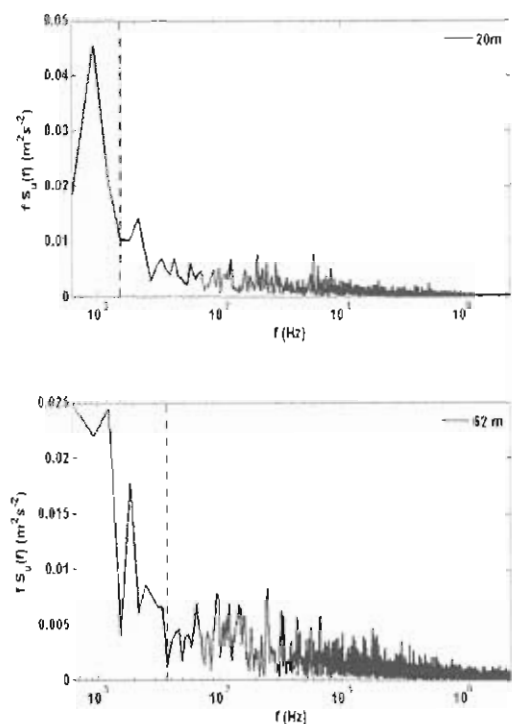


Figure 2. Mean power spectrum of u - component of the wind at 20 (top) and 62 m (bottom) above the ground level for 22 December 2008 from 00 to 04 LST. Vertical dashed-dotted line shows spectral energy gap.

Results of the Fourier analysis of the mean power spectrum of the u - component (first method) suggest averaging interval between 4 and 11 minutes. These results are in accordance with Vickers and Mahrt (2003) who showed that averaging time scale increase with instability due to the large convective eddies, while for the stable situation averaging time scale decreases because of the suppression of the large eddies by stratification. They found an appropriate averaging time scale of 9 min for neutral conditions, which decreased with increasing stability to 100 s for strong stability. On the other hand, based on the results of the second method (ogive), one cannot conclude on the length of the appropriate averaging time interval. However, further investigation, based on the longer input dataset, is needed.

Acknowledgements

This work has been supported by the Ministry of Science, Educational and Sport (Air Quality over a Complex Topography project No. 119-1193086-1323).

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EXPERIMENTAL MECHANICS

WIRELESS TIRE PRESSURE MONITORING BY EMPLOYING KINETIC ENERGY HARVESTING

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1. Introduction

Energy harvesting is the process of collecting low level ambient energy and converting it into electric power. Harvesting of kinetic energy of vibrations via piezoelectric bimorphs is advantageous as it is characterised by design simplicity, miniaturization and integration potential and high power densities. A metallic cantilever covered with piezoelectric material is thus placed in a vibrating environment. Charge is hence accumulated at the piezoelectric surface and used to power electric loads. A special interest in these devices is evident in the development of autonomous sensor nodes for ubiquitous wireless networks (Priya and Inman, 2009). On the other hand, tire pressure monitoring (TPM) has been proven to be extremely important when considering passenger safety. Correct tire pressures save lives and drastically improve tire lifetimes (Álvarez et al., 2008). Taking, however, into consideration the recent USA and EU acts, which impose TPMs in all new vehicles, a total of 10 batteries are to be disposed into the environment in a car lifetime. In this work an original solution of a wireless TPM system powered by a piezoelectric bimorph harvester is presented.

2. Piezoelectric energy harvesters

In order to have a quick yet accurate tool for tuning the response of a piezoelectric harvester, a modal model of the uncoupled mechanical behaviour of a tip-loaded cantilever is considered (Figure 1). Taking into account the Euler-Bernoulli beam theory and following the modal expansion method (Genta, 1998), the uncoupled eigenfrequencies of the cantilever are thus obtained.

The coupled electromechanical behaviour of the studied devices can be modelled following the recently introduced model (Erturk and Inman, 2009). For a vibration frequency ω close

to the eigenfrequency ω_n and a downstream resistive load R_L , the Frequency response function (FRF) voltage output α_v can hence be expressed, in terms of the electromechanical parameters κ_n , σ_n , C_p and χ_n , as:

$$\alpha_v(\omega) = \frac{\sum_{n=1}^{\infty} \frac{j\omega\kappa_n\sigma_n}{\omega_n^2 - \omega^2 + 2j\zeta\omega_n\omega}}{\frac{1}{R_L} + j\omega\frac{C_p}{2} + \sum_{n=1}^{\infty} \frac{j\omega\kappa_n\chi_n}{\omega_n^2 - \omega^2 + 2j\zeta\omega_n\omega}} \quad (1)$$

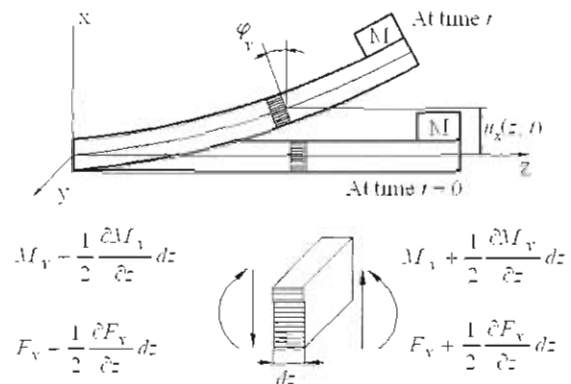


Figure 1. Uncoupled modal model.

The FRFs of output voltages (Figure 2) and powers of commercially available piezoelectric bimorphs, obtained on a suitable experimental set-up, correspond well to the coupled model results (Zelenika and Blažević, 2010). Due to electromechanical coupling, an increase of electrical loads has hence a nonlinear hardening effect on the dynamic behaviour.

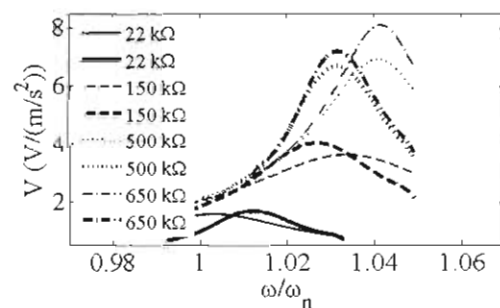


Figure 2. Experimental (thick) and coupled model (thin lines) voltage FRFs for $R_L = 22 \dots 650 \text{ k}\Omega$.

3. Wireless Tire Pressure Monitoring

Vibration as energy source is abundant around motor vehicles but on-road tests allowed establishing that its amplitudes and frequencies are far from the regular harmonic conditions considered in the design of piezoelectric scavenging devices – see Figure 3.

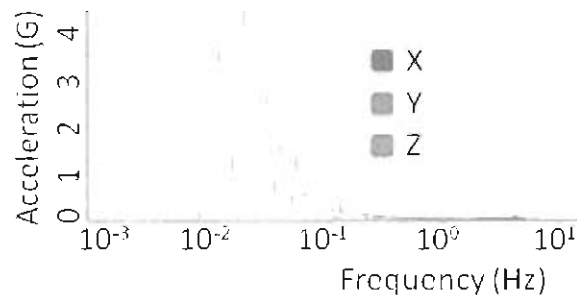


Figure 3. FFT diagrams of measurements on a car tire in open road conditions at 70 km/h.

Based on all of the above, an original Wireless Tire Pressure Monitoring System (WTPMS) was developed. It consists of a properly designed miniature piezoelectric bimorph with matched impedance; an off-the-shelf energy harvesting power supply chip (Linear Technology, 2011) that interfaces to the piezoelectric source, conditions and stores the produced energy and drives via a capacitor the pressure sensor; and a FreeScale MPXY8300 TPM module comprising a capacitive pressure sensor, a microcontroller and an RF transmitter (FreeScale, 2009). At specific time intervals, sensor data is hence accessed, analyzed and placed into the buffer for wireless transmission.



Figure 4. WTPMS device mounted on the car tire rim.

4. Obtained results

Experiments on the WTPMS were carried out on the set-up used for bimorphs' dynamic characterization. Tire pressure data could be

successfully acquired and transmitted to the receiver proving thus the concept in laboratory conditions. By designing suitable interfaces, the WTPMS was mounted then on a tire rim (Figure 4). About 6.5 mW were obtained in open road conditions, allowing the WTPMS to be powered while acquiring pressure data and transmitting it to the receiver in the car cockpit.

5. Concluding remarks

The uncoupled and coupled behaviour of piezoelectric scavengers was thoroughly studied. This has allowed integrating such devices into an innovative WTPMS module. Its full functionality was proven both in laboratory and in real on-road settings. The extension of the concept to a commercial solution that could be applied also in other fields (e.g. human or structural health monitoring) is being pursued.

Acknowledgements

The work was supported by the MSES project *Ultra-high precision compliant devices for micro and nanotechnology applications* and the BICRO project *Wireless autonomous tire pressure sensor – BAST*.

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INFLUENCE OF BOUNDARY CONDITIONS ON THE STATE OF STRESS AND STRAIN IN THE PLATE ARBITRARY FORM

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1. Introduction

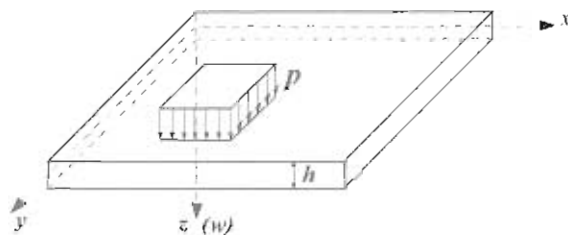


Figure 1. Plate loaded with transversal load

The equation for the plate in a rectangular coordinate system is shown in equation (1):

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{K} \quad (1)$$

The flexural rigidity of the plate

$$K = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)} \quad (2)$$

The solution of the equation (1) in closed form can be obtained only for plates of regular shape. For more complex forms of plate, equation (1) requires the numerical and experimental methods. In construction structures, some of the bearing members were plates of different shapes and different boundary relying conditions. This paper discussed the numerical and experimental procedure L-shaped plates for different boundary conditions and relying scheme loads to determine the influence of boundary conditions on the state of stress and strain in the plate.

2. Experimental Setup

Researches were conducted on L-shaped plates:

- Two adjacent edges are clamped, and other edges are completely free, Model "A", Figure 2.
 - One edge is clamped, and two opposite edges are simply supported, Model "B", Figure 3.
- Investigations were carried out for two load schemes: I scheme of the load plate with two concentrated forces $F_1 = 0,4625kN$, a scheme

II loading with a concentrated force $F_2 = 1,575kN$. The model plate is made of aluminium. Plate dimensions and dispositions of measurement points are shown in Figures 2 and 3.

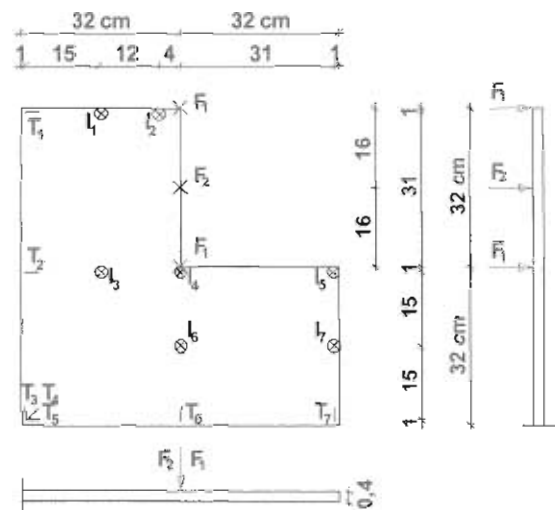


Figure 2. Model plate "A". Disposition of measurement points and load scheme

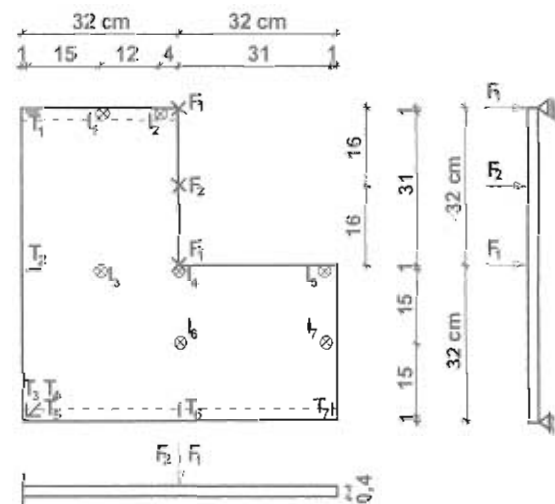


Figure 3. Model plate "B". Disposition of measurement points and load scheme

The relative deformation was measured by resistance strain gauges (T) and displacements (deflections) were measured by inductive strain gauges (I). Testing of aluminum samples' prescribed dimension is determined by the

modulus of elasticity $E=0,66 \cdot 10^5$ MPa and Poisson's ratio $\nu=0,30$. An experimental testing was conducted of L-plate model "A", Figure 2 for I and II loading scheme, Figure 4 and 5.



Figure 4. Model "A" plate loaded with I load scheme

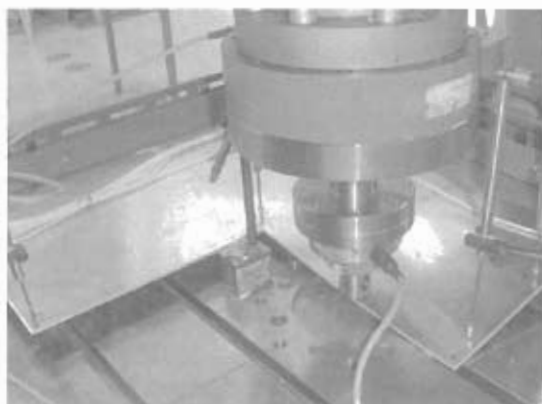


Figure 5. Model "A" plate loaded with II load scheme

Finite element method, the calculation of the model's plate "A" and "B" is in a computer program SAP 2000 V14.

3. Results and discussion

In Tables 1 and 2 are shown a comparison of deflection and stresses obtained by FEM and experiment for the Model "A" and "B" loaded with I load scheme.

4. Concluding remarks

Based on the analysis of the results obtained for Model "A" we can conclude on the good match of results of experimental research with theoretical results obtained by finite element method (FEM). The differences in the stresses results obtained by FEM and experimentally on

the measuring points T_2 and T_6 , placed in the similar disposition, occurred due to non-ideal execution of plates' border fixity on the area of above mentioned measuring places.

Model "A" - I load scheme			Model "B" - I load scheme
Measurement place	FEM	The experiment	FEM
Deflection in mm			
I1	17,4839	20,5599	0,00
I2	43,9697	40,8469	0,00
I3	6,0478	5,99	4,2283
I4	16,7678	12,8427	12,9362
I5	14,0482	13,5068	20,5271
I6	4,3075	5,6891	6,9392
I7	5,1251	5,224	10,8947

Table 1. Comparison of deflection results for Models "A" and "B" loaded with I load scheme

Model "A" - I load scheme			Model "B" - I load scheme
Measurement place	FEM	The experiment	FEM
Stresses in MPa			
T1	219,97	212,94	0,00
T2	78,53	37,14	60,14
T3	-0,09	-0,09	1,09
T4	-0,26	-1,25	2,62
T5	-0,09	-0,09	2,34
T6	42,21	60,82	-0,19
T7	68,36	63,45	-0,48

Table 2. Comparison of results of stress for models "A" and "B" loaded with I load scheme

For the model plate "B" we obtained a better state of stress and strain compared to the stresses and strains in the model plate "A". It can be applied in design of construction structures.

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EXPERIMENTAL DETECTION OF ADDED MASS OF THE BEAM PARTIALLY IMMERSSED IN A FLUID USING THE SYSTEM PROPERTIES AND EXPANDED MODE SHAPE

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1. Introduction

The behaviour of the structure partially immersed in a fluid is the subject of research for many years. The first studies (e.g. Lidholm 1965) have begun to explore changes in the natural frequencies of beam due to immersion in fluid. Most studies which involve the experiment and added mass were based on the detection of modal added mass (Rodriguez et al., 2006, Liang, 2007).

The scope of this experimental study is to determine the local added mass. This is the amount of mass that should be added to the beam in order to get the same effect as in the fluid. This mass is added only to the submerged part of the beam. The method applied in this paper is based on the detection of structural changes (Ge, 2005) using the finite element model. First, the method is applied to numerical simulation and then the model is validated, and at the end the method is applied to the experimental model in order to obtain local added mass from the measured data.

2. Mathematical model and numerical simulation

The well known equation which can describe the discretized undamped dynamic structure is written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \quad (1)$$

where $\mathbf{M}$ is the structural mass matrix, $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{f}$ is the applied force vector and $\mathbf{x}$ is nodal displacement vector. In theoretical terms, the vibrating structure that is submerged in water is commonly regarded as a fluid-structure interaction problem. In such problems, the structural dynamic equation has to be coupled with the fluid equations. But for a small vibration when the influence of structure

motion on flow field is very small therefore extended due to the influence of the surrounding fluid the problem can be described by equation (1)

$$(\mathbf{M} + \mathbf{M}_a)\ddot{\mathbf{x}} + (\mathbf{K} + \mathbf{K}_a)\mathbf{x} = \mathbf{f} \quad (2)$$

The matrices $\mathbf{M}_a$ and $\mathbf{K}_a$ are the mass and stiffness added to the existing structure matrices due to the immersion in a fluid. The added mass is a portion of the surrounding fluid that is accelerated as though it were rigidly attached to the structure. The added stiffness in most cases can be neglected. It is important only in special cases when it considering floating structures.

Under free vibration eigenvalue equations can be derived from equations (1) and (2). In the case of equation (2) corresponding eigenvalue equation can be written as

$$((\mathbf{K} + \mathbf{K}_a) - \omega_{ar}^2(\mathbf{M} + \mathbf{M}_a))\boldsymbol{\varphi}_{ar} = \mathbf{0} \quad (3)$$

where ω_{ar} is r th natural frequency and $\boldsymbol{\varphi}_{ar}$ is vector of r th mode shape for the structure immersed in a fluid. Upon expansion of equation (3) and neglecting higher order members, equation (3) can be simplified to

$$\boldsymbol{\varphi}_{ar}^T \mathbf{K}_a \boldsymbol{\varphi}_{ar} - \omega_{ar}^2 \boldsymbol{\varphi}_{ar}^T \mathbf{M}_a \boldsymbol{\varphi}_{ar} = \Delta\omega_r^2 \quad (4)$$

where the modal parameters are related to the structure without fluid, and $\Delta\omega_r$ is difference of natural frequencies for the structure with and without the surrounding fluid. If the members $\mathbf{M}_a$ and $\mathbf{K}_a$ in equation (4) are expressed as a function of the finite element matrices $\mathbf{M}^e$ and $\mathbf{K}^e$ of the structure without fluid

$$\mathbf{M}_a = \sum_{j=1}^{n_a} \mathbf{M}_j^e \delta m_j, \mathbf{K}_a = \sum_{j=1}^{n_a} \mathbf{K}_j^e \delta k_j \quad (5)$$

equation (4) can be used to determine proportional mass and stiffness factors δm_j and δk_j for immersed (j finite element) part of structure. The numerical simulation and modal analysis, Figure 1, was performed using fluid-structure interaction equations to compare with the results obtained by measurement.



Figure 1. Numerical simulation of the second bending mode for beam immersed in fluid

3. Experimental model and setup

Experiments were carried out in the laboratory environment. The measurement was performed in two steps, in the first step the measurement of vibration on the beam that is not immersed in water, and in the second step on the same beam that is partially immersed in water, Figure 2.



Figure 2. Impact test on the cantilever beam immersed in water and response is measured by laser vibrometer

The 650 mm long, with rectangular 50x10 mm cross-section beam, made of steel is used for experimental model. Tenth of the beam is immersed in water. Impact test modal analysis

was carried out using an instrumental hammer for excitation and laser vibrometer to measure the response.

The main problem that occurs when applying an experimental data in equation (4) is the difference in the number of degrees of freedom between the matrices of finite element and measured mode shape. To solve this problem the SEREP technique (O'Callahan, 1989) is applied to expand mode shapes, to fit the measured degree of freedoms to numerical one.

4. Concluding remarks

The possibility of determining the added mass of the beam immersed in water is experimentally explored. The results obtained by measuring have the same pattern as those numerically obtained. The size of experimentally calculated added mass corresponds to that obtained numerically. It is shown that the expansion of the mode forms can be used successfully in this application.

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EVALUATION OF METHODS FOR IDENTIFICATION OF STRUCTURAL DAMPING USING CONTINUOUS WAVELET TRANSFORMATION, APPLICATION TO NUMERICAL AND REAL DATA

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1. Introduction

Damping in mechanical systems is phenomena of energy dissipation. It comes in many forms and it is still not entirely studied. Generally, structural damping can be classified either as hysteretic or viscous. In practice many typed of damping acts simultaneously and concept of equivalent viscous damping is used instead. In this work we are testing methods that detect damping ratio using time-frequency signal decomposition. This is achieved using a CWT of free response signal. The evaluation of the damping identification methods is performed on numerically simulated and real laboratory models. The numerical simulation of the damping detection is performed on 2DOF model with a proportional viscous damping assumed. The damping identification on the practical model is verified by measuring longitudinal vibrations of the free bar.

2. CWT of signal

Wavelet transformation (WT) is an integral transformation method, which is achieved by an inner product of time signal with a basic wavelet function:

$$W_x^\psi(u, s) = \langle x, \psi_{u,s}^* \rangle = \int_{-\infty}^{\infty} x(t) \psi_{u,s}^*(t) dt$$

Coefficients of the WT are obtained by a dilatation s , and a translation u of the basic wavelet function $\psi(t)$:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right)$$

Normalised Gabor wavelet function is used:

$$\psi(t) = (\pi\sigma^2)^{0.25} e^{-t^2/2\sigma^2} e^{i\eta t}$$

Dilation parameter is related to the frequency using a following relation:

$$\omega(s) = \eta/s$$

Gabor wavelet belongs to the family of the analytic wavelet functions. This means that transformation of the real signal will also be analytic.

3. Ridge extraction and damping identification

Time-frequency localisation properties of the WT enable mode separation. This method uncouples the response of MDOF system, so that each mode can be treated as a one SDOF system, expressed as:

$$x(t) = A(t) \cos \varphi(t)$$

Basic procedure in order to identify damping is obtaining the ridge from the time-frequency decomposed signal. Ridges of the WT are defined as: $s = s(u)$. After the ridge has been extracted from the signal, the same principle is used for detection of the instantaneous amplitude and the instantaneous phase similarly to detecting those from a free response of a SDOF system using Hilbert transformation. Modal parameters (damping ratio and a natural frequency) from the ridge are extracted using a following expression:

$$\ln \left[2 |W_x^\psi(u, s(u))| / \sqrt[4]{4\pi\sigma^2 s(u)^2} \right] \approx \zeta \omega_n u + \ln A$$

Instantaneous amplitude of the ridge is:

$$A(u) = \arctan \left[2 |W_x^\psi(u, s(u))| / \sqrt[4]{4\pi\sigma^2 s(u)^2} \right]$$

Instant. phase of the ridge is expressed as:

$$\varphi(u) = \arctan \left[\text{Im}(W_x^\psi(u, s(u))) / \text{Re}(W_x^\psi(u, s(u))) \right]$$

Ridges are extracted using one of the following methods: Cross Section, Amplitude, Phase, and Impulse Response Reconstruction.

4. Experimental setup



Figure 1. Laboratory test model and Laser-Doppler vibrometer

Longitudinal vibrations in the free bar are measured using following equipment set:

- PolyTech type OFV1102, laser-Doppler vibrometer
- MMF type KF24, force sensor
- PC computer with a 24bit – NI PXI-4461 data acquisition (DAQ) card
- NI LabVIEW 8.2 DAQ software

The response of the steel bar on the impact is depicted in the following figure:

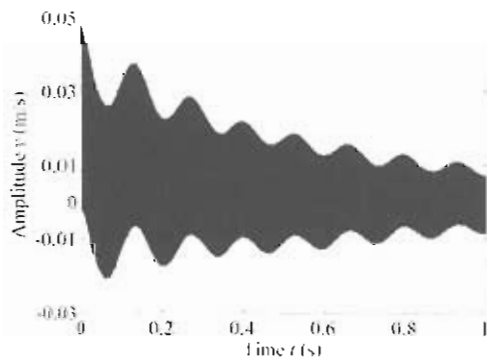


Figure 2. Time response of the steel bar

5. Results and discussion

Time responses from numerical example and the experimental examples are depicted in Figures 3 and 4. It can be seen that modes are well separated and ridges are extracted with no complications.

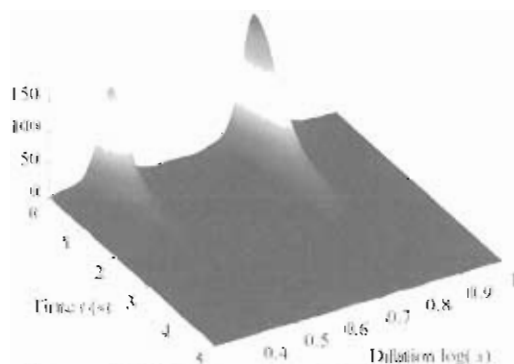


Figure 3. WT of the numerical 2DOF model

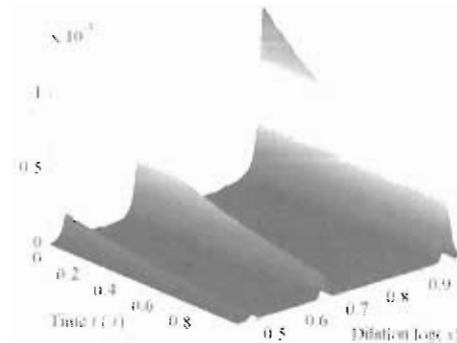


Figure 4. WT of the experimental model of the steel bar

6. Concluding remarks

Damping identification methods are presented and tested on the numerically simulated systems and experimental test case. Analytical solution is known for both: numerical system and the mathematical model of the experimental system. Results are evaluated to test the applicability of the evaluated time-frequency methods.

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EXPERIMENTAL TESTINGS OF GRANDSTAND RC GIRDERS

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1. Introduction

This paper presents the experimental static and dynamic testings of the grandstand reinforced concrete (RC) girders both as single elements and as monolithically assembled into grandstand structures of the Spaladium Arena in the city of Split [1], of the Sport Hall in the city of Makarska [2], and of the Sport Centre (Swimming Pool and Tennis Courts) in the city of Dugopolje [3].

The RC girders are simple supported beams over the span of 9.0, 6.08 and 9.66 meters, respectively. Total lengths of the girders are 9.99, 6.62 and 9.98 meters, respectively, and are supported in six, four and eight points, respectively, half of them on each end. Assembled girders are additionally connected in the thirds of the span. Dimensions of the cross-section can be seen in the Figures 1, 2 and 3, respectively.



Figure 1. Grandstand RC girder cross-section of the Spaladium Arena in the city of Split.



Figure 2. Grandstand RC girder cross-section of the Sport Hall in the city of Makarska.

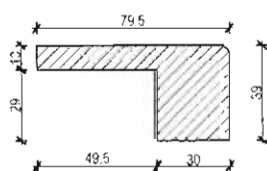


Figure 3. Grandstand RC girder cross-section of the Sport Centre in the city of Dugopolje.

2. Single Girder Testing

The experimental static and dynamic single girder testing was performed on site of the production. The testing load was determined to obtain design values of the internal cross-section forces. The whole testing was performed according to the Croatian rules [4, 5].

2.1 Static testing

The static load was performed with concrete blocks weighting approximately 500 kilos each placed in the mid-third of the girder over steel frame. Vertical and horizontal displacements were measured by dial gages positioned at the supports and in the mid-span, see Figure 4.



Figure 4. Positions of dial gages.

Strains of concrete were measured by Barry movable extensometer positioned at the supports and in the mid-span, see Figure 5.



Figure 5. Positions of strain gages.

2.2 Dynamic testing

The dynamic responses were induced by jumping groups of different number of humans. Dynamic response of the girder was registered in the time domain. The data acquisition was performed with uniaxial accelerometers (whose positions can be seen in Figure 6) and DEWETRON 2010 system. The FTT analysis was performed from the recorded accelelograms to obtain frequency responses. The natural frequencies were determined from the peaks of the FTT graphs using DEWESoft computer package [6].

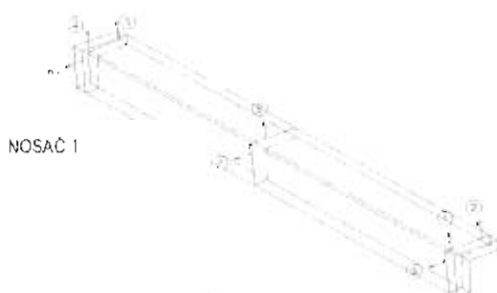


Figure 6. Positions of accelerometers.

3. Assembled Girder Testing

Assembled girders were connected in the thirds of the spans creating monolithically assembled grandstand structure. Dynamic testing was performed by jumping of humans.

4. Experimental Results

Considering space restrictions, only a very small part of the obtained experimental results [1, 2, 3] will be shown here.

Only one of the experimental results of the dynamic testing of the Spaladium Arena in the city of Split [1], i.e., measured accelerations and corresponding FTT diagrams is shown in Figures 7 and 8, respectively, for single girder testing. Measured accelerations of the assembled girders i.e. monolithically assembled grandstand structure are shown in Figure 9.



Figure 7. Measured accelerations – single girder.

Figure 8. Power spectral density function.

Figure 9. Measured accelerations – assembled girders.

5. Numerical Analyses

The numerical analyses were performed on PC with the models based on the finite element method with the software program FEAT2000 [7].

6. Conclusions

The obtained experimental results (maximal vertical displacements, strains i.e. stresses and dynamic responses) correspond well to the theoretical values [1-3]. The testing confirmed static and dynamic design hypotheses.

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FLUID MECHANICS

AERODYNAMICS SIMULATIONS OF GROUND VEHICLES IN UNSTEADY CROSSWIND

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1. Introduction

Crosswind stability is a problem for both today's and tomorrow's ground vehicles. Today, most types of ground transportation, such as buses, rail vehicles or cars, are sensitive to crosswind disturbances. It can even be a major safety issue for buses. The pressure for reducing the fuel consumption for vehicles due to environmental and financial concerns has pushed the car manufacturer to produce low-drag vehicles. Already in 1986, it was seen that streamlining vehicles to reduce the drag tends to deteriorate the crosswind sensitivity of the vehicle, see e.g. Gilhaus and Renn (1986) or Howell (1993), and further understanding of the above crosswind characteristics of the ground vehicles was necessary. In this study, numerical methods in conjunction of an advanced turbulence model (Detached-Eddy Simulations - DES -) are evaluated to simulate the time-development of the flow around a generic vehicle subjected to a sudden strong crosswind. The objective of the work presented is to understand some physical processes involved in the aerodynamics of bluff bodies in crosswind. An understanding of all the unsteady mechanisms is essential in order to develop a tool which would assess crosswind impacts with shorter simulation times.

2. Test Case and Numerical Setup

In this paper, we present results of Detached Eddy Simulations (DES) on two types of vehicle models, one box-like geometry and so called Windsor models. The box-like geometries are sharp edged (denoted SEVM) and radiused edged (denoted REVM), respectively. The Windsor model is a simplified car like geometry. The transient wind-gust is modelled via a time-dependent boundary condition. In all simulations, hexa-hedral

meshes are used. The Reynolds numbers are of the order 10^5 .

The computational domain together with boundary conditions is shown in Figure 1. The grids consist of hexahedral cells with 20 million cells for the Windsor model and 32 million cells for the box-like geometries. In all simulations, y^+ -values of 1-5 are used at the vehicle models. Also, the ratio of resolved turbulent kinetic energy in the wake region is studied in order to obtain grids that yield sufficiently resolved numerical solutions.

In all simulations, the incompressible Navier-Stokes equations were solved using DDES, see Spalart et al (2006), as turbulence model. A hybrid scheme as defined in Travin et al. (2002) was employed, that is a central scheme is used in the LES region of the flow and a higher order accurate scheme is used in the RANS region.

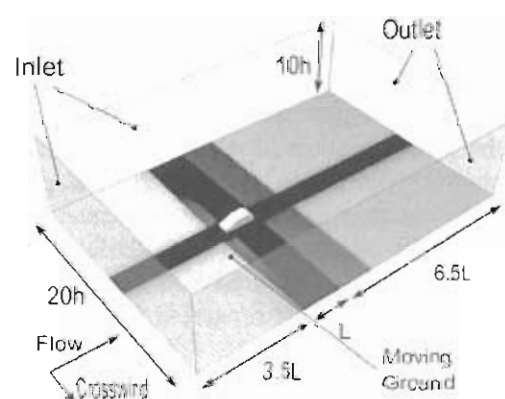


Figure 1. The computational domain dimensions and boundary conditions.

The dimensions of the different vehicle models can be found in Table 1.

Characteristic	SEVM	REVM	Windsor
Length, L	0.48 m	0.48 m	1.045 m
Width, l	0.2 m	0.2 m	0.390 m
Height, h	0.2 m	0.2 m	0.29 m
Ground Clearance	0.03 m	0.03 m	0.050 m
Radius of curvature, all edges	–	0.02 m	–
Radius of curvature, front leading edges	–	–	0.050 m
Radius of curvature, roof leading edge	–	–	0.200 m
Frontal Area, A	0.04 m ²	0.04 m ²	0.113 m ²

Table 1. The dimensions of the models.

3. Results and discussion

We present results for different wind gusts as well as parameter changes of the vehicles, such as radii of curvature and back light angle. The transient loads are studied in detail and are compared to experimental data, from Howell and le Good (2005), and Chadwick (1999).

In general, the DES manages to capture the essential transients of the flow, such that the aerodynamic loads are described in a satisfactory way. However, for the REVM, the separation at the leading edge is not fixed, and in this case, with the grids used, the DES fails to predict the separated flow correctly.

An overshoot in the yaw moment is identified and is seen to be linearly dependent on the steepness of the wind gust.

4. Concluding remarks

The inherent nature of the unsteady crosswind flows makes experiments difficult and numerical simulations can provide insight of the physics involved with more flexibility. The simple geometries studied in this paper for different numerical setups illustrate that DES are fully appropriate to simulate unsteady crosswind flows. The SEVM flow case with its sharp edges and the separated flow regions on the sides appear to be fully captured by the grids proposed in this study. A good agreement between the experimental data and the numerical data is reported and support that DES can be employed for transient crosswind flows.

The attention is drawn on the nature of the geometries. Investigating the first geometry using radiused edges instead of sharp leads to a more complex separation and might show the actual limit of the simulations involved, as complex industrial geometries are often a

combination of any kind of radius of curvature (e.g. the Windsor has already two types of edges). A different setup with a different RANS model for instance would provide a better understanding of the actual capacity of DDES for such flows.

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SIMULATION OF OFFSHORE WIND TURBINES INTEGRATING FULLY NONLINEAR IRREGULAR WAVES

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1. Introduction

The rapidly growing demand for offshore wind energy power production requires the development of more accurate numerical tools in order to investigate the effects of complex environmental conditions on wind turbine structures. A novel numerical package capable of predicting the nonlinear loads acting on offshore wind turbines exposed to severe sea states has been recently proposed by Marino et al, 2011. The model reproduces the structural response by coupling a fully nonlinear wave kinematic solver with a hydro-aero-elastic simulator of the entire system. Due to the large number of simulations required to set up a suitable probabilistic model for the design loads extrapolation, the main goal of the proposed simulation scheme is to increase the model accuracy without penalizing the computational time. To this aim, the hydrodynamic model is based on a domain decomposition strategy such that the numerical wave solver is invoked only on special sub-domains where highly nonlinear waves, such as breaking waves, are detected. The effects on the structural response of the impulsive wave loads associated with plunging breakers (Wienke and Oumeraci, 2005) was the main concern.

The novelty of the present contribution is the extension of the above global model also to less severe environmental conditions, when, according with the linear wave theory, no plunging breakers are detected. In particular, we will show that: i) the presence of steep non breaking waves, starting from sea state N° 5 on (Faltinsen, 1990), significantly influences the structural response as well; ii) breaking waves induced by the nonlinear waves interactions may occur even if they are not predicted by the linear criterion. With this improvement, the

numerical package permits now accounting for the effects of nonlinear hydrodynamics (from weakly to fully nonlinear) given a wider range of environmental conditions.

2. Model description

The global numerical framework is based on the coupling between the NREL open source FAST software (Jonkman, 2005) used for the hydro-aero-elastic simulation and the potential flow models (linear and fully nonlinear) used to predict the wave loads acting on the turbine substructure. The simulation scheme starts with the definition of the environmental parameters - mean wind speed, significant wave height and peak spectral period. After this, an irregular sea is reproduced in the time domain through a linear spectral approach. A zero-crossing analysis of the free surface elevation at the wind turbine location permits detecting when nonlinearities occur. Indeed, once all wave heights and wavelengths are known, each wave steepness can be compared with some steepness threshold so as critical time instants can be identified. Each of these time instants, along with the spatial location of the wind turbine, defines the central point of a time-space sub-domain on which the fully nonlinear solver is called for a more refined simulation. Thus, only locally in space and time, i.e. when necessary, the linear analytic solution is replaced by the numerical one. The 2-dimensional problem governing the propagation of gravity waves is formulated assuming the potential theory and a higher-order boundary element method is used to discretize Laplace's equation. For the time integration a 4th-order Runge-Kutta algorithm is used. The free surface time-depending potential problem is solved by means of a 2-step mixed Eulerian-Lagrangina scheme (Marino et al, 2011). We stress that the reproduction of

nonlinear irregular waves from linear initial an boundary conditions requires much care. Temporal and spatial ramps, along with a numerical absorber of nonlinear contributions in the transition zones, are developed and carefully tuned in order to avoid reflections and numerical instabilities.

3. Baseline turbine model

The turbine model is the NREL "5-MW Reference Wind Turbine for Offshore System Development" (Jonkman, 2009). A fixed-bottom monopile substructure is assumed to be located in a 20 m water-depth site. The most relevant features for our purpose concerning the turbine are listed in Table 1.

Rating Power	5 MW
Hub Height	90 m
Rated Wind Speed	11.4 m/s
Rotor Mass	110 t
Nacelle Mass	240 t
Tower base and pile diameter	6 m
Tower top diameter	3.87 m
Pile length (and water depth)	20 m
Pile mass	190 t

Table 1. Selected parameters of the baseline wind turbine model.

4. Results

After validating both the numerical solver and the global domain decomposition strategy, the effects of the nonlinear hydrodynamics are assessed by reproducing sea states with increasing severity. The main result the proposed model delivers is that from sea states N° 5 on, the effects on the structural response due to the nonlinear wave contributions are not negligible. They lead to a significant augmentation of the structural internal forces and displacements. Figure 1 shows the case of a plunging breaker occurring during 1-hour simulation of a SS7. Figure 2 shows the tower-top fore-aft displacement where all the five sub-domains are highlighted and compared with the

displacement induced by the linear hydrodynamics. Wave impacts occur on sub-domains 2, 4 and 5. The latter is shown in Figure 1.

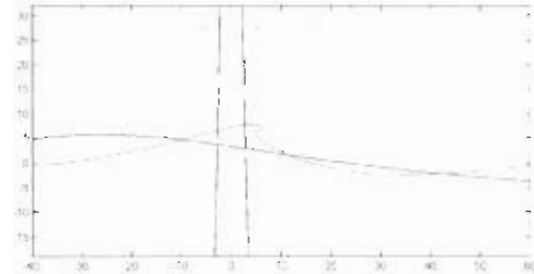


Figure 1. Snapshot at $t = 3168.9$ s of the fully nonlinear (red) and the linear analytical (blue) free surface evolution during a SS7.



Figure 2. Time series of the tower-top fore-aft displacement (SS7). Comparison of the response due to linear alone (blue) and linear with nonlinear sub-domains (red) hydrodynamics.

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MODELING OF FUEL PROPERTIES IN SIMULATION OF DIESEL INJECTION SYSTEMS

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1. Introduction

Stringent emission standards in light and heavy duty diesel engines impose high accuracy requirements on computational models for fuel injection, mixture formation and combustion. Accurate definition of the physical-chemical properties of diesel fuel is very important for the simulation of fuel injection systems. As real diesel fuels are complex mixtures of aliphatic and aromatic components, it is quite difficult to obtain their properties in the required pressure and temperature range, either theoretically or by experiment. However, the problem may be solved by modeling the diesel fuel as a mixture of the n-decane and 1-methylnaphthalene with a defined composition.

The present analysis focuses on the state behavior and vapor-liquid equilibrium of the n-decane and 1-methylnaphthalene mixture. Based on it, the paper attempts to derive the thermal, transport and other properties of the model mixture, such as sound velocity, bulk modulus, viscosity, heat capacity, thermal conductivity and others. Several examples of modern injection systems are discussing showing the influence of these properties on the simulation results at different operating conditions.

2. Thermal Liquid and Vapor Properties

Thermal properties and velocity of sound of n-decane and 1-methylnaphthalene mixture can be calculated from the equation of state, saturated vapor pressure and heat capacity using thermodynamic laws (Maier and Svoboda, 1985). The procedure consists of three steps. First, the properties of n-decane and 1-methylnaphthalene along the saturated liquid line are derived from isobaric heat capacities.

Next, the properties of compressed liquid and saturated vapor of both components are calculated by the equation of state. Then the mixture properties are evaluated using the volume additivity rule.

3. Cavitation, Air Solubility and Vapor-Liquid Equilibrium

Cavitation in diesel oil can be caused by the evaporation of the fuel or dissolved air, or by a combination of both these effects. The paper aims to describe thermodynamically the solubility of air and vapor-liquid equilibrium and to estimate the significance of these effects.

Typical diesel oil is saturated by air at 25 °C and 101.3 kPa. The air is modeled as a mixture of 79 mol% of nitrogen and 21 mol% of oxygen. For the solubility calculation of the air components Henry's law is applied. Phase equilibrium of the four component mixture is considered: n-decane (1), 1-methylnaphthalene (2), nitrogen (3) and oxygen (4). Vapor phase is modeled by the ideal gas and liquid phase by the state equation (Reid et al, 1977).

4. Viscosity and Surface Tension

Experimental data on low-pressure liquid viscosity is available for both n-decane and 1-methylnaphthalene (API Technical Handbook, 1986). Data on liquid viscosity of n-decane at elevated pressures is published by Krahn and Luft (1994). However, no data is available for the liquid viscosity of 1-methylnaphthalene at elevated pressures. There is no generalized equation for the prediction of liquid viscosity at high pressure. Hence empirical formulas have to be used. Empirical models are also applied for the surface tension and other properties.

5. Application Examples

Two examples of fuel property application in injection systems of light/heavy duty engines are presented. First example is an electronic unit pump system shown in Figure 1. It consists of a cam-driven pump, electro-magnetic delivery valve, high-pressure fuel line and injector nozzle. Second example is a solenoid-controlled high-pressure common rail injector shown in Figure 2. It operates at a pressure of up to 200 MPa. Typical problems in unit pump and common rail systems are high pressure oscillations and cavitation induced by wave propagation effects. To predict them accurately by calculation, an accurate definition of the fuel properties is necessary.

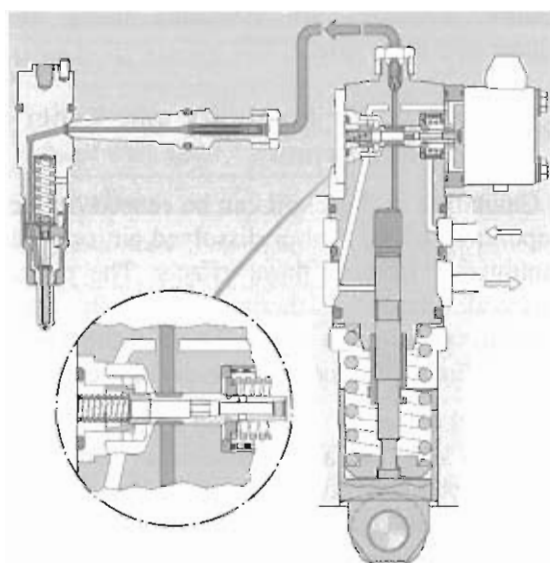


Figure 1. Electronic Unit Pump System (Bosch) for Heavy Duty Engines.

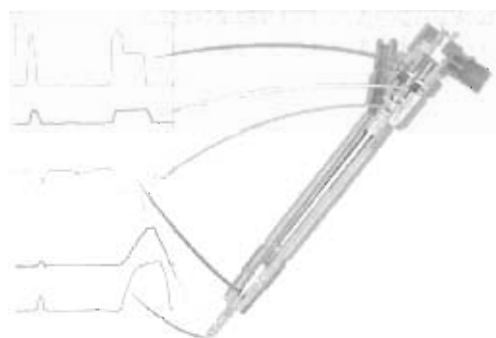


Figure 2. Electromagnetic Common Rail Injector for Light Duty Engines.

6. Results and Discussion

Modeling of fuel properties in the injection system simulation is discussed. Three cases are considered: fully variable fuel properties, properties based on constant sound velocity (linear acoustics) and constant (pressure-invariant) properties. Calculation is performed with AVL software Hydsim. The results can be summarized as follows:

- Using fully variable properties (calculated by thermodynamic laws), accurate estimates of bulk modulus, heat capacity and sound velocity can be obtained within the operating pressure and temperature range (0 to 250 MPa and -20 to 200 °C)
- Accurate viscosity prediction is limited to the lower pressure range. At high pressure empirical relationships have to be used
- Thermal conductivity and surface tension cannot be calculated precisely because of the lack of data for 1-methylnaphthalene at high pressure.

7. Concluding Remarks

Approximation of diesel oil as the mixture of n-decane (~90%) and 1-methylnaphthalene (~10%) yields reliable estimates for the major physical-chemical properties in the simulation of fuel injection systems.

Fuel density is not an independent property: generally it is a product of thermodynamic state governed by the bulk modulus and thermal expansion coefficient.

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AERODYNAMICS OF ICED AIRFOIL SIMULATED BY RANS CODE

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1. Introduction

Ice accretion on aerodynamic surfaces of the aircraft significantly impairs the designed performances. Ice accretions change designed airfoil shape and thereby introduce additional disturbances into flow field. This phenomenon brings to earlier separation causing drag to increase and lift to decrease.

In this work the numerical calculations were performed for two-dimensional flow around NACA23012 airfoil with and without rime ice accretion. Numerical calculations were performed with RANS (Reynolds Averaged Navier-Stokes) solver for incompressible flow with $k-\varepsilon$ turbulence model (realizable version - with standard wall functions) implemented in Fluent software package. The calculated results were compared with experimentally determined data (Bragg, 2008). In that report different ice type configurations were experimentally examined to provide data for validation and finding suitable numerical analysis of different ice type configurations. In this work the rime ice on airfoil leading edge configuration is examined.

2. Model Description

In this research only stream-wise shape is considered which is typical of a rime ice condition, having a smooth zone in the stagnation region with rime feathers and roughness downstream. In Figure 1 the shape of ice accretion on leading edge of NACA23012 airfoil is shown.

Numerical calculations were performed according to experimental setup where Reynolds number was equal to $Re = 1.8 \cdot 10^6$ referenced with airfoil chord and Mach number was equal to $Ma = 0.18$.

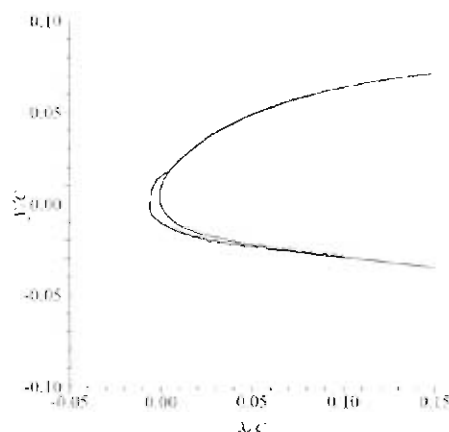


Figure 1. Ice accretion contour on leading edge of NACA23012 airfoil

3. Computational grid

The unstructured computational grid employed in this work is shown in Figure 2.

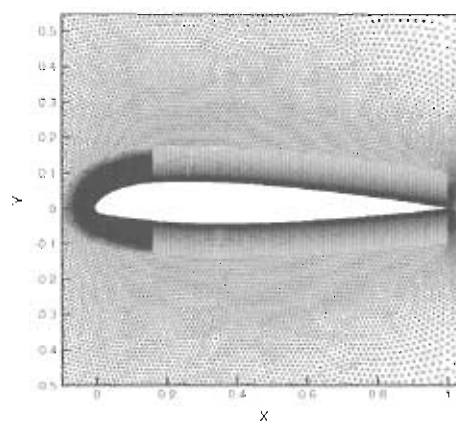


Figure 2. Computational grid around NACA23012 airfoil

4. Results

The results for lift coefficient, for airfoil with ice accretion configuration, as a function of angle of attack are shown in Figure 3. The calculated lift coefficient is in very good agreement with experimental data at majority angles of attack. At the angles of attack greater than $\alpha = 12^\circ$ sudden divergence between computed and experimental data is present. At these angles of attack the flow becomes

separated and RANS solver with $k-\varepsilon$ turbulence model was not able to predict lift coefficient value accurately.

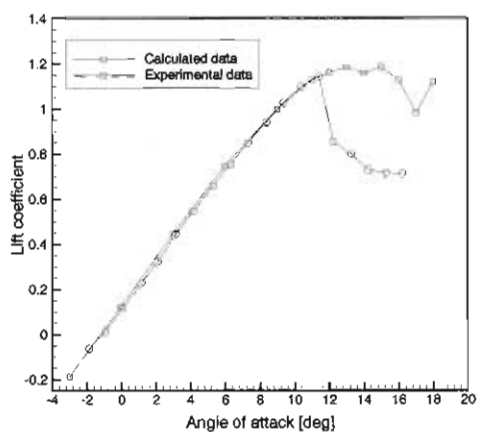


Figure 3. Lift coefficient versus angle of attack for airfoil configuration with ice

The results for drag coefficient, for airfoil configuration with ice accretion, as a function of angle of attack are shown in Figure 4. The calculated drag coefficient shows very good agreement with experimental data at majority angles of attack. A little divergence between computed and experimental data can be observed at negative angles of attack and between angle of attack $\alpha = 5^\circ$ and $\alpha = 9^\circ$. The good matching of the calculated and experimental data is more the consequence of drag increase in experimental data, while the calculated result showed very little sensitivity on the rime ice existence.

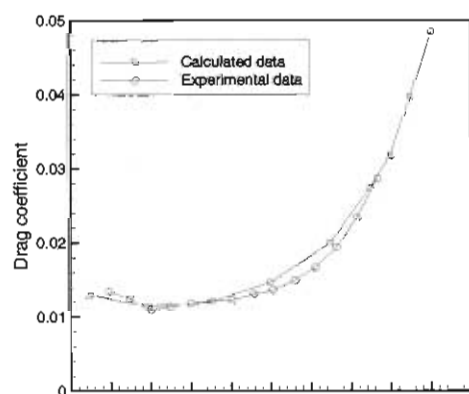


Figure 4. Drag coefficient versus angle of attack for airfoil configuration with ice

5. Conclusions

Calculated RANS results were in moderate agreement with experimental data for airfoil configuration with rime ice accreted at the airfoil leading edge. At angles of attack before occurrence of separated flow calculated results and experimental data for lift coefficient are in satisfactory agreement. Calculated lift coefficient does not show steep decrease at maximal values as experimental data do. Also, calculated maximum lift coefficient has not attained the same value as in experimental data.

Calculated drag results are in moderate agreement with experimental data for clean airfoil configuration. For this configuration the calculated drag coefficient is constantly overpredicted at all angles of attack. The calculated drag coefficient results for airfoil configuration with ice were in very good agreement compared to experimental data. The RANS calculations with $k-\varepsilon$ turbulence model showed little sensitivity on the rime ice existence.

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ANALYSIS OF THE STABILITY OF THE HELICOPTER WITH THE WATER CANNON

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1. Introduction

Water cannon has been recently used for the purposes of impulse fire extinguishing as a relatively new method of fighting fires with extremely small amounts of the fire extinguishing agent. The water fire extinguishing agent is discharged into the fire at very high velocity within an extremely short period of time. With high kinetic energy and the momentum water projectile will rapidly penetrate and extinguish the fire using only small amounts of water. Typical impulse device discharges the water in impulses with durations from 20 to 500 milliseconds with approximately exit speed of about 125 m/s depending on the size of the device.

Interest of this work is the analysis of the configuration of the water cannon mounted underside the helicopter on tilt mechanism that can be moved down from the horizontal position. Helicopter considered in this work is coaxial rotor configuration (Fig.1).

Process of firefighting with the mentioned helicopter is typical from the hover flight in close proximity of the building or another fire source. Helicopters with coaxial rotor configuration, compared to main rotor – tail rotor configuration, are more stable and easier for the pilot control, especially in hover flight regime but the wake from its rotors is stronger.

The impulse from the water cannon will influence the dynamics and the stability of the helicopter, primarily the pitch motion. This could have an affect on the precision of the water projectile regarding the vertical variation

of its trajectory. Furthermore, the wake of the coaxial rotor has influence on the water projectile: on the projectile itself, its dispersion as well as on its trajectory.

Helicopter configuration with water cannon may imply on a degradation of its handling qualities. Further steps on this topic may lead to evaluation of handling qualities to ensure are they good enough for the pilot to safely conduct the fire extinguishing operation, or if the presence of the cannon will require a modification of the flight control system.

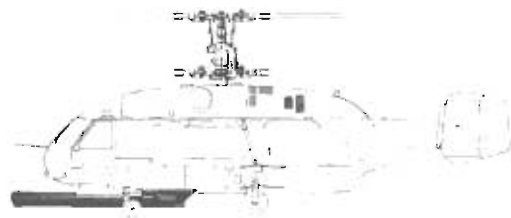


Figure 1. An example of schematic side view of possible water cannon installation on Kamov Ka-32A helicopter

In this preliminary report an emphasis will be on the influence of the impulse from the water cannon on the stability of the helicopter and its pitch motion.

2. Mathematical model of the helicopter motion

Linear model of the coaxial helicopter will be applied for the analysis. Influence of the impulse from the water cannon will be included as linear perturbation. In the presented analysis only longitudinal motion is discussed. Generally lateral and longitudinal motion of the

helicopter is strongly coupled due to the asymmetry of the aerodynamic forces and inertial asymmetry. With the assumption of the plane of symmetry, what is generally a good assumption for the coaxial rotor configuration, decoupling of the lateral and longitudinal motion is more eligible.

For hover flight that is of the main interest of this work, coupling between the longitudinal and lateral motion is even weaker. Degrees of freedom for longitudinal motion in hover are horizontal and vertical displacement and rotation about the y axis – pitch rate and pitch attitude. In hover flight rotor motion is simpler due to the symmetric aerodynamic forces and moments for symmetric flow, following constant feathering, flapping and lead-leg angles. Numerical example will be presented for the generic helicopter with coaxial rotor configuration, with assumption of rigid blades.

3. Modelling the impulse of the water cannon

The method of obtaining the jet in pulsed water cannon is based on the principle of compression, acceleration and ejection of a fluid through the top of the water cannon tube. In such regime of work reactive force developed, which may have influence on helicopter stability, need to be suppressed. In this work basic principles of both water cannon and water projectile kinematics and dynamics are studied. The fire distinguishing efficiency is more reliable by using water projectiles rather than steady water impacts. Water cannon model development is divided on interior and exterior ballistics considerations. Interior ballistics model will analyze interactions of forces on water projectile inside of the cannon tube. Main force which acts on water projectile has nonlinear development in time along water cannon tube and need to be obtained by numerical approach. This force is characteristic for first period of the cannon kick. Exterior ballistics principles are applied here only in the moment when water projectile just left cannon tube. In this short period high compressed air from the tube still bleeds and reactive force is developing. In this region of exterior ballistics

main force can be reliably expressed by analytical form. This force is significant for second period of the cannon kick. Both forces from two mentioned periods analyzed by interior and exterior ballistics principles have influence on helicopter stability. Second stage of external ballistics principles leads to calculation of the flight parameters of the water projectile with influence from the rotor wake and here will not be studied.

4. Expected results

It is expected that the linearized model of the helicopter with the water cannon will give first assessment of the analyzed configuration. Beside the stability analysis of the helicopter under the influence of the water cannon, the work will present the guidelines regarding the required parameters of the water cannon and its construction.

Possible utilization of the results could lead to the synthesis of the helicopter stabilization system about the pitch axis that would improve the stability of the aircraft and also precision of application of the water cannon.

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HEAT AND MASS TRANSFER

HEAT CONDUCTION IN FRACTAL GEOMETRIE BY MIXED BOUNDARY ELEMENT METHOD

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1. Introduction

In the paper the numerical simulation of heat diffusion in the fractal geometry of Koch snowflake is presented using mixed Boundary Element Method (BEM). The idea and motivation of work is to improve the cooling of small electronic devices using fractal geometry of surface similar to cooling ribs. The heat diffusion is assumed as the only principle of heat transfer. The results are compared to the heat rate of flat surface.

The mathematical formulation of fractals originated in 17th century by Gottfried Leibniz. In 1975 Benoit Mandelbrot coined the word "fractal" to denote an object whose Hausdorff-Besicovitch dimension is greater than its topological dimension. He illustrated this mathematical definition with amazing computer visualizations.

There are many fields where fractals are used in engineering applications such as porous media modelling, nanofluids, fracture mechanics and many more. For example, Huang et al 2011 used fractals to model surface morphology in order to numerically compute thermal contact resistance in the field of heat transfer as in our case. His principle assumption is to neglect heat convection and radiation thus the heat conduction is the only mechanism for heat transfer in small air pockets between two surfaces.

The idea of using fractal to model cooling surface raise up from the fundamental question: "How long is the coast of Britain?" stated in paper by Mandelbrot. Using smaller and smaller ruler the length limits to infinite. The area is obvious finite. If this logic is applied to small electronic device cooler, large efficiencies must be obtained.

The BEM has a long tradition in our research group headed by prof. Škerget.

Presented numerical scheme is based on our work dated in 2004.

The full length article will be organized as follows. After Introduction in section 2 the problem definition is stated together with governing equations. In section 3, the numerical algorithm is explained briefly. The section 4 is dedicated to Results. First, the developed numerical algorithm will be verified using an analytical solved case of a flat surface cooler represented the first starting fractal. Different meshes are tested in detail. The second case is a Koch snowflake. The paper finished with conclusions.

2. Problem definition

The problem considered is a cut out of a small electronic device as shown in Figure 1.

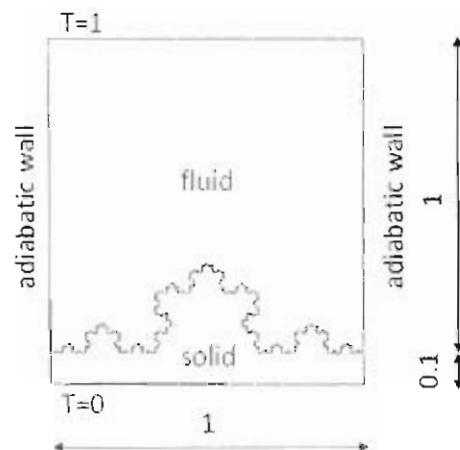


Figure 1: The geometry and boundary conditions of the air cooled fractal electronic device.

The physical model assumptions follow. First, the major assumption is that the heat conduction is the only mechanism of heat transfer thus neglecting heat convection in air and radiation. The basis of this assumption is that the dimension of the problem is small, for example 1 cm and still air around this device.

Another reason is small temperature difference causing small buoyant forces and causing minimal heat radiation. Second assumption is a steady state condition.

The problem has been non-dimensionalised as follows. Length quantities x and y are scaled with the length of the cut-off. Normalized temperature is obtained by scaling with the temperature difference between hot and cold walls. The material properties could be treated in two ways. Since only steady state is computed, the solution is only heat conductivity depended. If governing equation is non-dimensionalised using air conductivity, eg fluid non-dimensional conductivity becomes 1 and solid conductivity is equal to the ratio of solid and air conductivity. In the case of Cu, Al or steel cooler this ratio is 16000, 9000 and 2300 respectively. Such high values could cause numerical instabilities. The second possibility of non-dimensionalising is to use heat diffusivity a instead conductivity. In this manner the diffusivity ratio is only 5.4, 4.4 and 0.8. Anyway, interface boundary condition equals heat flux between solid and fluid resulting in high value heat conductivity ratio in system matrix in both cases of non-dimensionalising. This is the case, where complicated physic couldn't be overcome by numerical trick.

The non-dimensional governing equation is a Laplace equation

$$\frac{\partial}{\partial x_j} \left(a \frac{\partial T}{\partial x_j} \right) = 0$$

The boundary conditions are strait forward as shown in Figure 1.

3. Numerical algorithm

Multidomain mixed Boundary Element Method (BEM) numerical algorithm for solving 2D diffusion problems will be described in this section.

The developed solver has to fulfil following demands: accuracy, stability at high conductivity ratios, economy of the solution

and ability to solve high grid density with high longest/shortest element ratio.

The accuracy of BEM is a well-known quality of BEM. In comparison to the other numerical methods, the accuracy is more evident using low grid densities, see Ramšak et al 2004.

The stability is a common problem using BEM. The stability of our BEM numerical algorithm is the main topic in our previous work Ramšak and Škerget 2004, where considerable effort was done to increase the stability at a convection dominated laminar flows. The result was a stable algorithm solving a driven cavity case up to Reynolds number value 50 000. The usual maximal value is 10 000 to 15 000 using BEM.

The numerical method will be presented in detail in full length paper.

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MESHLESS AND OTHER NOVEL COMPUTATIONAL METHODS

THREE-DIMENSIONAL SOLUTION OF BENDING OF FUNCTIONALLY GRADED MAGNETOELECTROELASTIC CIRCULAR PLATES

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1. Introduction

A meshless method based on the local Petrov-Galerkin approach is proposed for plate bending analysis with magneto-electro-elastic properties. Material properties are considered to be continuously varying along the plate thickness. Axial symmetry of geometry and boundary conditions for a circular plate reduces the original 3-D boundary value problem into a 2-D problem in axial cross section. Both mechanical and thermal loads with stationary and transient dynamic conditions are considered in this paper. The local weak formulation is employed on circular subdomains on the axial cross section. Subdomains surrounding nodes are randomly spread over the analyzed domain. The test functions are taken as unit step functions in derivation of the local integral equations (LIEs). The moving least-squares (MLS) method is adopted for the approximation of the physical quantities in the LIEs. After performing the spatial integrations, one obtains a system of ordinary differential equations for certain nodal unknowns. That system is solved numerically by the Houbolt finite-difference scheme as a time-stepping method.

2. Local integral equations

The uncoupled thermoelasticity is considered here. It means that a heat generation by the mechanical field is neglected. Recently, we have considered the heat generation by mechanical field in the coupled thermoelasticity (Sladek et al., 2006). For most materials the inverse thermoelastic, pyroelectric and pyromagnetic effects are very weak, i.e. the heat generation by mechanical, electrical and magnetic fields can be neglected. However, stresses, electrical displacements and magnetic induction can be influenced by temperature.

Then, the constitutive equations involving the general electro-magneto-thermo-elastic interaction (Huang and Kuo, 1997) for

continuously non-homogeneous media with material coefficients are given by spatially dependent material coefficients as follows

$$\sigma_{ij}(\mathbf{x}, \tau) = c_{ijkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x}, \tau) - e_{kij}(\mathbf{x})E_k(\mathbf{x}, \tau) - d_{kij}(\mathbf{x})H_k(\mathbf{x}, \tau) - \lambda_{ij}(\mathbf{x})\theta(\mathbf{x}, \tau) \quad (1)$$

$$D_j(\mathbf{x}, \tau) = e_{jkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x}, \tau) + h_{jk}(\mathbf{x})E_k(\mathbf{x}, \tau) + \alpha_{jk}(\mathbf{x})H_k(\mathbf{x}, \tau) + p_j(\mathbf{x})\theta(\mathbf{x}, \tau) \quad (2)$$

$$B_j(\mathbf{x}, \tau) = d_{jkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x}, \tau) + \alpha_{kj}(\mathbf{x})E_k(\mathbf{x}, \tau) + \gamma_{jk}(\mathbf{x})H_k(\mathbf{x}, \tau) + m_j(\mathbf{x})\theta(\mathbf{x}, \tau). \quad (3)$$

The complete set of governing equations consists of the heat conduction, equilibrium of momentum and Maxwell equations:

$$[k_{ij}(\mathbf{x})\theta_{,j}(\mathbf{x}, \tau)]_{,i} - \rho(\mathbf{x})c(\mathbf{x})\dot{\theta}(\mathbf{x}, \tau) = 0, \quad (4)$$

$$\sigma_{ij,j}(\mathbf{x}, \tau) + X_i(\mathbf{x}, \tau) = \rho\ddot{u}_i(\mathbf{x}, \tau), \quad (5)$$

$$D_{j,j}(\mathbf{x}, \tau) = 0, \quad (6)$$

$$B_{j,j}(\mathbf{x}, \tau) = 0. \quad (7)$$

In cylindrical coordinate system, they are transformed into the form

$$\begin{aligned} &\sigma_{rr,r}(r, z, \tau) + \sigma_{rz,z}(r, z, \tau) + \\ &+ \frac{1}{r}[\sigma_{rr}(r, z, \tau) - \sigma_{\varphi\varphi}(r, z, \tau)] - \\ &- \rho(\mathbf{x})\ddot{u}_r(r, z, \tau) = -X_r(r, z, \tau), \\ &\sigma_{rz,r}(r, z, \tau) + \sigma_{zz,z}(r, z, \tau) + \frac{1}{r}\sigma_{rz}(r, z, \tau) - \\ &- \rho(\mathbf{x})\ddot{u}_z(r, z, \tau) = -X_z(r, z, \tau), \\ &D_{r,r}(r, z, \tau) + D_{z,z}(r, z, \tau) + \frac{1}{r}D_r(r, z, \tau) = 0, \\ &B_{r,r}(r, z, \tau) + B_{z,z}(r, z, \tau) + \frac{1}{r}B_r(r, z, \tau) = 0, \end{aligned}$$

$$[k_{ab}(\mathbf{x})\theta_{,b}(r, z, \tau)]_{,a} + \frac{k_{rz}(\mathbf{x})}{r}\theta_{,z}(r, z, \tau) +$$

$$+\frac{k_{rr}(\mathbf{x})}{r}\theta_{,r}(r,z,\tau)=\rho(\mathbf{x})c(\mathbf{x})\frac{\partial\theta}{\partial t}(r,z,\tau). \quad (8)$$

In the MLPG method the test and trial function are not necessarily from the same functional spaces. Applying the Gauss divergence theorem to the weak formulation of governing equations on local subdomains Ω_s and selecting Heaviside unit step functions as test functions there (Sladek et al., 2010), one can recast equations into the following form

$$\begin{aligned} & \int_{\partial\Omega_s} \sigma_{rb}(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}(\sigma_{rr}-\sigma_{\varphi\varphi})d\Omega - \\ & - \int_{\Omega_s} \rho(\mathbf{x})\ddot{u}_r(r,z,\tau)d\Omega = - \int_{\Omega_s} X_r(r,z,\tau)d\Omega, \\ & \int_{\partial\Omega_s} \sigma_{zb}(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}\sigma_{rz}(r,z,\tau)d\Omega - \\ & - \int_{\Omega_s} \rho(\mathbf{x})\ddot{u}_z(r,z,\tau)d\Omega = - \int_{\Omega_s} X_z(r,z,\tau)d\Omega, \\ & \int_{\partial\Omega_s} D_b(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}D_r(r,z,\tau)d\Omega = 0 \\ & \int_{\partial\Omega_s} B_b(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}B_r(r,z,\tau)d\Omega = 0, \\ & \int_{\partial\Omega_s} q(r,z,\tau)d\Gamma - \int_{\Omega_s} \rho(\mathbf{x})c(\mathbf{x})\dot{\theta}(r,z,\tau)d\Omega + \\ & + \int_{\Omega_s} \left[\frac{k_{rz}(\mathbf{x})}{r}\theta_{,z}(r,z,\tau) + \frac{k_{rr}(\mathbf{x})}{r}\theta_{,r}(r,z,\tau) \right] d\Omega = 0 \end{aligned}$$

where q is the heat flux

$$q(r,z,\tau)=k_{ab}(\mathbf{x})\theta_{,b}(r,z,\tau)n_a(\mathbf{x}).$$

The MLS approximation is applied for the mechanical displacements, the electric and magnetic potentials, and the temperature

$$\mathbf{u}(\mathbf{x},\tau) \approx \mathbf{u}(\cdot) \cdot \hat{\mathbf{u}} = \sum_{a=1}^n \phi^a(\cdot) \hat{\mathbf{u}}^a(\tau),$$

$$\psi(\mathbf{x},\tau) = \sum_{a=1}^n \phi^a(\mathbf{x}) \hat{\psi}^a(\tau),$$

$$\mu(\mathbf{x},\tau) = \sum_{a=1}^n \phi^a(\mathbf{x}) \hat{\mu}^a(\tau),$$

$$\theta(\mathbf{x},\tau) = \sum_{a=1}^n \phi^a(\mathbf{x}) \hat{\theta}^a(\tau), \quad (9)$$

Substituting the spatial approximations into the local integral equation, we obtain a system of ordinary differential equations (ODE), which is solved by the Houbolt method.

Acknowledgements

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A MESHFREE METHOD FOR ELASTICITY PROBLEMS WITH INTERFACES

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1. Introduction

In our days, even with the advances in computers, there is a strong need for efficient numerical solvers for many engineering problems. If in one hand, the numerical solvers must be fast, accurate and easy to implement, in the other they should maintain all these features, regardless the spatial dimension of the problem. From this point of view, meshfree methods present many advantages over more traditional methods, such as the Finite Element Method (FEM).

In this paper, we study a meshfree method as a solver to a transmission problem in linear elasticity. These problems have wide applications in continuum mechanics (see [6] and the references therein).

Consider an isotropic and homogeneous elastic domain, Ω , with positive Lamé parameters λ_0, μ_0 and an elastic inclusion ω strictly contained Ω , with elastic parameters λ_1, μ_1 . We shall assume that these domains are regular, open, bounded and simply connected. In absence of body forces, the governing transmission problem is

$$\begin{cases} \nabla \cdot \sigma_0(u_0) = 0 & \text{in } \Omega_c := \Omega \setminus \bar{\omega} \\ u_0 = g_0 & \text{on } \Gamma := \partial(\Omega \setminus \bar{\omega}) \setminus \partial\omega \\ \nabla \cdot \sigma_1(u_1) = 0 & \text{in } \omega \\ u_1 - u_0 = [u] = g_1 & \text{on } \gamma := \partial\omega \\ \sigma_1(u_1)n - \sigma_0(u_0)n = [\sigma(u)n] = g_1^n & \text{on } \gamma \end{cases}$$

Where u_i is the displacement and the stress tensor σ_i is given by

$$\sigma_i(u_i) = \lambda_i (\nabla \cdot u_i) I + \mu_i (\nabla u_i + (\nabla u_i)^T).$$

Last equation is a transmission equation on the traction vector $\sigma(u)n$, where n is the normal

vector on γ , pointing outwards with respect to Ω_c (hence pointing inwards with respect to ω).

This transmission problem has been addressed by several authors in the literature, where numerical methods based on FEM have been studied and implemented (eg. [5, 6]). The main disadvantage with these methods are that they are not easy to implement and that they can only apply to 2D problems.

We propose the method of fundamental solutions (MFS) (see [2, 3]) for this problem. This meshfree method, is a particular case of Radial Basis Functions approximation methods (RBF) (see [1,4]) and, as we show is particularly suited for the above type of systems of elliptic equations.

2. MFS for elasticity problems with interfaces

The main idea behind the MFS is to consider representations

$$\begin{aligned} u_0(x) &= \sum_k \Phi^0(x - y_k) \alpha_k, \quad y_k \notin \bar{\Omega}_c \\ u_1(x) &= \sum_k \Phi^1(x - z_k) \beta_k, \quad z_k \notin \bar{\omega} \end{aligned}$$

that is, linear combinations of point source fundamental tensors centered at points y_k, z_k placed outside the domain of interest. Recall that the fundamental tensor for the Lamé system satisfies the equation (in the distribution sense)

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where δ is the Dirac delta distribution, and we consider

$$\Phi^i(x) = \begin{cases} \frac{\lambda_i + 3\mu_i}{4\pi\mu_i(\lambda_i + 2\mu_i)} \left[-\ln|x| \delta_{ij} + \frac{\lambda_i + \mu_i}{\lambda_i + 3\mu_i} \frac{x_i x_j}{|x|^2} \right], & x \in \mathbb{R}^2 \setminus \{0\} \\ \frac{\lambda_i + 3\mu_i}{8\pi\mu_i(\lambda_i + 2\mu_i)} \left[\frac{1}{|x|} \delta_{ij} + \frac{\lambda_i + \mu_i}{\lambda_i + 3\mu_i} \frac{x_i x_j}{|x|^3} \right], & x \in \mathbb{R}^3 \setminus \{0\} \end{cases}$$

The vectorial coefficients α_k , β_k can be computed by imposing, on some boundary points, the boundary conditions of the above transmission problem. This leads to a system of linear equations on the above coefficients. Note that since the point sources are placed outside the domains of interest, the corresponding point source fundamental basis functions satisfy the Lamé system. On the other hand, the method can also be applied to 3D problems.

3. Numerical Simulations

In the following we present some plots in order to illustrate the performance of the method. In this 2D case the geometry of the domain is presented in Fig. 1.

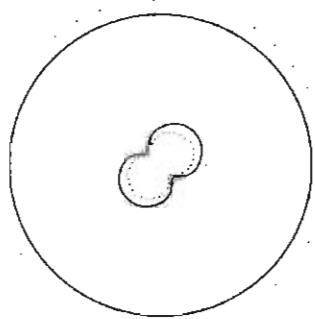


Figure 1. Geometry of the domain. Boundary in black; point sources for u_0 in blue; point sources for u_1 in red.

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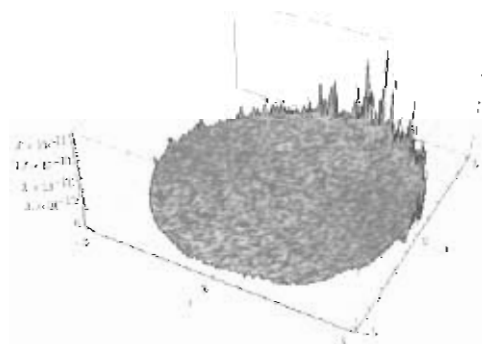


Figure 2. Absolute error on the first component (order of error is up to 10^{-11})

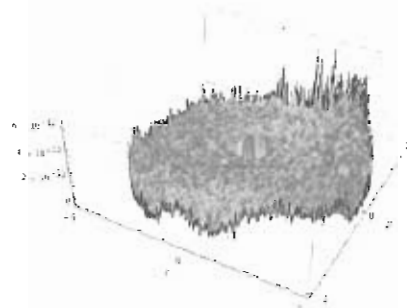


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A MESHFREE METHOD FOR ELASTICITY PROBLEMS WITH INTERFACES

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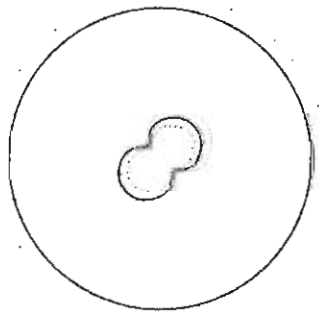


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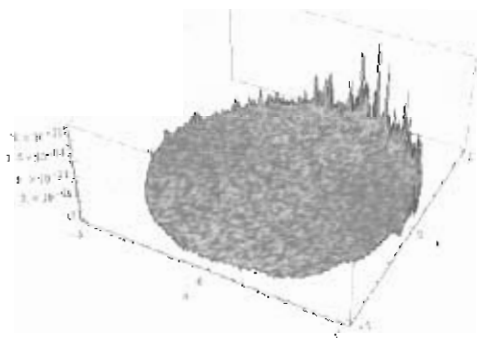


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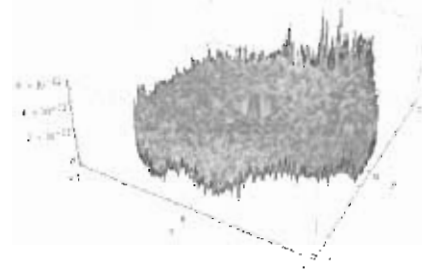


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SOLUTION OF THE TORSION PROBLEM USING THE R-FUNCTION METHOD

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1. Introduction

R-function method (Rvachev, 1982) was developed in Kharkiv during the second half of the last century. It is based on the idea of the reverse procedure for determining analytic and approximate solutions of mathematical physics problems. Rvachev developed so called "semi-algebra" which allows accurate description of the geometry of the area, all boundary conditions and zones with different properties of material, and set foundations for practical applications in solving engineering problems. To describe the problem mathematically, the so called "solution structure" (Shapiro, 2007) is constructed which depends on the shape of the domain, boundary conditions and differential component. The first two components are given accurately using R-function method, and the third differential component is determined as linear combination of classical or finite basis functions. R-function method provides simple application of the strong formulation that allows good results in the problems involving large local gradients. Also, this approach can ensure algorithms for obtaining numerical solutions with desired level of accuracy.

2. Numerical Solution

Numerical procedure for solving different boundary problems is described with differential equation and associated boundary conditions:

$$Au - f = 0, \quad u|_{\Gamma_1} = \alpha, \quad \partial u / \partial n|_{\Gamma_2} = \beta \quad (1)$$

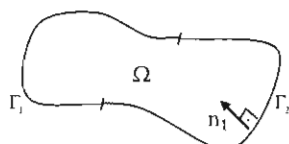


Figure 1. Geometry of the domain and boundary

Generally, the solution can be written in the form of the solution structure in which some

components are unknowns, while the other components can be determined accurately in advance:

$$u = u(\omega, \Psi, \Phi) \quad (2)$$

The solution component ω in the solution structure (2) exactly introduces all informations on the domain geometry. So, ω is called the primary function of the solution and is constructed using the rules of Boolean algebra and the following formulas of Rvachev's "semi-algebra" (Shapiro, 2007) which are also called R-operations of conjunction, disjunction and complement:

$$\begin{aligned} F_1(f_1, f_2) &= f_1 \wedge f_2 = f_1 + f_2 - \sqrt{f_1^2 + f_2^2} \\ F_2(f_1, f_2) &= f_1 \vee f_2 = f_1 + f_2 + \sqrt{f_1^2 + f_2^2} \\ F_3(f) &= -f \end{aligned} \quad (3)$$

Figure 2 illustrates analogy of Boolean algebra with (3).

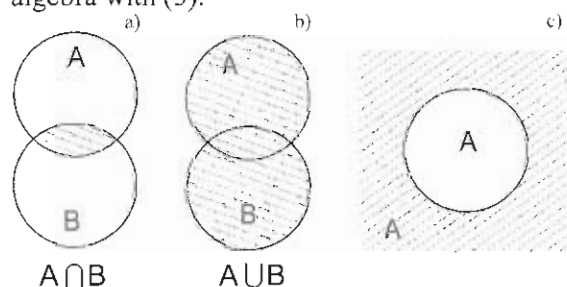


Figure 2. a) Section and b) Union of the sets f_1 and f_2 , and c) Complement of set f .

The second component Ψ accurately introduces information on all boundary conditions and on some part of boundary corresponds to the given boundary conditions. This component represents a smooth function defined in the whole domain and is determined by the following expression:

$$\Psi(x, y) = \frac{\sum_{i=1}^n u_i \prod_{j=1, j \neq i}^n \omega_j}{\sum_{i=1}^n \prod_{j=1, j \neq i}^n \omega_j} \quad (4)$$

The third component Φ is the unknown component in the solution structure (2) and can be determined as a linear combination of finite or infinite number of chosen basis functions from appropriate vector space.

We usually use a finite number of basis functions in the form:

$$\Phi(x, y) = \sum_{i=1}^n c_i \cdot \phi_i(x, y) \quad (5)$$

The best choice of coefficients c_i in the linear combination (5) is determined by applying one of the standard numerical procedures.

3. Results and discussion

The torsion problem of prismatic bars (Gotovac and Kozulic, 2000), Figure 3, is analyzed. It is described with differential equation and associated boundary conditions:

$$\nabla^2 u = -2, \quad u|_{\Gamma_0} = 0, \quad u|_{\Gamma_1} = C^{(i)} \quad (6)$$

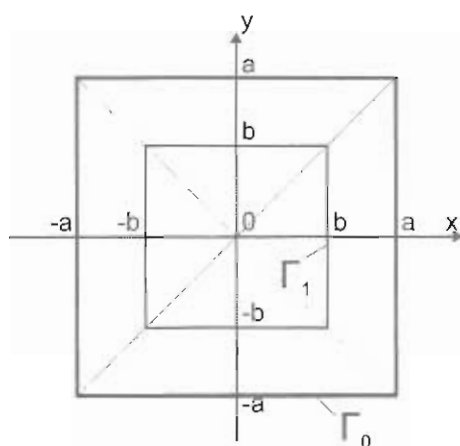


Figure 3. The cross-section of prismatic bar

The solution structure of the problem (6) is:

$$u \approx u_n = \frac{C^{(i)} \omega_0}{\omega_0 + \omega_1} + \omega_0 \cdot \omega_1 [C_0 + C_1(x^2 + y^2) + C_2 x^2 y^2 + C_3(x^4 + y^4) + \dots] \quad (7)$$

where

$$\omega_0 \equiv (a^2 - x^2)(a^2 - y^2) \geq 0$$

$$\omega_1 \equiv (b^2 - x^2)(b^2 - y^2) \geq 0$$

4. Concluding remarks

In this paper, the application of R- function method for solving torsion problem of prismatic bar is presented. Solutions are obtained using both formulations: with stress function and with warping function. Accurate solution is certainly between obtained two solutions, which allows easy determination of the numerical accuracy of the obtained numerical results. Comparison of results obtained with the proposed procedure and with the finite element method is also performed.

Acknowledgements

Authors acknowledge the support of the Ministry of Science, Education and Sports of the Republic of Croatia within the research project nb. 083-081541-1534.

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COMPARISON OF NUMERICAL MODELS FOR DYNAMIC ANALYSIS OF ARCH STRUCTURES

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1. Introduction

This paper presents two different numerical approaches for dynamic analysis of arch structures. By introducing inertial forces in the system of three differential equilibrium equations of the second order (Gotovac, Sesartić and Kozulić, 2009) the differential formulation for obtaining dynamic response of the curved beam is obtained.

The first numerical model is based on the finite element methodology by using exact curved beam elements with six degrees of freedom. The second numerical model is meshless model based on a procedure of strong formulation and using the Fup basis functions. The numerical results obtained with the curved finite elements are compared with the results obtained with the Fup basis functions and the other conventional numerical methods.

2. Finite element model for a curved girder

Dynamic analyses of arch structures have been carried out by several investigators using various types of curved beam elements. The equation of motion formulated in matrix notation is derived from the principle of virtual work (Przemieniecki, 1968):

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{P}(t) \quad (1)$$

Based on the algebraic-trigonometric shape functions (Figure 1.), the exact stiffness matrix $\mathbf{K}$ of curved girder element with constant curvature was derived (Gotovac, Sesartić and Kozulić, 2009; Litewka and Rakowski, 1998). This stiffness matrix is exact, totally free of the shear and membrane locking effects (Stolarski and Belytscho, 1983). By implementing exact stiffness matrix $\mathbf{K}$ and diagonal mass matrix $\mathbf{M}$ into eq. (1), a numerical model for dynamic analysis is developed. The system of dynamic differential equilibrium equations is formed by

the adaptive Runge-Kutta Verner method (Verner, 1994). Adaptive methods are designed to produce an estimate of the local truncation error of a single Runge-Kutta step (Simos and Papakaliatakis, 1998). This is done by having two methods in the tableau, one with the sixth order and one with the fifth order.

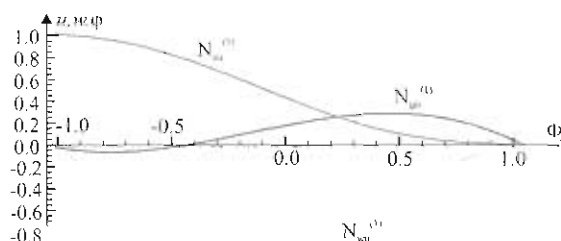


Figure 1. Exact basis functions due to displacement $u_1=1$

3. Meshless model for a curved beam

This model is based on the collocation method by using $Fup_2(x)$ basis functions (Figure 2.) which belong to the class of atomic basis functions (Rvachev and Rvachev, 1979).

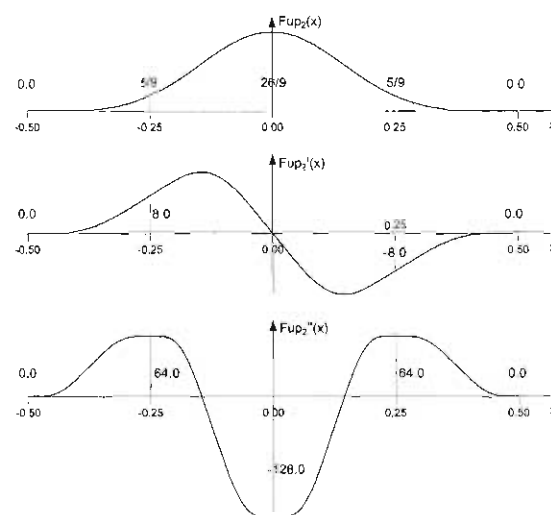


Figure 2. Fup_2 basis functions and its first two derivatives

The Fup basis functions are infinitely differentiable finite functions with compact support. These basis functions have excellent

approximation features as well as the very important feature of universality, which enables the consistency of the adaptive procedure (Gotovac and Kozulić, 1999). The system of three differential equilibrium equations of the second order for dynamic analysis is simultaneously satisfied in collocation points of closed domain, while boundary conditions are satisfied exactly at the domain boundary.

4. Numerical examples

In order to demonstrate the accuracy, reliability and applicability of the present numerical models the computer programs have been developed. Presented numerical models enable monitoring of shear and membrane influences on displacements (Figure 3.). Sample problems of a curved beam with various cases of boundary conditions and dynamic loads are solved with both numerical models. The obtained numerical results are then compared.

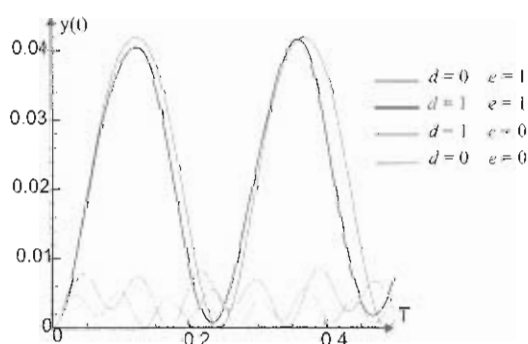


Figure 3. Influences of shear (d) and membrane (e) lockings on radial oscillations

5. Concluding remarks

In this paper two numerical models of arch structures in dynamic analysis are developed. Direct comparison of numerical results based on these two different numerical methods was presented. We can conclude that shear and membrane effects have a vital influence on dynamic responses of the arch structures.

Acknowledgements

This research has been supported by project number 083-0831541-1534 of the Ministry of Science, Education and Sport of the Republic of Croatia.

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MOVING LOAD ANALYSIS USING SPECTRAL METHOD

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1. Introduction

Moving vehicles analysis is an important and demanding task in civil engineering characterized by its non-conservative nature. In its strong form the problem is described as a system of differential or partial differential equations. Time discretization is usually based on finite difference technique of the Newmark type (like in Podworna, 2011). Due to certain difficulties a modified Newmark could be advantageous (see Kožar, 2009). Space discretization used in this paper is based on Chebyshev spectral method (Trefethen, 2000). Mathematically Chebyshev spectral method has been somewhat modified so that it is completely formulated using matrix operators and its' use resembles stiffness matrix approach in structural analysis. Boundary conditions of Dirichlet and Neumann type are taken into account using Lagrange multipliers.

2. Model formulation

Chebyshev polynomial is a polynomial of degree N defined in points x_j according to the equation

$$p(x) = \prod_{j=0}^N (x - x_j) \quad x_j = \cos\left(j \frac{\pi}{N}\right), \quad j = 0, 1, \dots, N$$

Detailed description can be found in specialized literature (e.g. Trefethen, 2000). Here we will address some details specific to analysis of plates under the moving load.

Application of spectral method consists in solution of the strong formulation (differential equation) of the problem (static or dynamic). The method can be formulated using matrix differentiation operator D_N . Spectral methods produce full differentiation matrix, which means that all points are involved in getting the result. One may argue that that is the reason for high accuracy of spectral methods.

Combination of of D'Alembert's principle and constraints (e.g. from boundary conditions) leads to the expanded differential equation describing constrained structure under the moving load

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} \frac{\partial^2}{\partial t^2} \begin{pmatrix} u \\ \lambda \end{pmatrix} + \begin{pmatrix} \Delta\Delta & C^T \\ C & 0 \end{pmatrix} \begin{pmatrix} u \\ \lambda \end{pmatrix} = \begin{pmatrix} F(x,t) \\ 0 \end{pmatrix}$$

$$F(x,t) = p(t) \cdot \delta(x - v_x \cdot t)$$

M is the mass matrix, 0 s are zero matrices of the appropriate size, $\Delta\Delta$ is spectral operator playing the role of the stiffness matrix, λ is vector of Lagrange multipliers, u is the displacement and $F(x,t)$ is loading function. In order to apply Newmark class of integration schemes the equation has to be rewritten in the incremental form. Results for moving constant force are shown in Figure 1.

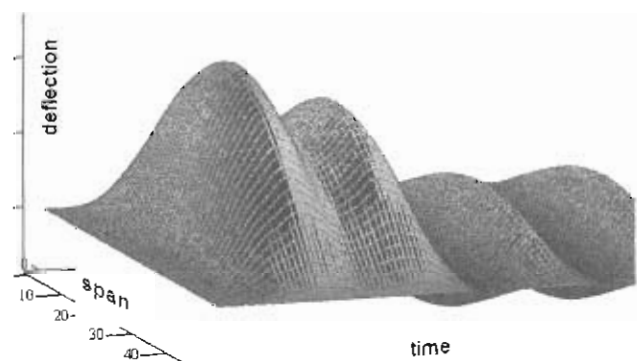


Figure 1. Displacement in time of beam under the moving load.

Mass and "stiffness" matrices could be rather stiff for some boundary conditions. Introduction of 0^{-20} (very small number in place of 0 s) can sometimes improve the behavior of numerical procedures (especially eigenvalue analysis). Additionally $\Delta\Delta$ matrix can be

stabilized by introduction of rigid body modes (Ibrahimbegovic and Mamouri, 2002). In our case numerical procedures in MathCAD 13 performed well with 0s but those in MathCAD 11 required the trick. In order to asses quality of the solution, condition number for spectral operators have been calculated. Among many possibilities we have chosen L1 norm based on singular values of a matrix. In contrast to condition numbers based on other norms it can be evaluated for singular matrices as well. Actually, we are dealing with nearly singular matrices bnt due to limited calculation capabilities the practical performance can be similar to singular matrices. In Table 1 there are condition numbers for some examples. Beam is simply supported beam, plate 4 is plate simply supported on 4 sides (with different aspect ratios).

	Beam	Plate 4 1:1	Plate 4 1:5
Dirichlet	N=49 $1.493 \cdot 10^{10}$	N=961 $4.263 \cdot 10^8$	N=713 $5.025 \cdot 10^7$
singular	N=51 $7.761 \cdot 10^{20}$	N=1089 $7.411 \cdot 10^{19}$	N=825 $1.362 \cdot 10^{20}$
r.b.m.	N=51 $1.750 \cdot 10^{20}$	N=1089 $1.633 \cdot 10^{21}$	N=825 $4.480 \cdot 10^{19}$
Lagrange	N=55 $8.804 \cdot 10^{15}$	N=1345 $1.226 \cdot 10^{27}$	N=1049 $1.813 \cdot 10^{27}$
Penalty	N=51 $1.235 \cdot 10^{17}$	N=1089 $2.889 \cdot 10^{16}$	N=825 $2.630 \cdot 10^{16}$

Table 1. Sizes and condition numbers of spectral stiffness matrices for various structures

In the first column of Table 1 there is an explanation of boundary conditions: "Dirichlet" – homogenous boundary conditions imposed through removal of points where displacement values is known to be zero, "singular" – full sized matrix without any boundary conditions, it is singular, "r.b.m." – above matrix with removal of rigid body modes, not singular any more, "Lagrange" – r.b.m. matrix expanded with boundary conditions and Lagrange multipliers, "Penalty" – r.b.m. matrix with boundary conditions imposed through penalty number (α from 10^8 to 10^{10}).

3. Results and discussion

Condition number alone can not give the whole picture about matrix usability. It is evident from Table 1 that the removal of rigid

body modes can worsen the condition number but at the same time matrix becomes non-singular and can be used in calculations. Also, introduction of Lagrange multipliers seem to deteriorate the performance but it is only the multiplier part of the matrix that is badly conditioned, the rest behaves well and the results are quite acceptable. Introduction of techniques that would separate Lagrange multipliers from the rest of the matrix would further confirm this statement.

4. Concluding remarks

Discretization based on spectral method in its form resembles the stiffness matrix and time discretization can be performed using any suitable method. The spectral differential matrix operator is fast and simple to construct. Boundary conditions can be treated in several ways but Lagrange multipliers offer the most general approach suitable even for the most complicated boundary conditions. Several methods have been tried out and the condition numbers of the resulting matrix operators are presented. Penalty method requires special mentioning since it has the best condition number but fails in complicated cases where the number of constraints is large (compared to size of the stiffness operator). That remains the subject of further investigation.

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NUMERICAL ANALYSIS OF 2-D LINEAR ELASTIC PROBLEMS BY MLPG METHOD

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1. Introduction

Although meshless methods have attracted considerable attention of the scientific community over the last two decades, high numerical costs associated with the calculation of meshless approximation functions and inefficient numerical integration still represent serious obstacles in their commercial use. The mixed Meshless Local Petrov-Galerkin (MLPG) Method paradigm (Atluri et al., 2004) represents an efficient remedy for these deficiencies, and has been successfully applied for solving certain demanding engineering problems, such as the bending of thin shells (Sorić and Jarak, 2010). This contribution deals with the application of the MLPG approach in two-dimensional (2-D) linear elasticity, whereby special attention is dedicated to the mixed strategy.

2. MLPG method for 2-D elasticity

For the stationary state of a linear elastic 2-D solid continuum shown in Fig. 1, the strong form of governing equations is given by the equilibrium equations, written in the global Cartesian coordinate system as

$$\sigma_{ij,j} + b_i = 0; i = 1, 2; j \in \{1, 2\}; \text{ in } \Omega. \quad (1)$$

In the above equation, b_i are the components of the body force vector and σ_{ij} stands for the components of the stress tensor. Ω denotes a global computational domain, which is bounded by a global boundary Γ , $\Gamma = \Gamma_u \cup \Gamma_t$, on which the following boundary conditions (BCs) are prescribed

$$u_i = \bar{u}_i, \text{ on } \Gamma_u; t_i = \sigma_{ij} n_j = \bar{t}_i, \text{ on } \Gamma_t. \quad (2)$$

Herein, $\bar{u}_i$ and $\bar{t}_i$ are the prescribed displacements and surface tractions, respectively, while n_j denotes the direction cosines of the unit outward normal vector to Γ .

A set consisting on N field nodes $\mathbf{X}_I; I = 1, 2, \dots, N$, are used to discretize Ω . Unknown field variables are approximated by employing meshless approximations, such as the Moving Least Squares (MLS) scheme (Atluri 2004) or the Polynomial Point Interpolation Method (PPIM) (Liu, 2003), where the nodes do not have to be connected in the global meshes of adjacent elements.

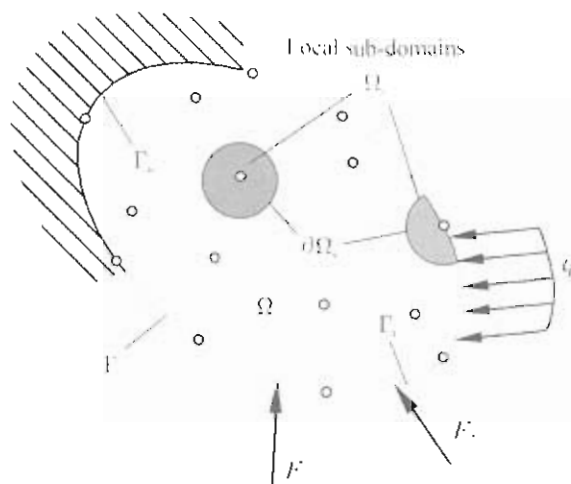


Figure 1. Global domain Ω with local sub-domains Ω_s

According to the MLPG method, a small domain, called a local sub-domain and denoted as Ω_s^I , is defined around each node $\mathbf{X}_I$, as shown in Fig. 1. Over each Ω_s^I , the governing equations (1) are written in a local weak form (LWF) by employing the weighted residual approach,

$$\int_{\Omega_s^I} v_{ki} (\sigma_{ij,j} + b_i) d\Omega = 0, \quad I = 1, 2, \dots, N. \quad (3)$$

Herein, $v_{ki} = \delta_{ki} v$; $k = 1, 2$, describes two linearly independent sets of test functions. δ_{ki} stands for the Kronecker delta symbol and v denotes an arbitrary kinematically admissible test function. As a special case, if the Dirac delta function $v = \delta(\mathbf{X} - \mathbf{X}_I)$ is employed for each Ω_s^I , LWF (3) transforms to

$$\sigma_{ij,x^i}(\mathbf{X}_I) + b_i(\mathbf{X}_I) = 0, \quad (4)$$

and a classical collocation method is obtained.

In purely displacement-based (primal) formulations, a displacement field appears as the only unknown independent field, and $\sigma_{ij} = \sigma(u_i^{(h)})$, where $u_i^{(h)}$ denotes the approximated displacements. On the other hand, in the mixed MLPG strategy, some additional field variables, such as certain strain or stress components, are approximated separately from the displacements. Depending on the choice of test functions and trial functions, as well as their supporting domains, various different meshless methods may be derived (Atluri, 2004).

3. Results and discussion

A thick beam with the dimensions $L \times D = 24 \times 4$ and the unit thickness is subjected to the BCs indicated in Fig. 2. The Young's modulus is $E = 2 \times 10^5$, while the Poisson's ratio is $\nu = 0.3$.

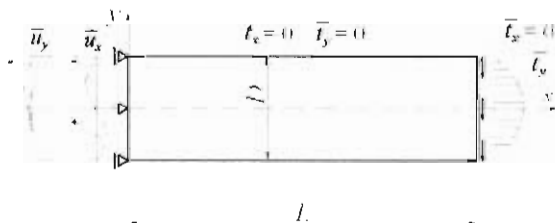


Figure 2. BCs for thick beam

Fig. 3 depicts the results of a convergence test, obtained by the mixed formulations that employ the separate approximation of stresses and displacements. Therein, the relative error in the L2-norms of nodal displacements, denoted as r_u , is used. The tested formulations are based either on the weak forms (WF) or the collocation (COL) of the governing equations (1). The meshless functions schemes with the second-order basis are applied. MLS-T denotes the MLS functions where the delta property is achieved by the transformation of the nodal variables (Atluri, 2004), while IMLS stands for the MLS functions obtained by utilizing regularized weight functions (Sorić and Jarak, 2010). In the collocation algorithms, the force

BCs are enforced by employing a penalty method.

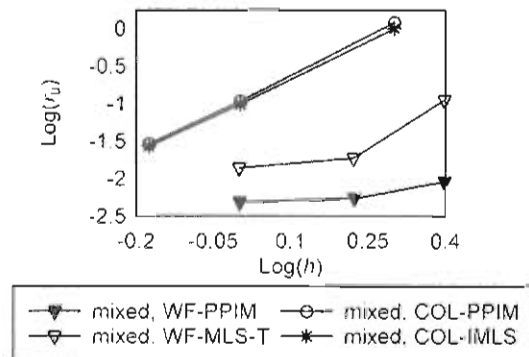


Figure 3. Results of convergence study

Higher accuracy is obtained if the WF approach is used. Both applied approximation schemes yield similar results in the collocation, but in the case of WF, PPIM performs significantly better than MLS-T. This can be attributed to the complications aroused by the transformations of nodal variables.

4. Concluding remarks

The MLPG method has been applied to the linear elastic 2-D problems. By employing the local Petrov-Galerkin approach various different meshless formulations have been developed. Their efficiency has been compared by appropriate numerical tests, and the obtained results imply the superiority of the mixed formulations based on WF.

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A NOVEL PRESSURE-VELOCITY COUPLING ALGORITHM

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1. Introduction

During last two decades research efforts within the field of CFD has been directed to solving physically and computationally more demanding problems, leaving the problem of pressure-velocity coupling in solving Navier Stokes equations somewhat neglected. Finite volume methods for solving fluid flow problems are mostly based on SIMPLE-like methods. SIMPLE algorithm was originally defined on staggered grids [1] on which it is consistent and physically clear. On collocated grids SIMPLE-like methods require non-physical pressure boundary conditions and use of arbitrary interpolations [2] with a possible negative consequence to the solution accuracy. Their convergence rate strongly relies on method's underrelaxation factors. The theoretical research [3] here presented has led to a new pressure-velocity coupling method – FLOP algorithm (Flux LOoping for Pressure drop) which does not suffer from such drawbacks on collocated grids, nor requires underrelaxation factors and that has a significantly higher convergence rate than SIMPLE-like methods, while keeping the segregated solving procedure.

2. Mathematical model

Incompressible fluid flow is governed by the continuity and momentum equations:

$$\frac{\partial v_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_j v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left(\mu \frac{\partial v_i}{\partial x_i} \right), \quad (2)$$

here, written with denotations: t – time, x_i – Cartesian coordinate, ρ and μ – constant fluid density and dynamic viscosity, v_i – velocity field and p – pressure field.

3. FLOP algorithm

Following the idea of the original formulation of the SIMPLE algorithm on staggered grids, in the new algorithm on collocated grids the mass fluxes F through the finite volume's faces are chosen to be the main variables. When mass fluxes satisfy the continuity equation, the velocities at the volume's centres may be obtained by the following interpolation:

$$v_i = \frac{1}{\rho \Delta V} \sum_{k=1}^{nb} x_i^k F^k, \quad (3)$$

where nb designates the number of volume's faces, while x_i^k designates the vectored distance from the volume's centre to the respective face centre. Once the F and v_i are known, the pressure gradient at volume's centres may be calculated from the discretized form of equation (2). By applying (3) to (2) for a finite volume with centre A, this is formally written as:

$$\left. \frac{\partial p}{\partial x_i} \right|_A = f_i(F^k), \quad k = 1, 2, \dots, nb_A \quad (4)$$

From all possible solutions for mass fluxes that satisfy continuity equations, the solution of a given problem is the one that defines a unique pressure field.

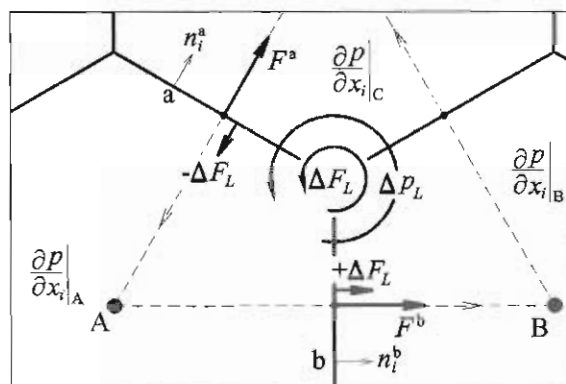


Figure 1. FLOP algorithm variables assembly

The pressure gradient field defines unique pressure field if it is irrotational. The

irrotationality condition is rewritten in form of zero circulation condition ($\Delta p_L=0$) for (4) integrated along a sufficient number of closed loops. Next, Δp_L -s are rewritten in form of functions of ΔF_L -s – the unknown corrections of fluxes required to achieve $\Delta p_L = 0$. The resulting system is solved by a Newton's method. As depicted on Fig.1, the ΔF_L -s are such assembled that continuity equation remains satisfied after each correction.

By solving the Laplace's equation, the algorithm's procedure starts from mass fluxes F which satisfy the continuity equation. Thereafter, the algorithm (due to linearization of the convection terms) iterates on the solution for corrections ΔF_L -s, to obtain a zero (sufficiently small) Δp_L in all loops, which then is - the solution of the problem. Finally, the actual pressure field is reconstructed from such obtained pressure gradient field.

4. Comparison of FLOP and SIMPLE algorithms

The new FLOP algorithm was coded into own computer program and compared with the SIMPLE algorithm, professionally (presumably -optimally) coded within the FLUENT software package. In comparisons performed, both algorithms used the same EDS [4] differencing scheme and identical grids. The comparisons were made on a basis of solution for the velocity field at the same level of accuracy in respect to the solution obtained at the limit of machine precision. The conducted tests covered incompressible, inviscid and laminar flow problems ranging from low to high (at the verge of steadiness) Reynolds numbers for 2D and 3D problems, with various boundary conditions and grid sizes. A typical example is the one of a well known 2D lid-driven cavity laminar flow at $Re = 100$, for which comparison is depicted on Fig. 2.

5. Some concluding remarks

For all conducted comparisons, the algorithms behavior is similar to one depicted on Fig. 2.: a significantly higher convergence rate and efficiency of the FLOP algorithm is noted, along with independency of No. of iterations on grid size. Due to only physical interpolations used and fulfillment of the continuity equation constraint during the

iterative process, the new FLOP algorithm does not require any stabilization procedure. This is opposite to all SIMPLE-like algorithms that require underrelaxation factors for which optimal values are problem dependent. Being a novel method, current adversity is the matrix of the linear system for ΔF_L -s, for which's solution a direct solver is used. That issue should be hopefully resolved within future researches.

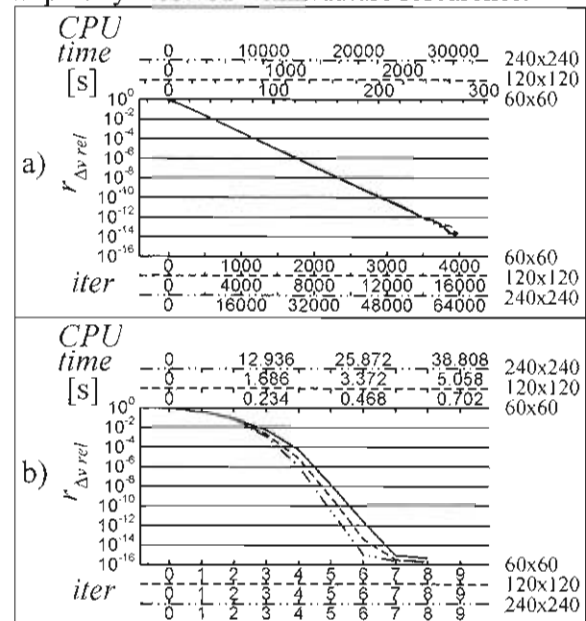


Figure 2. 2D lid-driven cavity laminar flow at $Re = 100$, solved for three different grid sizes. The convergence histories of own solutions in terms of the average relative difference in velocity field in respect to the own solution obtained at machine precision - $r_{\Delta v_{rel}}$. Histories given for: a) SIMPLE algorithm; b) FLOP algorithm.

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MULTY-PHYSICS AND MULTIPLE-SCALE MODELLING

FLUID-STRUCTURE INTERACTION ANALYSIS OF A DARRIEUS WATER TURBINE

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1. Introduction

The lack of a new stable green energy resources there have caused adapted wind energy extraction technologies being used for river stream energy extraction, which extract kinetic energy of a free stream as well. Their development is further enhanced by using modern computer simulations. In order to improve the technology for energy extraction, they provide us a valuable tool for numerous virtual experiments prior to real one, so the testing can be made on already pre optimised versions of design. The latest simulation procedures, like the FSI enables us to include numerous influential factors into single simulation, which corresponds better to reality where all the factors determine behaviour of a structure simultaneously. The scope of present study is to test a procedure for predicting important design parameters of a Darrieus water turbine regarding structural integrity and due to hydrodynamic loading in mechanical part, as well as performance change due to deformation of that structure in fluid dynamics part. The results will show us if the flexible blades produce any improvement in overall turbine output.

2. Simulation Setup

Simulations were carried out in Ansys software package, which allowed us straightforward transient FSI analyses in one package. The model of a 3 blades Darrieus turbine with horizontal axis was 1.8 m wide with 1 m in diameter. Its airfoil was Naca0012S profile projected on a circle of motion and it was made of steel with 3 mm thick wall without any reinforcements, so we could observe some large deformations. We wanted to inspect the highest stresses and deformations resulted by

hydrodynamic loading, as well as performance alterations due to the deformation of whole blade length and also their profile. The Darrieus turbine is known to operate with oscillating torque which causes oscillations in angular velocity as well. To imitate this behaviour, we had to include the results of an angular velocity from a previous study, where we simulated starting and operation behaviour of a flow driven Darrieus turbine using 6dof solver in Ansys CFX. We then compared the results of a FSI simulation with the control simulation, which had its blades modelled stiff. The comparison was made regarding overall efficiency, so we can estimate the contribution of structural deformation effects.



Figure 1. Post processed results of a simulation; the streamlines represent the stream velocity and blade deformations are 20× enlarged.

3. Results and discussion

The deformation of an airfoil profile was relatively small, so their contribution to overall efficiency was minimal. At a given length of the blades they deformed for a maximum of 4.6 mm in the middle, but they also twisted, so the angle of attack was changed as well. The results showed, that the overall torque of a turbine with flexible blades increased by 17%. However at

longer blades, which are needed to cross a large part of a river cross section, their deformation would increase significantly and FSI simulation would provide valuable design guidelines from structural point of view, to predict how big deformations and twisting would increase turbine performance as well as assure high efficiency and safety of operation.

4. Concluding remarks

FSI simulation of a Darrieus water turbine represents a valuable tool for more thorough study of its operating parameters. Darrieus turbine itself was first intended for wind energy extraction with vertical axis configuration. Its water application appeared recently in tidal energy extraction projects. Its use on rivers can improve their exploitation, especially where building of dams is impossible due to environmental or other grounds.

Acknowledgements

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EFFECT OF MICROSTRUCTURE ON THE FRACTURE TOUGHNESS OF POLYCRYSTALLINE DIAMOND AND POLYCRYSTALLINE CUBIC BORON NITRIDE

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1. Introduction

Polycrystalline diamond (PCD) and polycrystalline cubic boron nitride (PCBN) are super-hard materials used in demanding material removal operations such as rock drilling and high speed turning of aerospace alloys. They are composed of a sintered mass of diamond (or cubic boron nitride) particles of the order of 1-30 micron together with a binding agent. The binding agent is any one of a number of metal or ceramic phases. The advantage a polycrystalline material offers over a single crystal material is greater toughness due to the random orientation of the crystalline cleavage planes in the sintered mass. However failure due to fracture and chipping is still a major problem in the industry today.

2. Experimental Setup

In this study a combined experimental-numerical investigation was used to examine the effect of microstructure on the fracture toughness of both PCD and PCBN. The effect of grain size and binder quantity was examined independently using an adapted three-point bend fracture test method (Carolan *et al.* 2011). The cohesive zone parameters for each material investigated were extracted using the method described in (Carolan *et al.* 2011). Numerical modeling of the TPB test was carried out using the finite volume (FV) method.

Representative microstructural volumes were created in OpenFOAM-1.6-ext (OpenFOAM, 2010) to numerically examine the effect of change in microstructure on the bulk cohesive zone models determined experimentally. A typical microstructural mesh is given in Figure 1.

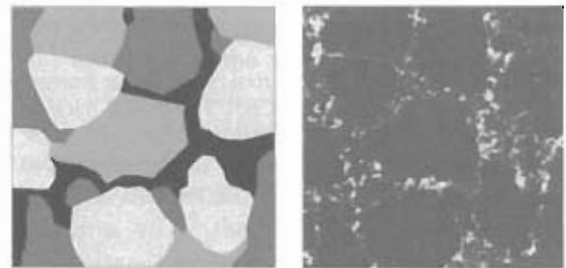


Figure 1. Numerical representation of PCD microstructure and associated SEM micrograph. The larger diamond grains are meshed directly while the binder rich region is modelled using representative material models.

3. Results and discussion

The effect of notch root radius observed experimentally was reproduced numerically using a unique CZM for each material as shown in Figure 2. It was found that the process zone size was approximately equal to the CBN grain size.

The results demonstrate clearly that the fracture properties of both PCD and PCBN are influenced by the underlying microstructure of the material under investigation. In particular the critical fracture toughness was found to increase with increasing grain size as shown in Figure 3.

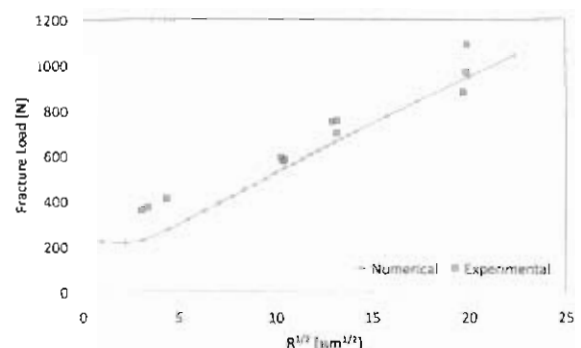


Figure 2. Experimental results and CZM predictions for 'apparent' fracture toughness versus notch root radius for PCBN of varying grain size (Carolan, 2011).

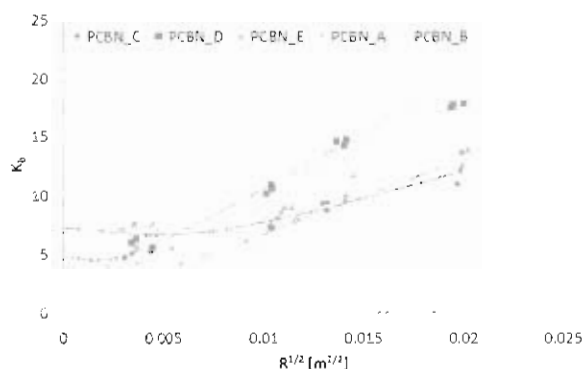


Figure 3. The effect of notch root radius on the fracture toughness of five grades of PCBN. The critical fracture toughness is largest for the PCBN with the largest CBN, while the effect of notch root radius is more pronounced for the material with smaller CBN grain size (Carolan *et al.*, 2011).

It is proposed that local thermal effects ahead of a dynamically loading crack tip are the source of the experimentally observed drop in fracture toughness for these materials at high rates of loading. The properties and quantity of the binders are found to play a key role in governing this behaviour.

4. Concluding remarks

The structure property relationship of super-hard materials is very complex. The composite nature of the bulk material introduces several extra variables than for single phase materials. The difficulties in producing large quantities of material also mean that experiments must be carried out on extremely small samples. The current work has demonstrated, using a combined experimental-numerical approach, the effect of the most important of these variables on the bulk properties of the material.

The next step is to move from using numerical modelling as a tool for explaining how the behaviour of existing grades of material and use it as a design tool, i.e. virtual material design.

Acknowledgements

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MULTISCALE MODELLING OF DEFORMATION PROCESSES OF NODULAR CAST IRON

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1. Introduction

In recent years, a special attention has been paid to investigate relations between the microstructure and mechanical properties of materials since almost all materials used in engineering are micro-heterogeneous. It is well-known that the classical continuum mechanics does not consider structural effects in the material at the micro-level. To overcome this problem, multiscale techniques have been developed for modelling of macroscopic behaviour of heterogeneous materials where the results obtained by analysis of a representative volume element (RVE) (micro-level) are used as an input to the continuum model (macro-level) [1]. To solve the problem at the micro-level the finite element (FE) method is mostly applied [1], [2], [3].

This paper provides an overview of the studies that are performed in order to investigate the influence of the size and distribution of graphite nodules in a ferrite matrix on the mechanical behaviour of the ductile cast iron EN-GJS-400-18-LT. To obtain more accurate results second order homogenization algorithm is applied, as shown in [1]. The aforementioned algorithm is derived for the plane stress and plane strain state assuming the elastoplastic behaviour of the material constituents. For the computational homogenization purposes on macro-level, a strain gradient finite element with C^1 continuity is applied, originally proposed in [4] for plate bending problems. The finite element and computational homogenization algorithm derived are implemented into the FE program ABAQUS/Standard using user-defined subroutines [5].

2. Second-order computational homogenization algorithm

Figure 1 presents the scheme of the multiscale computational algorithm based on

the second-order homogenization technique. The left side of the figure shows the macro-level model, while the right side depicts a representative volume element (RVE) on the micro-level. Herein, the RVE is discretized by the first-order quadrilateral four-node finite elements, while the strain gradient three-node triangular elements are used on the macro-level. The RVE is the smallest material volume that contains all essential information about microstructure, and therefore, the RVE has to be defined as the statistically representative sample of the material on the micro structural level. The computational homogenization algorithm based on the linear displacement boundary condition on the RVE, consists of the steps shown in Table 1.

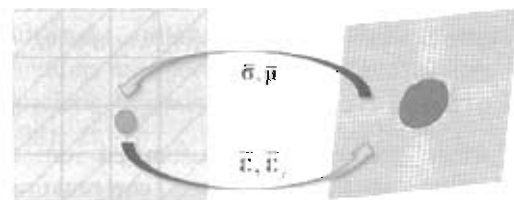


Figure 1. Multi-scale computational scheme

As may be seen from the Table 1, the micro-level analysis is performed using the macroscopic strain, $\bar{\epsilon}$, and the strain gradient, $\bar{\epsilon}_{,i}$, which are then transformed into the boundary nodes displacements of the RVE. Thereafter, the macroscopic stress $\bar{\sigma}$, the double stress $\bar{\mu}$ and the tangent stiffness $\bar{C}$ are obtained from the solution of the RVE boundary value problem at each macro-level integration point by averaging over the RVE.

3. FE formulation and verification

The 2D strain gradient triangular FE used in the present work has three nodes with six degrees of freedom per node for each displacement component (the displacement, two first and three second derivatives).

Table 1. Computational homogenization algorithm

1. Define strain increment, $\Delta \bar{\epsilon}$, and strain gradient, $\Delta \bar{\epsilon}_{,i}$, at the macro level.
2. Calculation of the displacement increment of RVE boundary nodes.
3. Solution of the RVE boundary value problem.
4. Calculation of the macroscopic stress, $\bar{\sigma}$, the double stress, $\bar{\mu}$, and the tangent stiffness, $\bar{C}$.

In order to verify the FE formulation, several benchmark examples available in the literature are considered. Firstly, a cantilever beam subjected to bending is analysed as shown in Fig. 2. Here, the vertical displacement at point A of presented FE discretization is compared with the displacements obtained by the triangular and quadrilateral plane elements from ABAQUS library. It is shown that the element passes the patch tests and it can accurately describe strain gradient problems.

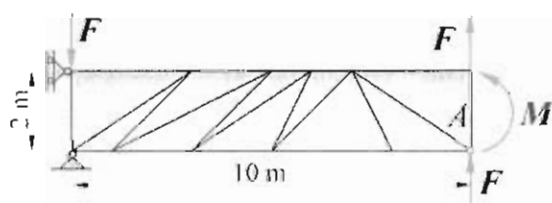


Figure 2. A cantilever beam subjected to bending

4. Further investigations

Further study will be focused on the application of the developed computational homogenization algorithm to simulate the deformation process of the nodular cast iron EN-GJS-400-18-LT with four different microstructures. These microstructures are obtained by four casting technologies: Osmosis, Flotret, Tundish and Inmould.

For the multiscale simulation, mechanical properties of the micro-constituents (Fig. 3) will be taken from the literature [6]. Besides, based on a comprehensive metallographic analysis, the averaged microstructural characteristics will be determined to define the RVE for all investigated material types.



Figure 3. Nodular cast iron microstructure counterparts (Flotret process)

5. Conclusion

The paper presents a strain gradient triangular finite element and the second-order computational homogenization algorithm derived for plane strain and plane stress problems assuming the elastoplastic behavior of the material micro-constituents. The FE presented and the computational algorithm are implemented into the FE program ABAQUS. The robustness of the FE formulation is verified by using standard patch tests. The proposed numerical formulation will be used to model deformation processes of the nodular cast iron with four different microstructures. Finally, the numerical model will be verified by comparing the numerical solutions with the experimental data.

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SPATIAL COUPLING OF MULTIBODY SYSTEM WITH VISCOUS FLOW DYNAMICS IN PARTITIONED ALGORITHMS

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1. Introduction

The analysis of aeroelastic phenomena is one of the very important topics in aeronautical engineering. These phenomena include critical high frequency effects such as flutter, but low frequency aeroelastic effects can also have a significant influence in branches such as flight mechanics and flight control. Theoretical analysis of flutter is typically constrained to linear problems, so numerical procedures for an analysis of a more complex case studies are of high importance.

Since the structural and aerodynamic partitions possess quite different dynamic characteristics, the partitioned approach is typically used for integration of the coupled fluid-structure (FS) interaction problem. Generally, partitioned coupling can be accomplished in a weak and strong manner. After convergence of the coupling algorithm strong coupling conserves forces and energy transferred in each time step. Weak coupling, on the other hand, is performed without outer iteration in every time step, and overall conservation properties of weakly coupled method depend both on time and spatial coupling used. For spatial coupling, if aeroelastic interface is conserving virtual work (Farhat et al, 1998) it is referred to as conservative aeroelastic interface. Interface that conserves only force and moment, but not the virtual work, will be conveniently referred to as consistent aeroelastic interface, as done by (de Boer, van Zuijlen and Bijl, 2008).

Weakly coupled partitioned algorithm is depicted schematically in Figure 1. Used CSS method renders simulation first order consistent in time, and spatial interface is considered in virtual displacement terms.

In this paper the problem is observed in 2D, using a model of a wing with control surface in fluid flow, but the formulation is general so it is equally valid in 3D.

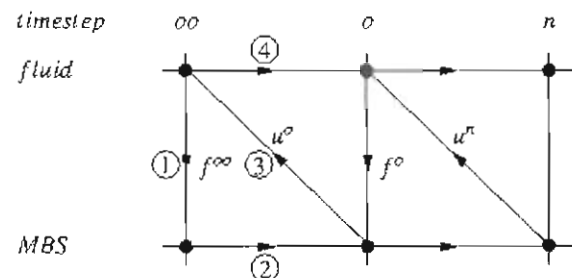


Figure 1. Conventional Serial Staggering (CSS).

2. Numerical model

A structural system is modelled as a multibody system (MBS), whose dynamical equations are modelled as differential-algebraic equations of index 3 (i.e. Terze and Naudet, 2008).

$$M(\mathbf{x})\ddot{\mathbf{x}} + \Phi_x^T(\mathbf{x}, t)\lambda = Q(\dot{\mathbf{x}}, \mathbf{x}, t),$$

$$\Phi(\mathbf{x}, t) = 0.$$

The Navier-Stokes mathematical model of incompressible viscous fluid is discretised with cell-centred basic finite volume (FV) method in the ALE formulation (i.e. Tuković and Jasak, 2012). The coupling of velocity and pressure field on collocated grid arrangement is accomplished by an approximate projection method (Rhie and Chow, 1983).

$$\frac{d}{dt} \int_V dV + \oint_{\partial V} \vec{n} \cdot (\vec{v} - \vec{v}_s) dS = 0,$$

$$\frac{d}{dt} \int_V \vec{v} dV + \oint_{\partial V} \vec{n} \cdot (\vec{v} - \vec{v}_s) \vec{v} dS = \oint_{\partial V} \vec{n} \cdot \nabla \vec{v} dS - \int_V \frac{1}{\rho} \nabla p dV.$$

A mathematical model used for the deformable mesh is pseudo-solid, described by Lamé-Navier (displacement formulation elastostatic problem) equations, which are also discretised by the cell-centred finite volume method.

In contrast to the work done in (Terze et al., 2009), where all MBS was represented with radial basis function interpolation (RBF) as one deformable pseudo-body, here interface is mapped rigidly on majority of FS interface. Only near the hinge, that connects airfoil and control surface in used example, a RBF is used to "patch" between the rigid bodies of MBS. That zone, which spreads on both bodies, absorbs topological discontinuities that occur in vicinity of the control surface joint. By following this route, elastic deformation is localised to a minor part of FS interface.

Particularity of the cell-centred collocated FV mesh arrangement is that the fluid-load evaluation points and the vertices at which mesh is defined do not coincide. So, on the polyhedral mesh the aeroelastic formulation is generally only approximately conservative. It will be shown in the paper that the consistent coupling on the rigid part of the interface is also conservative on any FV mesh arrangement.

3. Results and discussion

The described methodology is applied for a motion simulation of an airfoil with control surface in incompressible viscous fluid flow. In order to eliminate turbulence model influence, Reynolds number was chosen low enough to retain laminar flow on this type of airfoil for all angles of attack that occur during simulation.

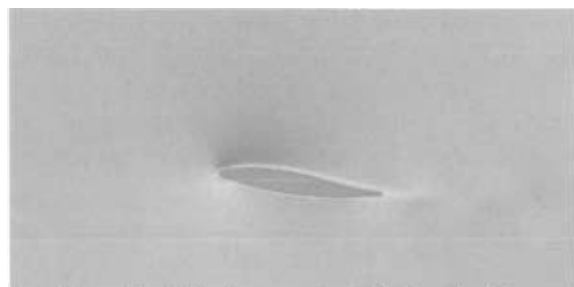


Figure 2. Multibody system in fluid flow. Red line indicates reference chord position.

4. Concluding remarks

Main goal of the paper is to develop spatial coupling interface that can be successfully applied for MBS on the structural partition side.

For that purpose, consistent RBF aeroelastic interface is constructed on minor parts of the interface. With the conservative rigid parts, by-major-parts conservative weakly coupled FSI solution algorithm is constructed for the simulation of MBS in viscous incompressible fluid flow. For each partition, a A-stable backward differencing formula is used.

Acknowledgements

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NANOMECHANICS AND NANOSTRUC- TURED MATERIALS

ESTIMATING HURST PARAMETER WITH PERIODOGRAM METHOD IN ROBOT LASER HARDENED PATTERNS GGG 60 AND GGG 60 L

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1. Introduction

In the microstructure of robot laser hardened material we see roughness surface. Hurst parameter is understood as the correlation of two random steps x_1 and x_2 , which are followed by time to time difference Δt . We focused on the point of robotic laser hardened pattern of standard codes to the DIN standard GGG 60 and GGG 60 L. We describes the experimental research process. Robotic laser hardening is used in the automotive, aerospace and military industries. Hurst parameter occurs in many fields of applied mathematics, including fractals and chaos theory. Hurst parameter is used in fields ranging from biophysics to network computers. Hurst parameter was originally developed in hydrology. However, modern techniques for assessing Hurst parameter coming from fractal mathematics. Hard martensitic microstructure provides improved surface properties such as abrasion resistance and high strength. In this work we have used scanning electronic microscopy (SEM), search and analyze the fractal structure of the robotic laser-hardened material.

2. Experimental Setup

We made samples of the materials of standard codes to the standard DIN GGG 60 and GGG 60 L. Robotic laser materials were ardened at different temperatures. So we change the parameter of temperature $T \in [800, 2000]^\circ \text{C}$ with a step of 100°C . Each sample was etching and were observed under the microscope. Pictures we made with the field scanning electron Emission Microscope JMS-7600F JEOL company.

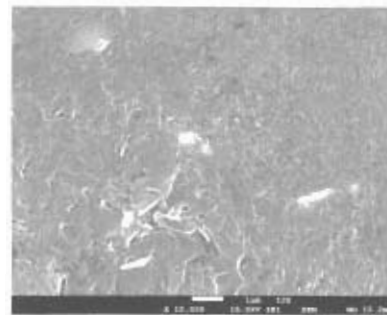


Figure 1. Microstructure of material GGG 60

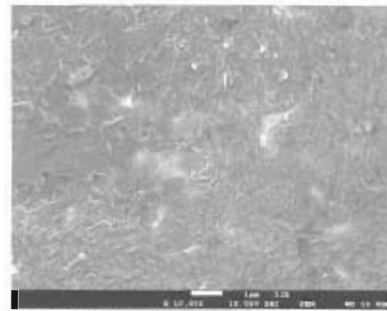
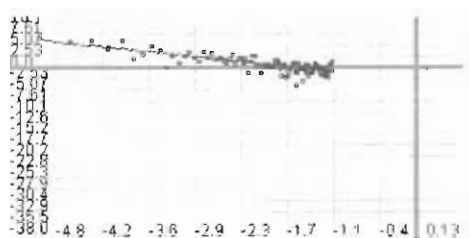


Figure 2. Microstructure of material GGG 60 L.

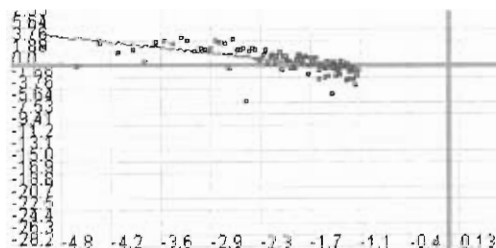
On picture 1 and Picture 2 we see microstructure of hardened material GGG 60 in GGG 60 L.

3. Results and discussion

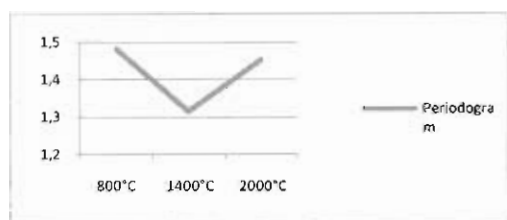
Methods for evaluation Hurst parameter H can be 'robust' basis divided into two categories, namely the assessment in the time domain and evaluation in the frequency or wavelet space. Assessment methods in time domain based on a comparison of the original process and the average process with the method of aggregation: variance method or adjusted scale is also a graphical method based on the properties Hurst phenomenon.



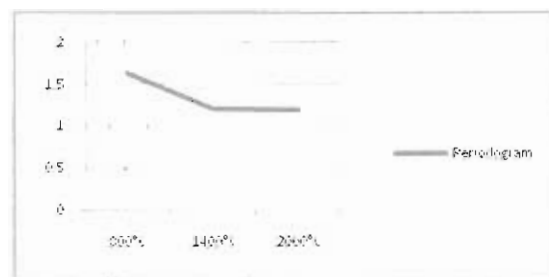
Graph 1: Periodogram method for material GGG 60 L
 $H=1.421$



Graph 2: Periodogram method for material GGG 60
 $H=1.175$



Graph 3: Hurst parameter H for material GGG 60 L



Graph 4: Hurst parameter H for material GGG 60

4. Concluding remarks

We did experiments on two different materials GGG 60 and GGG 60 L. We found the following:

1. Hurst parameter H we calculated with periodogram method.

2. Using relation $D = 2 - H$ can be calculated fractal dimension for all patterns.

3. Relationship between the microstructure of laser hardened patterns we can better understand through research Hurst parameter H .

Acknowledgements

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NUMERICAL MODELING OF THE NANOSTRUCTURES

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1. Introduction

Atomistic/molecular simulations have been performed by many researchers to confirm the strong size-dependency of nanostructures. However, the length and time scales for the atomistic simulations are still limited by the available computational power, and such methods are prohibitive in the analysis of nanoscale devices with complex geometries. It is therefore important to extend the continuum mechanics concepts as an efficient tool to analyze the mechanical response of nanoscale structures. On other hands there are growing interest to overcome this difficulties by multiscale simulation philosophy.

2. Numerical models

For the analysis of complex nanostructures geometries and boundary conditions encountered in nanotechnology applications, a versatile numerical model such as the coarse-grain method, quasi-continuum method concurrent and hierarchical multiscale method was developed. In this work carbon nanotube (CNT) is taken as example for numerical modelling of nanostructure.

A CNT is viewed as a geometrical frame-like structure and the primary bonds between two nearest-neighbouring atoms act like load-bearing beam members. Every carbon atom acts as the joint of the related load-bearing beam members. The elastic and sectional properties of the beam members (the carbon-carbon bonds) can be obtained by establishing a linkage between the microscopic computational chemistry and the macroscopic structural mechanics.

From the viewpoint of molecular mechanics, a CNT can be regarded as a large molecule consisting of carbon atoms. The general

expression of the total potential energy, omitting the electrostatic interaction, is a sum of energies due to valence or bonded interactions and non-bonded interactions.

$$U = \sum U_r + \sum U_\theta + \sum U_\phi + \sum U_\omega + \sum U_{vdw} \quad (1)$$

where U_r is for a bond stretch interaction, U_θ for a bond angle bending, U_ϕ for a dihedral angle torsion, U_ω for an out-of-plane torsion, and U_{vdw} for a non-bonded van der Waals interaction. For covalent systems, the main contributions to the total energy come from the first four terms of eq. (1). Under the assumption of small deformation, the harmonic approximation is adequate for describing the energy.

$$U = \frac{1}{2} k_r (\Delta r)^2 + \frac{1}{2} k_\theta (\Delta \theta)^2 + \frac{1}{2} k_\phi (\Delta \phi)^2 \quad (2)$$

where k_r , k_θ and k_ϕ are the bond stretching force constant, bond angle bending force constant and torsion resistance respectively and Δr , $\Delta \theta$ and $\Delta \phi$ are bond stretching increment, the bond angle change and the angle change of bond twisting, respectively. According to the theory of classic structural mechanics, the strain energy of a beam member is

$$U = \frac{1}{2} \int_0^L \frac{N^2}{EA} ds + \frac{1}{2} \int_0^L \frac{M^2}{EI} ds + \frac{1}{2} \int_0^L \frac{T^2}{GJ} ds \quad (3)$$

where N , M and T are axial force, bending moment, and torsion applied on the beam, respectively. A linkage between structural mechanics and molecular mechanics using equivalence of energies, the following relationships are derived:

$$\begin{aligned}\frac{EA}{\ell} &= k, \\ \frac{EI}{\ell} &= k_\theta, \\ \frac{GJ}{\ell} &= k_t,\end{aligned}\quad (4)$$

where A is cross sectional area, E modulus of elasticity, I and J moments of inertia and ℓ is length.

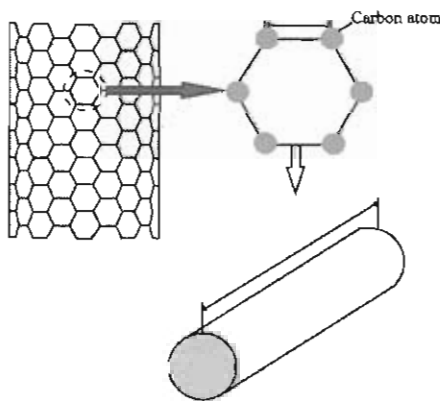


Figure 1. The molecular and structural mechanics analogy

The atomic finite element procedure in stiffness matrix method for CNT as space frame structures, result with matrix equation

$$[K]\{U\} = \{F\} \quad (5)$$

where $[K]$, $\{U\}$ and $\{F\}$ are stiffness matrix, displacement vector and force vector, respectively.

3. Results and discussion

A space-frame of the carbon nanotube (CNT) model is constructed by two different types of circular beam finite elements. The primary beams are used to model the bond stretching energy (link the nearest carbon atoms), while the secondary beams are employed to account for the potential associated with bond angle variation (bridging the next-nearest carbon atoms). A different CNTs type with representative volume element is chosen in the

present work, including zigzag and armchair. The influence of chiralities and CNT radii on effective Young modulus and Poisson ratio are presented by Fig. 2 and Fig. 3, respectively.

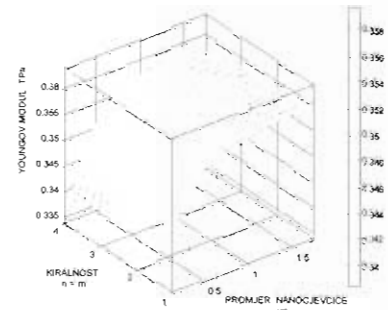


Figure 2. The effective Young modulus dependence on chiralities and CNT radii

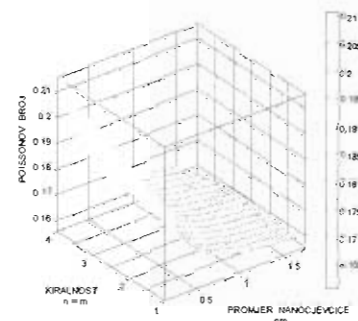


Figure 3. The effective Poisson ratio dependence on chiralities and CNT radii

4. Concluding remarks

The underlying cause of the size dependence of these properties at the nanoscale is the competition between surface and bulk energies. The effective elastic properties of carbon nanotube are determined by atomic finite element method.

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OPTIMAL DESIGN OF ENGINEERING STRUCTURES

PRECISION AND STABILITY OF MAXWELL-TYPE KINEMATIC MOUNTS

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1. Introduction

Kinematic mounts (KMs) are often used in high-precision applications since they are self-locating and free from backlash, allow sub-micrometric re-positioning, can accommodate differential thermal expansions and their behaviour can be represented in closed form (Slocum, 1992). Their main drawback is constituted by the high contact stresses that are to be analysed via the nonlinear Hertz theory (Hertz, 1895). The most common KM design configuration is the Maxwell-type KM constituted by 3 V-grooves on one end (typically the support) and 3 balls on the other end of the mount (typically the supported piece) so as to achieve an exact constraint of all 6 spatial degrees of freedom (Figure 1).

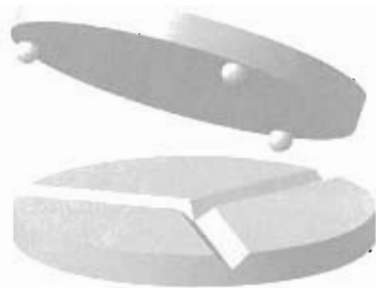


Figure 1. Maxwell type 3-V groove KM.

The aim of this work is to analyse the influence of mechanical parameters on KMs' behaviour and especially on their positioning precision and stability. An example of a KM used to support a large structure is considered.

2. Analysis of KM precision and stability

The analysis of the considered KMs comprises force and moment balance equations, expressions for the calculation of stresses and deflections at the contact points and error motion calculation. Knowing the external loads and the geometry, the loads at each groove-ball interface and the respective contact point reactions can be computed from the overall force and moment balances. The nonlinear behaviour

of point contacts between elastic isotropic solids, where the contact area is small compared to the radii of curvature and the dimensions of the involved bodies, encompasses then the calculation of the equivalent Young's modulus and contact radius. The elliptical contact area, the interpenetration distances and the contact stresses can hence be obtained.

It was shown that the approximate methods of calculation of the characteristic KM parameters are appropriate for most high-precision applications (Zelenika, 2004). By performing experimental measurements (Figure 2), it is thus proven that in the whole elastic deformations range the correspondence of the theoretical values with the experimental ones is within the intervals of uncertainty of the latter. The KMs' precision is shown to be comparable to the surface finish (100nm range).

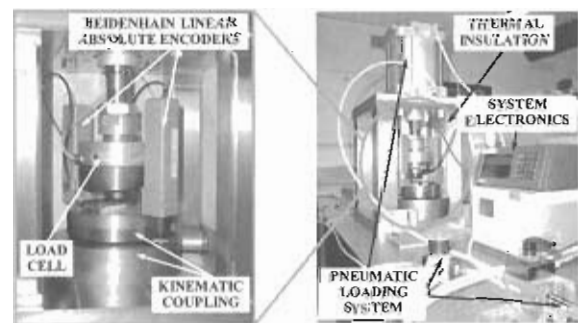


Figure 2. Used experimental set-up.

The calculated stresses and strains allow next, under the assumptions that the change of the distances between the KM supports is small, to calculate the couplings' error motions about its centroid (Slocum, 1992) –Figure 3.

3. Design example

The calculation procedure is hence implemented in structured software algorithms and used to assess the characteristics of a KM aimed at supporting a large vacuum chamber of a particle accelerator facility. The input data define the maximal support area diameter of 1 m and the load to be supported as 5 kN with an

additional 300 N lateral load. The KM elements are chosen having silicon nitride balls and tungsten carbide grooves.

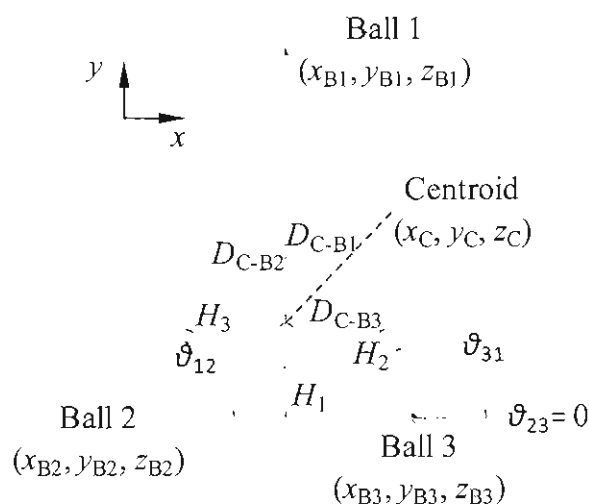


Figure 3. Top view of KM's geometry.

The stress calculations allow establishing that, for a certain coupling radius R_C and a maximal allowable stress q_{all} (Esk; Gimex), suitable ball radii are $R_b = 18$ mm (Figure 4) and V-groove arch radii are $R_g = -21.6$ mm.

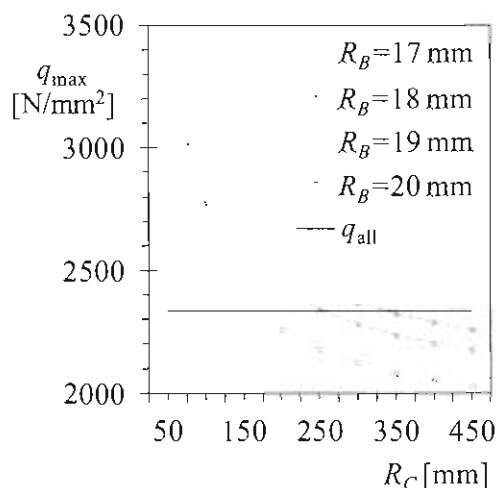


Figure 4. Dimensioning of the KM elements.

In terms of KM's stability, two design configurations are considered, i.e. one where the normals to the planes containing the contact forces are directed towards the centroid of the coupling (case A) and the other (case B) where the normals bisect the angles between the balls (Slocum, 1992). Obviously, the KM will lose its stability when one of the contact forces becomes negative. The stability conditions

could thus be determined for different KM geometries and lateral load orientations. In Figure 5, as an example, are shown the stability regions when the lateral load is directed as the positive KM x-axis and passes through the KM centroid. It could hence be shown that, when the length of the KM is extended with respect to its width, the configuration of case B is generally better from the stability point of view.

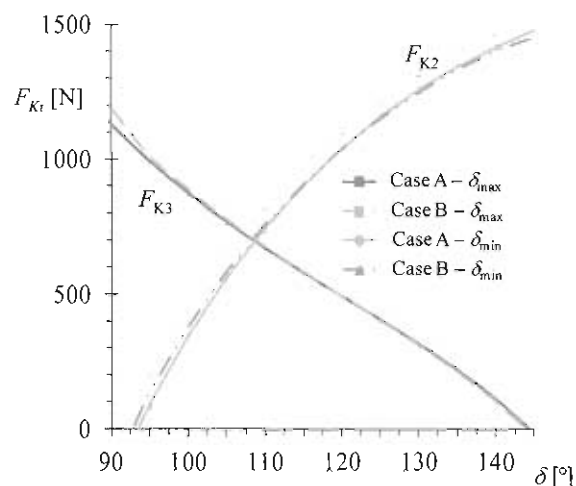


Figure 5. Stability region of various KM configurations.

4. Conclusions

A thorough analysis of the precision and stability of Maxwell-type KMs is performed. Suitable algorithms are implemented and applied to a design example, allowing the stability regions for different designs to be established.

Acknowledgements

The work is supported through the MSES project *Ultra-high precision compliant devices for micro and nanotechnology applications*.

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UNCERTAINTY OF ROUGHNESS PARAMETER PREDICTION

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1. Introduction

In many cases, there are strict requirements regarding surface roughness of mechanical components being produced. The surface roughness of details is defined in the drawing and measured after machining. It is known that the surface roughness depends on parameters of machining. The main problem of the technological process lies in the fact that first is the machining process and then is the measurement process of surface roughness. Therefore, we do not know in advance what roughness we obtain after the machining process.

There are various methods for prediction of surface roughness based on regression analysis or fuzzy logic. In order to predict the surface roughness it is necessary to have:

- prediction method,
- set of experimental data (surface roughness measured after different machining parameters and parameters themselves),
- set of technological process parameters for which we wish to predict the surface roughness.

For example, the following input parameters are used to predict surface roughness after milling: cutting speed v (m/min); feed f (mm/rev. or mm/tooth); depth of cut d (mm).

Surface roughness prediction methods facilitate selection the optimal technological parameters for machining process to obtain desired surface roughness.

2. Uncertainty estimation in general

To evaluate accuracy and precision of the measurement result, we are using the concept of

uncertainty. Each measured value must be accompanied by an estimate of its uncertainty.

Uncertainty is component of the measurement result which indicates the range in which true value lies with a given probability (usually 95%). Uncertainty is the best way of expressing the accuracy of measurement, as it includes both random and systematic errors.

In other words, uncertainty guarantees measurement results. For example, if measurement result is 0,5 mm and the uncertainty of this measurement result is 0.005 mm, then in this case we guarantee that true value of measurand may vary from (0,5-0,005 mm) to (0,5+0,005 mm) with the probability 95% (unless different probability is given) (see Figure 1).

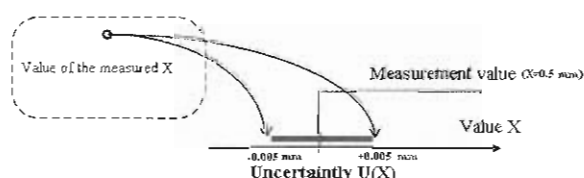


Figure 1. Measurement result X and its measurement uncertainty $U(X)$

3. Uncertainty of predicted roughness parameters

Regardless of prediction method, predicted value of roughness parameter can be represented as a function of many variables: experimental data (each technological process parameter and corresponding measured roughness parameter for each experiment) and a set of technological process parameters for which roughness parameter should be predicted.

$$Ra = f(x_1, x_2, \dots, x_n) \quad (1)$$

In its simplest form, the uncertainty of predicted roughness parameter depends on derivatives of the prediction with known input variables:

$$u(Ra) = \sqrt{\left(\frac{dRa}{dx_1} \cdot u(x_1)\right)^2 + \left(\frac{dRa}{dx_2} \cdot u(x_2)\right)^2 + \dots + \left(\frac{dRa}{dx_n} \cdot u(x_n)\right)^2} \quad (2)$$

Analytical determination of these derivatives is extremely complicated, especially in the case of using a model based on fuzzy logic. For this reason, our proposal is the use of numerical method to find the uncertainty $u(Ra)$ with known input parameters and their uncertainties.

4. Concluding remarks

Methods of surface roughness prediction do not provide uncertainty estimation of predicted value themselves. In this work, we are making an attempt to outline surface roughness prediction uncertainty estimation method.

Acknowledgements

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PLASTICITY

NUMERICAL SIMULATIONS OF S-RAIL FORMING FOR 6111-T4 SHEET BASED ON HILL STRESS FUNCTION

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1. Introduction

Constitutive modelling of the anisotropic plastic behaviour of sheet material is of great importance in the design and analysis of forming processes intended for these materials. The phenomenological plasticity theories based on associated flow rule are most frequently applied for metal materials. A realistic constitutive model for a sheet material based on associated flow rule requires a yield function with complex functional form and with considerable number of parameters in order to simultaneously describe anisotropy of yielding and anisotropy of plastic flow. Recently, finite element formulations based on non-associated flow rule and simple four parametric stress functions have been developed and successfully applied to sheet forming processes (Cvitanic et al., 2007, 2008, 2011). The objective of the present paper is to further investigate the associated and non-associated formulations based on simple quadratic four parametric Hill stress function (Hill, 1948). Analyzed formulations are evaluated in predicting S-rail forming process for sheet sample of aluminium 6111-T4 alloy.

2. Numerical modelling

Simulations are performed using update Lagrangian formulation of continuum based resultant (CBR) shell element coupled with algorithmic formulations of the analyzed constitutive models based on implicit return mapping procedure (Cvitanic et al., 2008, Yoon et al., 1999). The developed formulations are implemented into the finite element code ADINA 8.6 via CUSERG subroutine (ADINA, 2009). The blank shape, tool data, process and material parameters for S-rail benchmark

forming problem are provided by NUMISHEET'96 (NUMISHEET'96, 1996). Figure 1. shows the specific dimensions of the blank and tools as well as the initial setup where y axis is aligned with the rolling direction.

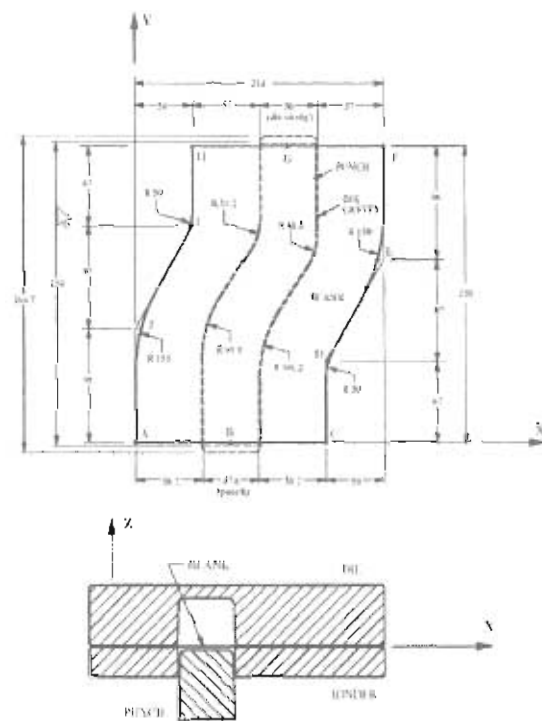


Figure 1. S-rail forming problem geometry. Blank and tools dimensions and initial setup.

Die, punch and blank holder are modelled as rigid surfaces while blank is modelled with 1160 4-node CBR shell elements. During the forming process the die is fixed and the constant blank holder force of 10 kN is applied. For the frictional contact treatment, the rigid target algorithm available in ADINA 8.6 is employed and constant Coulomb friction coefficient of 0.1 is used. For the considered sheet of 0.92 mm initial thickness, isotropic

hardening rule is adopted. For the associated formulation, Hill function adjusted to measured Lankford parameters is used as yield function and plastic potential function. For the non-associated formulation, Hill function adjusted to Lankford parameters is used as plastic potential while Hill function adjusted to measured yield stresses serves as yield function.

3. Results and discussion

The results of the analyzed formulations are evaluated by the comparison with experimental data. Figure 2. shows the deformed shapes at punch stroke of 37 mm before and after springback obtained by the non-associated formulation. In the article deformed shapes of the referent lines shown in Figure 1, IE, JD and BG before and after springback are compared with experimental data. Predictions obtained show reasonably good agreement with measurements.

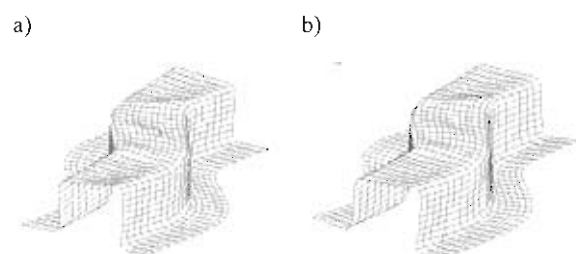


Figure 2. Predictions of the deformed shape at punch stroke of 37 mm obtained by non-associated formulation.
a) Shape before springback; b) Shape after springback.

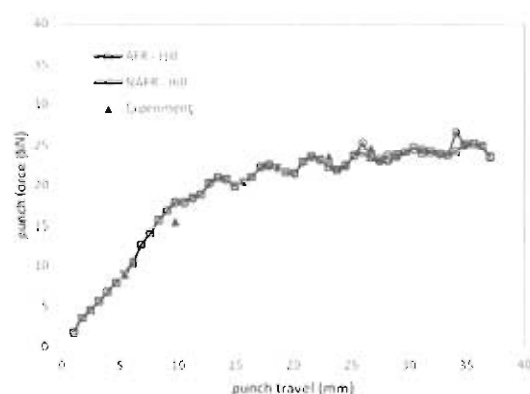


Figure 3. Predicted and measured punch load vs. punch displacement curves in S-rail forming problem.

Figure 3. shows the comparison of the predicted and measured punch load vs. punch displacement curves. It can be observed that the analyzed associated formulation (AFR-Hill) and non-associated formulation (NAFR-Hill) result in almost identical predictions that are in good agreement with the experimental data.

4. Concluding remarks

Although not recommended for aluminium alloys, due to its quadratic form, simple four parametric Hill stress function under associated or non-associated flow rule enables satisfactory predictions of the S-rail forming process for the considered alloy. Also it can be concluded that in this example with complex geometry, differently calibrated Hill yield functions result in almost identical predictions.

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A VARIATIONAL CONSTITUTIVE UPDATE FOR FINITE STRAIN PLASTICITY THEORY COUPLED TO CONTINUUM DAMAGE MECHANICS

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1. Introduction

A variational constitutive update for finite strain plasticity theory coupled to continuum damage mechanics is presented. The starting point is a coupled formulation originally introduced by Lemaitre. Within this formulation, ductile damage is described by continuum damage mechanics model using a scalar-valued damage variable (Kintzel and Mosler, 2010; Kintzel and Mosler, 2011). In the present work, this model is re-written into finite strain plasticity theory with a multiplicative decomposition of deformation gradient into elastic and plastic parts. Kinematic hardening effects are captured by means of an Armstrong-Frederick-type model (Mosler, 2010; Čanadija and Mosler, 2011). In sharp contrast to conventional methods, the resulting material description is reformulated in a variationally consistent manner. This reformulation allows to compute the evolution equations of all internal variables by minimizing the governing energy functional of the considered system. The numerical efficiency of this model is demonstrated by means of selected numerical examples.

2. Kinematics

A body occupies the reference configuration $B_0 \subset \mathbb{R}^{n_{\text{dim}}}$ at time t_0 and the current configuration $B \subset \mathbb{R}^{n_{\text{dim}}}$ at time T . Motion $x = \varphi(X, t)$ is a one-to-one smooth mapping and defines a deformation from the reference to the current configuration of the body. Therefore, motion maps material points $\mathbf{X}$ to spatial points $\mathbf{x}$ within the time period $t \in [t_0, T]$.

As a measure of deformation, the deformation gradient is used:

$$\mathbf{F}(\mathbf{X}, t) = \partial \varphi(\mathbf{X}, t) / \partial \mathbf{X} \quad (1)$$

Since the paper at hand deals with inelastic strains, at this point now standard multiplicative decomposition of deformation gradient into a recoverable elastic part $\mathbf{F}^e$ and an irrecoverable plastic part $\mathbf{F}^p$ is established:

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p. \quad (2)$$

Elastic $\mathbf{L}^e$ and plastic $\mathbf{L}^p$ velocity gradients are specified as:

$$\dot{\mathbf{F}}^e = \mathbf{L}^e \mathbf{F}^e, \quad \dot{\mathbf{F}}^p = \mathbf{L}^p \mathbf{F}^p. \quad (3)$$

As a deformation tensor, the right Cauchy-Green tensor $\mathbf{C}$ is used. Similarly to the definition of $\mathbf{C}$, the elastic right Cauchy-Green tensor $\mathbf{C}^e$ is also defined:

$$\mathbf{C} = \mathbf{F}^T \mathbf{F}, \quad \mathbf{C}^e = (\mathbf{F}^e)^T \mathbf{F}^e. \quad (4)$$

3. Balance laws

Thermodynamical processes are governed by the balance laws. For the sake of completeness, balance laws are briefly summarized in the local form. Balance of momentum law is:

$$\text{DIV } \mathbf{P} + \rho_0 \mathbf{B} = 0, \quad (5)$$

where inertial effects were neglected. Here $\mathbf{P}$ is first Piola-Kirchhoff stress tensor and $\mathbf{B}$ are body forces. This equation is accompanied by the suitable set of boundary conditions. First law of thermodynamics or balance of energy law is stated in the form:

$$\dot{E} + \text{DIV } \mathbf{H} = \mathbf{P} : \dot{\mathbf{F}} + \rho_0 Q, \quad (6)$$

also augmented with the usual boundary conditions. In eq. (6), E is internal energy, H heat flux and Q specific heat source.

4. Isotropic damage model

Material is characterized by the following free energy function Ψ (Grammenoudis et al., 2009):

$$\Psi = (1 - D) \Psi_{orig}, \quad (7)$$

where

$$\Psi_{orig} = W(\bar{\mathbf{C}}\bar{\boldsymbol{\alpha}}) + U(J) + K^{iso}(\alpha_i) + K^{km}(\mathbf{k}), \quad (8)$$

and $D \in [0, 1]$ is a standard scalar-valued damage variable, α_k set of strain-like kinematic hardening variables, α_i strain-like isotropic hardening variables. Therefore, plastic part of the Helmholtz free energy is also influenced by the damage evolution. In the text that follows $\mathbf{Q}$ denotes a set of stress-like variables conjugated to $\boldsymbol{\alpha}$.

If effective stress variables $\bar{\boldsymbol{\Sigma}}, \bar{\mathbf{Q}}_k, \bar{Q}_i$ are now defined as:

$$\bar{\boldsymbol{\Sigma}} = (1 - D) \boldsymbol{\Sigma}, \bar{\mathbf{Q}}_k = (1 - D) \mathbf{Q}_k, \bar{Q}_i = (1 - D) Q_i \quad (9)$$

then yield function is expressed by means of the effective stress variables:

$$\phi = \sqrt{\frac{3}{2}} \left\| (\bar{\boldsymbol{\Sigma}}^{dev} - \bar{\mathbf{Q}}_k^{dev}) \right\| - \bar{Q}_i(\alpha_i) - Q_0^{eq} = 0. \quad (10)$$

5. Variational constitutive updates

The stress power is defined as:

$$\mathbf{E} = \dot{\Psi} + J^*, \quad (11)$$

where J^* is:

$$J^* = \sup_{\mathbf{P}, \mathbf{Q}} (D + J)$$

$$J = \begin{cases} 0 & \forall (\mathbf{P}, \mathbf{Q}) \in \bar{\mathbf{E}}_{\bar{\mathbf{P}}} \\ \infty & \text{otherwise} \end{cases} \quad (12)$$

where D is dissipation and space of admissible stresses is $\bar{\mathbf{E}}_{\bar{\mathbf{P}}} \equiv \{(\mathbf{P}, \mathbf{Q}) \in \square^{9+n_{int}} \mid \phi(\mathbf{P}, \mathbf{Q}) \leq 0\}$.

Based on eq. (11), incremental energy functional I_{loc} is defined and the solution is obtained from the variational principle:

$$(\boldsymbol{\Sigma}_{n+1}, \Delta p) = \arg \inf_{\boldsymbol{\Sigma}_{n+1}, \Delta p} I_{loc}, \quad (13)$$

where

$$\Delta p = \frac{\Delta \lambda}{1 - D_{n+1}} \quad (14)$$

and $\Delta \lambda$ is consistency parameter.

Acknowledgements

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MECHANICAL BEHAVIOUR CHARACTERIZATION OF SHEET METAL AT LARGE PLASTIC STRAINS

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1 Introduction

Standard tensile test is a well known and frequently used test for a determination of the hardening behaviour. Unfortunately, the experienced plastic instability at relatively low plastic strain, referred frequently as neck occurrence or necking, destroys uniformity of the stress-strain field in a tensile specimen [1]. In engineering practice more or less appropriate extrapolation techniques based on different analytical hardening laws (Swift, Voce, Gosh etc.) are used which, as it is also shown in this paper, often fail to predict correctly the hardening behaviour of sheet metal at large plastic strain. To overcome this problem and to enable also at such strain conditions a more reliable hardening curve identification we enlarge the uniaxial experimental data set with the results obtained from a newly developed shear testing. The main comparative advantage of this test is its simple implementation which requires no further development of existing experimental equipment for standard tensile testing. A proper material characterization, based on the acquired experimental data, is done by means of simulating the experiment numerically. In the study the Yld2000-2d [2] and BBC2003 [3] yield criteria are adopted to model plastic anisotropy, whereas the hardening curve is modelled using cubic spline technique. On associate plastic flow theory based constitutive models are implemented in ABAQUS/Explicit via VUMAT subroutine, while integration is performed by the NICE, an explicit integration scheme developed recently by the authors [4, 5].

2 A new shear specimen

It is well known, that large plastic strain can be achieved in sheet metal when the latter is exposed to shear loading. However, classical shear tests are not used in engineering practice since their implementation demands particular experimental equipment and measurement instruments. In this paper we present a purposely designed shear

specimen to be tested in a common standard tensile environment (testing machine and measurement sensors). Accordingly, the overall dimensions of the developed shear specimen are the same as those of a standard tensile specimen, the difference being in the central part which is designed in a way to obtain shear stress-strain state in two symmetrically located shear regions, as shown in Fig. 1. In order to get simple shear loading conditions in those regions the contour was carefully optimized using finite element based simulations.

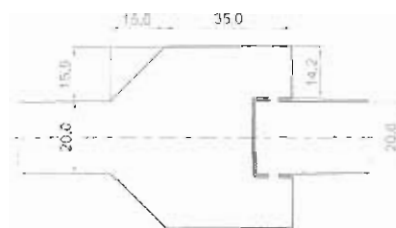


Figure 1. Shear specimen, designed for use on tensile machine

The shear specimen is loaded in the same way as standard tensile specimen; it is being clamped into a tensile test machine and then loaded by imposing edge displacement (see Fig. 2).

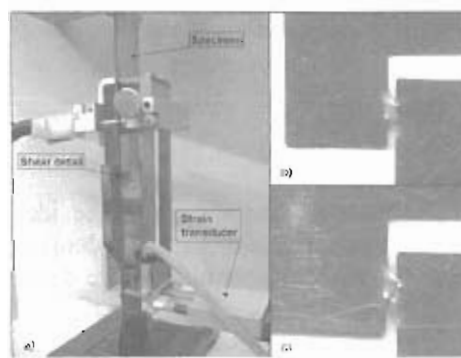


Figure 2. Shear test on tensile test machine

The indicator showing the degree of loading in each material point is the equivalent plastic strain ϵ_{eq}^p . In Fig. 3 it can be seen that the evolution of the equivalent plastic strain ϵ_{eq}^p in the shear region is

severe (pay attention on the scale). Wide area of large plastic strain which is manifested in the shear zone is advantageous also from the numerical point of view. If only a few finite elements would exhibit large plastic strains, the inverse problem would be ill-conditioned and identification results uncertain. It can be also seen, that distribution of the largest simulated equivalent plastic strain is in good agreement with the fracture path (Fig. 2c), which gives the simulation greater physical objectivity.

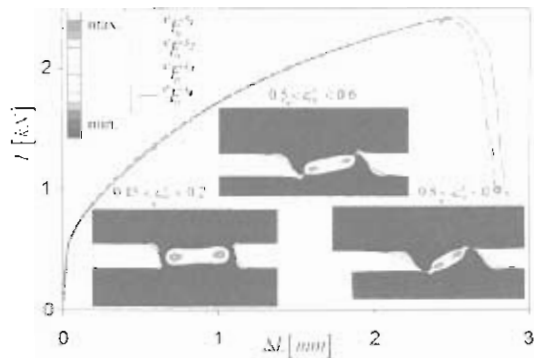


FIGURE 3. Measured forces vs. elongation for four specimens and respective simulated equivalent plastic strain

Considering the equivalent plastic strain in such shear test attains rather large values before fracture, it enables, in contrast to tensile tests, the hardening identification of sheet metal at larger plastic strains.

3 Hardening curve identification

Along with all parameters of the Yld2000-2d and BBC2003 constitutive models the hardening parameters should be identified as well. On a cubic splines based determination of the hardening curve the following set of parameters is used: $\sigma_y^0, \sigma_y^{0.05}, \sigma_y^{0.15}, \sigma_y^{0.25}, \sigma_y^{0.425}, \sigma_y^{0.60}, \sigma_y^{0.90}$ where superscripts denote the equivalent plastic strain values at which yield stresses are considered.

For the sake of comparison of the identified hardening curves with analytical hardening laws, parameters for all considered laws were determined from tensile test $F-\Delta L$ experimental data registered prior to the occurrence of necking at $\epsilon_{eq}^p = 0.42$. In Fig. 4 a comparison between the identified and analytically given hardening curves, the latter being fitted to the identified function but only up to the value $\epsilon_{eq}^p = 0.42$, is shown.

Low ability of analytical hardening laws to predict the hardening behaviour at large plastic strains is experienced. All of them fit very accurately the hardening behaviour up to the value

0.42 of the equivalent plastic strain, which is expected since it is considered in the fitting (Fig. 4). A prediction of the hardening behaviour with analytical hardening laws at large plastic strains is however unsatisfactory, what justifies the need of additional experiment to identify the hardening behaviour at large strains.

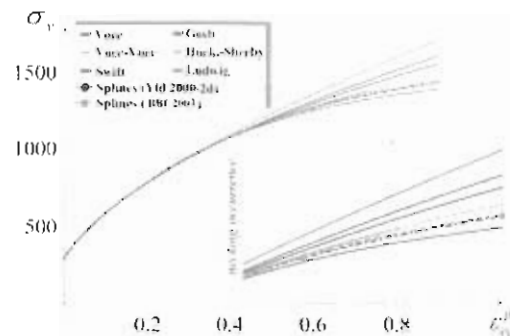


FIGURE 4. Comparison of identified hardening at large plastic strains with analytical extrapolations

4 Conclusions

In this contribution a new approach to identification of the hardening behaviour of sheet metals after onset of necking has been proposed. Let us point out the following:

- A new shear specimen of a specific geometry was presented, for which there is no need for additional testing devices, because the proposed shear test can be performed by classical tensile testing equipment. The proposed shear test gives information about yielding up to 0.9 of the equivalent plastic strain for stainless sheet metal.
- For large plastic straining we discourage usage of analytical hardening laws, extrapolated from pre-necking behaviour of sheet.

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SOLID AND STRUCTURAL MECHANICS

ANALYSIS OF GEOMETRICALLY EXACT LAYERED COMPOSITE BEAMS WITH A RIGID INTERLAYER CONNECTION

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1. Introduction

Research and application of layered composite structures in many areas of engineering has increased considerably over the past couple of decades and continues to be a topic of undiminished interest in the computational mechanics community. In the present work a finite-element formulation including a non-linear solution procedure for a geometrically exact planar layered composite beam is presented which is applicable to the problems involving large displacements and rotations. In this work, the interlayer slip and uplift effects are not considered, i.e. it is assumed that the interlayer connection only allows for the occurrence of independent rotations of each layer. The layers are assumed to be made of linear elastic material and initially straight with the cross-sections of the layers symmetric with respect to a common vertical principal axis (a condition for a plane problem). The composite beam reference axis and the reference axes of each layer may be chosen arbitrarily. Thus, loading can be applied on each layer separately, distributed over the length of the beam or concentrated at the beam ends. In this way the fundamental loading mechanism needed to test a layered beam's behaviour for various delamination modes (e.g. the double-cantilever beam test) is provided. The actual delamination tests as well as a full dynamic analysis, however, are not subject of the present work.

2. Governing equations

The deformed geometry of a straight planar beam is fully described by the two unknown displacement functions (one for each of the coordinate directions) and one unknown cross-sectional rotation function. Here we assemble a layered composite beam by firstly considering each of the layers as a separate beam. The governing equations for such a layer consist of

the standard kinematic and constitutive equations and the principle of virtual work (Reissner). Next, we relate the unknown functions of the horizontal and vertical displacements of a layer ($2n$ unknown functions for a composite beam with n layers) to the new unknown functions of the horizontal and vertical displacements of the composite beam as a whole from the condition of a rigid stick between the layers. Finally, in the compound principle of virtual work of the layers, we embed the geometric relations between the layers in the virtual displacements. In addition to the two unknown displacement functions in the composite beam there also exist the unknown functions of the cross-sectional rotations of each layer, which makes a total number of $2+n$ unknown functions. The governing equations of the composite beam are highly non-linear in terms of unknown functions and they cannot be solved analytically. In contrast to some other formulations for the geometrically non-linear layered beams (Vu-Quoc and Deng) here we consistently apply the above geometric considerations to account for the possibility that the external loading may be off-set with respect to the chosen reference axis of the composite beam. Obviously, the relations between the virtual displacements of the layers and those of the composite beam thus also influence the vector of the applied loading, which effectively becomes configuration-dependent.

3. Solution procedure

The resulting unknown displacement and rotation functions are approximated using a finite number of nodal values and interpolation functions between the nodes. The same interpolations are used for the functions of virtual displacements and rotations. The solution is obtained numerically using the Newton-Raphson solution procedure. The vector of residual forces of the composite beam follows in a standard way from the principle of

virtual work approximated in the above manner. The tangent stiffness matrix is obtained by the consistent linearization of the residual force vector with respect to the unknown nodal parameters. The nodal values of displacements and rotations are calculated iteratively until a desirable accuracy is achieved.

4. Results and discussion

Numerical examples are used to compare the presented model with other similar models. The results are presented and discussed.

5. Concluding remarks

The analysis of a composite layered beam is presented with the solution procedure explained in detail. Since the geometry of the composite beam has been defined exactly, this theory is applicable also for large displacements and rotations. The connection between the layers analysed here is rigid, but the next step will be to define the contact stiffness. This will allow for both interlayer slip and uplift to occur. Our final goal is to introduce the interlayer delamination and non-linear constitutive law of the interlayer connection.

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LINKED INTERPOLATION IN HIGHER-ORDER TRIANGULAR MINDLIN PLATE FINITE ELEMENTS

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1. Introduction

In this work higher-order linked interpolation applied to triangular plate finite elements is analyzed in detail. Benefits of the linked interpolation are well known in the Timoshenko (thick) beam finite elements, where not only is the shear locking with such an interpolation completely avoided, but also the exact solution is achieved provided the applied loading is static and expressed in terms of the polynomial functions [1]. In the case of Mindlin (thick) plate finite elements, the exact solution for a genuine planar behaviour cannot be obtained in terms of a finite number of parameters. Additionally, with linked interpolation shear locking may be considerably reduced, but it cannot be completely avoided [2, 3, 4]. In this paper the basis for development of higher-order triangular Mindlin plate elements is found in the analogy between the Timoshenko beam theory and the Mindlin plate theory in a manner closely related to our earlier studies of arbitrary quadrilateral plate elements [5]. The results obtained on standard test examples are compared and numerically assessed against the reference results from literature and from FEAP program by R.L.Taylor [6], using various mesh densities and various order of interpolation.

2. Linked interpolation formulation

The well known differential equations of the Timoshenko beam:

$$EI\theta''' = -q, \quad GA_s(w' + \theta') = -q$$

relating the transversal load q with the second derivative of the bending moment $M = -EI\theta'$ and the first derivative of the shear force $T = -GA_s(w' + \theta')$, can be exactly integrated for the transverse displacement w

and the section's rotation θ , without involving bending and shear stiffnesses EI and GA_s .

The expressions that are exact for the beam differential equations are now used to interpolate the displacement w and the two section's rotations θ_x and θ_y of the Mindlin plate triangular element.

In the linked interpolation formulation we propose here, the expression for shear along an element side is taken to be a polynomial two orders lower than the displacement interpolation polynomial. Every direction parallel to the element side retains this property. In contrast to the procedure proposed by Auricchio and Taylor [3], we remain within the realm of pure displacement-based finite-element design procedures.

The elements obtained in this way are compared with the Liu-Riggs family of elements [7], also developed on the pure displacement-based concept and we find out that the two families are very similar (equal for the lowest-order element). However, the generation of the higher-order elements is easier using the linked interpolation formulation than the Liu-Riggs concept.

A number of standard tests are performed (the patch test for constant bending and constant shear, the clamped and simply supported square plate, the skew Morley plate and circular plate) for the moderate and extremely small plate thicknesses and compared with some of the known elements based on the mixed formulation.

3. Results and discussion

The elements based on the linked interpolation formulation are capable of reproducing the exact analytical result for the case of cylindrical bending of certain order (quadratic for the three-node elements, cubic

for the six-node elements and so on). This means that the six-node and ten-node triangular elements are able to reproduce the exact solution in the constant shear patch test [4] regardless of the plate thickness (*Table 1*).

The lowest-order (three-node) element of the family suffers from the shear-locking effect for very coarse meshes and thin plates, but its performance improves with mesh refinement.

Performance of the elements derived is generally good for a number of standard benchmark tests as the finite-element mesh is refined, one notable exception being the skewed plate, for which the presented elements do not perform so well as the element of Auricchio and Taylor [3].

Otherwise, the six-node and the ten-node element, in general, are clearly successful when compared to the lower-order elements

for the problems with the same total number of the degrees of freedom.

4. Concluding remarks

The triangular elements based on the linked interpolation derived here perform comparably well for a number of standard benchmark tests and are competitive to the elements based on the mixed formulation in the Mindlin-type plate analysis.

Acknowledgements

The results shown here have been obtained within the scientific project No 114-0000000-3025: 'Improved accuracy in non-linear beam elements with finite 3D rotations' financially supported by the Ministry of Science, Education and Sports of the Republic of Croatia.

Table 1: The standard patch test [4] results for the proposed elements (displacement at control point)

Elements	Patch test for constant curvature			Patch test for constant shear		
	Thick plate	Thin plate	result	Thick plate	Thin plate	result
3-node	0.5414000	0.5414000	pass	-0.2455871	0.000318188	fail
6-node	0.5414000	0.5414000	pass	-0.2450933	0.000215467	pass
10-node	0.5414000	0.5414000	pass	-0.2450933	0.000215467	pass
Analytical solution	0.5414	0.5414		-0.245093333	0.000215467	

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COMPUTATIONAL ASPECTS OF CONFIGURATION DEPENDENT INTERPOLATION IN NON-LINEAR HIGHER-ORDER 2D BEAM FINITE ELEMENTS

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1. Introduction

In non-linear 3D beam theory with rotational degrees of freedom [1] configuration-dependent interpolation may be utilized to provide a result invariant to the choice of the beam reference axis [2] or invariant to a rigid-body rotation [3]. For 2D beam elements, the latter issue vanishes, and such elements are more illustrative for the study of accuracy of the configuration-dependent interpolation in higher-order elements.

Since the approximate character of the finite-element method stems from the introduction of interpolation functions for the field variables and their variations, the actual choice of the interpolation functions is of a great importance for the accuracy of the finite element method. The most commonly used standard Lagrangian interpolation procedure (with reduced integration used to eliminate the shear locking effect) is satisfactory if our attention is limited to finding the results for the field variables at nodal points in linear analysis. However, if we want to obtain the exact field distribution this kind of interpolation is unable to do so, even in linear analysis. To obtain the exact solutions in linear analysis simply using the right interpolation function and without applying reduced integration, a different kind of interpolation, called the *linked interpolation* is needed, in which the unknown displacement function do not depend only on the nodal displacements, but also on the nodal rotations [4]. Marco Borri and Carlo Bottasso have developed a so-called fixed-pole formulation which effectively generalizes the idea of linked interpolation to non-linear two-noded beam elements. This interpolation is called the *helicoidal interpolation* by the authors and is based on the fact that the tangent to the beam centroidal axis and the normal of the cross section follow the same transformation rule [2].

This kind of interpolation can also be obtained starting from the condition of constant Reissner's strain measures [5].

2. Reissner's beam theory

In Reissner's beam theory, normal strain ε , shear strain γ and the change of the sectional orientation along the length of the beam κ are given by the following expressions:

$$\varepsilon = (\cos \psi + u') \cos(\psi + \varphi) + (\sin \psi + w') \sin(\psi + \varphi) - 1$$

$$\gamma = -(\cos \psi + u') \sin(\psi + \varphi) + (\sin \psi + w') \cos(\psi + \varphi)$$

$$\kappa = \varphi',$$

where ψ is the slope of the beam in the initial configuration, φ is the function describing the slope of the cross sections in the deformed configuration as a function of the arc-length coordinate x , $u(x)$ and $w(x)$ are the horizontal and vertical displacements that can be written in terms of the components of the initial and the deformed position vectors $R_x(x)$, $R_y(x)$, $r_x(x)$ and $r_y(x)$, respectively, and a dash denotes a differentiation with respect to the arc-length coordinate x (see Figure 1). Taking the above strain measures as constant results in the same interpolation that was introduced by Borri and Bottasso. In this way we obtain the following interpolation:

$$\begin{Bmatrix} r_x \\ r_y \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{N} \begin{Bmatrix} R_{x1} \\ R_{y1} \end{Bmatrix} + \mathbf{N} \begin{Bmatrix} R_{x2} \\ R_{y2} \end{Bmatrix}, \text{ with}$$

$$\mathbf{N} = \frac{\sin\left(\frac{\varphi_2 - \varphi_1}{2} I_2\right)}{\sin\left(\frac{\varphi_2 - \varphi_1}{2}\right)} \begin{bmatrix} \cos\left(\frac{\varphi_2 - \varphi_1}{2} I_1\right) & \sin\left(\frac{\varphi_2 - \varphi_1}{2} I_1\right) \\ -\sin\left(\frac{\varphi_2 - \varphi_1}{2} I_1\right) & \cos\left(\frac{\varphi_2 - \varphi_1}{2} I_1\right) \end{bmatrix}$$

where I_i is the standard Lagrangian polynomial and R_i is the initial position of the node i with its horizontal and vertical components. This interpolation is *non-linear* in

the field variables (*configuration-dependent*) and has the following limiting value in the case of the non-linear analysis becoming linear:

$$\begin{Bmatrix} u \\ w \\ \varphi \end{Bmatrix} = \sum_{i=1}^n \begin{bmatrix} I_i & 0 & -\frac{I_i}{n}(R_y - R_{yi}) \\ 0 & I_i & -\frac{I_i}{n}(R_x - R_{xi}) \\ 0 & 0 & I_i \end{bmatrix}$$

where $n=2$ here is the number of nodes. This result coincides with the exact solution of the linear Timoshenko beam problem for arbitrary number of nodes n [4] (the linked interpolation).

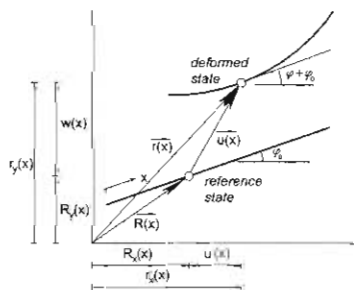


Figure 1. Kinematics of the problem.

In order to generalize the non-linear Borri-Bottasso result for two-noded elements to higher-order elements we may take the strain measures to be linearly distributed rather than constant, which however complicates the procedure considerably. Alternatively, we may attempt to generalize the linear higher-order result obtained by Jelenić and Papa to non-linear analysis.

3. Configuration-dependent higher-order interpolation

The simplest idea of how to perform this generalization and come up with a configuration-dependent interpolation is to substitute the initial positions of the nodal points in the linked interpolation [4] with their current positions:

$$\begin{Bmatrix} u \\ w \\ \varphi \end{Bmatrix} = \sum_{i=1}^n \begin{bmatrix} I_i & 0 & -\frac{I_i}{n}(r_y - r_{yi}) \\ 0 & I_i & -\frac{I_i}{n}(r_x - r_{xi}) \\ 0 & 0 & I_i \end{bmatrix}$$

As it can be seen, the unknown displacement fields depend on the position vectors in the deformed state.

4. Concluding remarks

In this paper higher-order configuration-dependent interpolation (i.e. nonlinear in the field variables) has been introduced and the results compared with the results from the literature that include the helicoidal interpolation of Borri and Bottasso as well as the results when the interpolation functions are obtained by taking a linear distribution of the Reissner strain measures.

5. Acknowledgements

The results shown here were obtained within the scientific project No 114-0000000-3025: „Improved accuracy in non-linear beam elements with finite 3D rotations” financially supported by the Ministry of Science, Education and Sports of the Republic of Croatia.

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A NON-LINEAR PLANAR BEAM FINITE ELEMENT BASED ON FIXED POLE APPROACH

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1. Introduction

The fixed-pole approach (Borri and Bottasso, 1994; Bottasso and Borri, 1998) is mainly motivated by dynamic analysis of non-linear beam structures, i.e. conservation of mechanical constants in numerical time integration. By omitting the inertia terms (the nodal vector of inertial forces) the static analysis formulation is obtained, which will be discussed in this work.

We consider a geometrically non-linear planar beam with kinematic equations following from Eric Reissner's beam theory (Reissner, 1972). The constitutive law is linear, and the position vector of a point along the beam, $\mathbf{r}$ is defined as follows:

$$\mathbf{r} = \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} = \begin{Bmatrix} x + u \\ v \end{Bmatrix} \quad (1)$$

where x is the co-ordinate of the point along the centroid axis of the beam, while u and v are the horizontal and vertical displacements, respectively. We also note that θ is the rotation of the cross section. The cross-sectional stress resultants are defined as follows:

$$\begin{aligned} \mathbf{n} &= \begin{Bmatrix} n_1 \\ n_2 \end{Bmatrix} = \\ &= \begin{Bmatrix} EA[\cos\theta \cdot (1 + r'_1) + \sin\theta \cdot r'_2 - 1] \\ GA_s[-\sin\theta \cdot (1 + r'_1) + \cos\theta \cdot r'_2] \end{Bmatrix} \quad (2) \\ m &= EI\theta' \end{aligned}$$

where E and G are Young's and shear moduli, A , A_s and I are the cross-sectional area, shear area and second moment of area, while n_1 , n_2 , and m are the normal and shear forces and the bending moment, respectively.

2. Fixed-pole approach

Following the fixed-pole theory (Borri; Bottasso, 1994), the fixed-pole nodal static residual is obtained. This is performed by reducing the standard stress resultants to "fixed-

pole" stress resultants (see Figure 1) and requesting the specific virtual work to remain invariant to the change of the stress resultants:

$$\begin{aligned} n_1 \cdot \delta r_1 + n_2 \cdot \delta r_2 + m \cdot \delta \theta &= \\ = \bar{n}_1 \cdot \delta \bar{r}_1 + \bar{n}_2 \cdot \delta \bar{r}_2 + \bar{m} \cdot \delta \bar{\theta} \end{aligned} \quad (3)$$

from where follows the relationship between the fixed-pole and standard virtual quantities as

$$\begin{Bmatrix} \delta \bar{\mathbf{r}} \\ \delta \bar{\theta} \end{Bmatrix} = \begin{Bmatrix} \delta \mathbf{r} + \mathbf{r} \times \delta \theta \\ \delta \theta \end{Bmatrix} \quad (4)$$

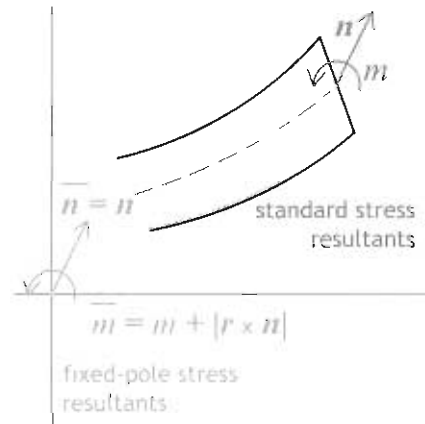


Figure 1 A planar beam in a deformed state, with standard and fixed-pole stress resultants shown

By applying the principle of virtual work with fixed-pole virtual quantities:

$$\begin{aligned} G \equiv \int_0^L \langle \delta \bar{\mathbf{r}}^T \quad \delta \bar{\theta} \rangle \left(\frac{d}{dx} \right) \begin{Bmatrix} \bar{\mathbf{n}} \\ \bar{m} \end{Bmatrix} dx \\ - \int_0^L \langle \delta \bar{\mathbf{r}}^T \quad \delta \bar{\theta} \rangle \begin{Bmatrix} \bar{\mathbf{n}}_e \\ \bar{m}_e \end{Bmatrix} dx \end{aligned} \quad (5)$$

$$-\langle \delta \bar{\mathbf{r}}_0^T \quad \delta \bar{\theta}_0 \rangle \begin{Bmatrix} \bar{\mathbf{F}}_0 \\ \bar{M}_0 \end{Bmatrix} - \langle \delta \bar{\mathbf{r}}_L^T \quad \delta \bar{\theta}_L \rangle \begin{Bmatrix} \bar{\mathbf{F}}_L \\ \bar{M}_L \end{Bmatrix} = 0$$

we obtain the fixed-pole nodal static residual after applying the Lagrangian polynomials, $I^l(x)$ for the interpolation of the fixed-pole virtual displacements:

$$\bar{\mathbf{g}}^l \equiv \bar{\mathbf{q}}_l^l - \bar{\mathbf{q}}_e^l = 0 \quad (6)$$

Where $\bar{q}_i^i$ and $\bar{q}_e^i$ are the fixed-pole vectors of internal and external forces at node i , respectively and defined as follows:

$$\begin{aligned}\bar{q}_i^i &= \int_0^L I^i \left\{ \frac{\bar{n}}{\bar{m}} \right\} dx \\ \bar{q}_e^i &= \int_0^L I^i \left\{ \frac{\bar{n}_e}{\bar{m}_e} \right\} dx + \delta_1^i \left\{ \frac{\bar{F}_0}{\bar{M}_0} \right\} + \delta_N^i \left\{ \frac{\bar{F}_L}{\bar{M}_L} \right\}\end{aligned}\quad (7)$$

The finite element derived from this approach has the fixed-pole degrees of freedom $\bar{r}$, $\bar{\theta}$ and therefore it cannot be combined with standard finite elements with displacements and rotations as the degrees of freedom.

Conversion back to standard unknowns is performed by introducing the relationship between the fixed-pole and standard nodal virtual quantities (4) which in turn gives the modified nodal static residual

$$\bar{g}^i \equiv \bar{q}_i^i - \bar{q}_e^i = 0 \quad (8)$$

Where $\tilde{q}_i^i$ and $\tilde{q}_e^i$ are the modified nodal vectors of internal and external forces, respectively and defined as follows:

$$\begin{aligned}\tilde{q}_i^i &= \int_0^L I^i \left[\begin{array}{cc} I & 0 \\ \bar{r} - \bar{r}_i & I \end{array} \right] \left\{ \frac{\bar{n}}{\bar{m}} \right\} dx \\ \tilde{q}_e^i &= \int_0^L I^i \left[\begin{array}{cc} I & 0 \\ \bar{r} - \bar{r}_i & I \end{array} \right] \left\{ \frac{\bar{n}_e}{\bar{m}_e} \right\} dx + \delta_1^i \left\{ \frac{\bar{F}_0}{\bar{M}_0} \right\} \\ &\quad + \delta_N^i \left\{ \frac{\bar{F}_L}{\bar{M}_L} \right\}\end{aligned}\quad (9)$$

and enables combining these elements with the standard ones in a finite element mesh.

3. Algorithm

The modified nodal static residual is consistently linearised in line with the Newton - Raphson solution procedure, from where the expression for the tangent stiffness matrix is obtained. In order to algorithmically define the stiffness matrix, various interpolation options for the displacement unknowns and their respective iterative changes must be clearly defined.

Depending on the interpolation option used, there are three possible algorithms which will be numerically tested and compared.

4. Concluding remarks

A brief explanation of the fixed-pole approach in non-linear beam analysis is given, followed by the numerical algorithm for a non-linear planar beam finite element derived from the fixed-pole theory but combinable with standard beam finite elements.

Three interpolation options for problem unknowns and their respective iterative changes are presented and implemented in the algorithm. A numerical example is given with comparisons between different interpolation options.

The theoretical formulations for a spatial non-linear beam element under dynamic loading have been derived and the numerical examples will be shown in future works.

Acknowledgements

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STEADY-STATE CREEP ANALYSIS OF THICK-WALLED PRESSURE VESSELS AND PIPING

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1. Introduction

Distribution of stresses, strains and displacements across the thickness of a thick-walled pressure vessel and a thick-walled pipe is considered in this paper. The pressure vessel is loaded by the internal pressure p of the fluid and also by an axial force N of the fluid pressure exerted on the vessel heads. Any component of strain (radial, circumferential, and axial) at a given point of the thick-walled pressure vessel is composed of elastic strain (e), plastic strain (p), and creep strain (c). It is assumed that stresses do not change with time (stresses are *constant* at any moment). Such a process is called *steady-state creep*. In this process of creeping, elastic strains can be neglected with respect to creep strains, especially when an element is exposed to creep during a long period of time. An analysis of stress distribution has been carried out by means of analytical methods, and results are presented in the form of diagrams.

2. Theoretical background

In the secondary stage, creep develops at a *constant* and *minimum creep strain rate* $\xi_{c,min}$, which depends on the *stress* σ and *temperature* T , i.e.

$$\xi_{c,min} = Q(\sigma, T). \quad (1)$$

It has been proven experimentally that the best approximation of this dependence is when the minimum creep strain rate depends on the stress to the power of n , e.g.

$$\xi_{c,min} = Q(\sigma) = k\sigma^n, \quad n = 1, 2, 3, \dots \quad (2)$$

The quantities k and n depend on the material and temperature of testing. The values of these coefficients in alloyed steels (Cr-Ni-Mo-W) depend on the heat treatment applied and the temperature of testing. The coefficient n ranges from 1 to 10. According to [2], the dependence of *creep strain* ε_c on *stress* and *time* at a given

temperature can be written as a product of two functions

$$\varepsilon_c = Q(\sigma, T) \cdot \Omega(t, T), \quad (3)$$

where the first function, $Q(\sigma, T)$, is a function of stress and temperature, and the second, $\Omega(t, T)$, is a function of time and temperature.

The function $\Omega(t, T)$ does not have to be approximated by an analytical expression since, in the calculation of creep in machine elements, it is possible to determine stress distributions without an explicit expression for the function Ω . This will be shown in this study.

3. Results and discussion

Analytical expressions for the distribution of the circumferential σ_ϕ and the axial σ_z normal stresses across the wall of thick-walled tube and/or pressure vessel in the conditions of steady-state creep of a material can be written in the following form

$$\sigma_\phi = p \cdot \frac{r_1^{2/n}}{r_2^{2/n} - r_1^{2/n}} \cdot \left(1 + \frac{2-n}{n} \cdot \frac{r_2^{2/n}}{r^{2/n}} \right), \quad (4)$$

$$\sigma_z = p \cdot \frac{r_1^{2/n}}{r_2^{2/n} - r_1^{2/n}} \cdot \left(1 - \frac{n-1}{n} \cdot \frac{r_2^{2/n}}{r^{2/n}} \right). \quad (5)$$

It is assumed that the ratio between the internal and the external radius of the pressure vessel / tube is $r_1/r_2 = 0.5$. The magnitude of the exponent n in the relations (4) and (5) has been varied and has taken the values $n = 3, 5, 7, 10, 25$, and 1000. The Figures 1 and 2 give comparative presentations of stress distribution in the conditions of *steady-state creep* and in the elastic range. The distribution of circumferential stress σ_ϕ and axial stress σ_z across the wall thickness of a pressure vessel and/or pipe in the conditions of steady-state creep differs significantly from the distribution in the elastic range. While the circumferential stresses σ_ϕ in the elastic range have the

maximum values on the internal wall of the tube / pressure vessel, in the conditions of steady-state creep, these stresses are at their maximum on the external wall of the pressure vessel / tube. On the other hand, axial stresses σ_z are constant across the wall thickness of the tube / pressure vessel in the elastic range, while in the condition of steady-state creep they are variable, with the maximum values on the external wall of the pressure vessel and/or tube.

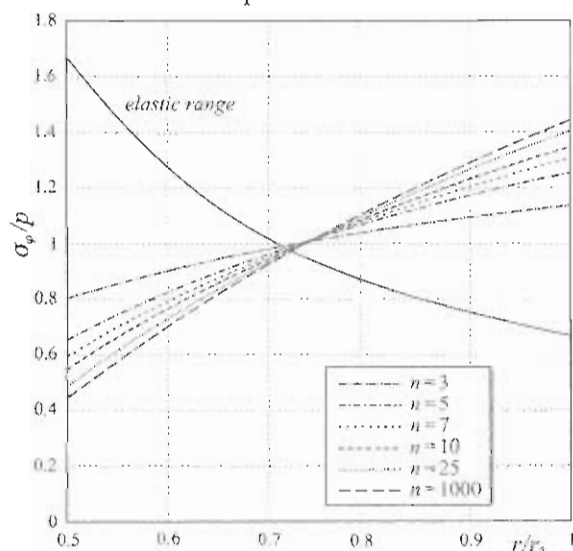


Figure 1. Distribution of circumferential stresses σ_ϕ across the wall thickness of a tube / pressure vessel in the steady-state creep conditions and in the elastic range.

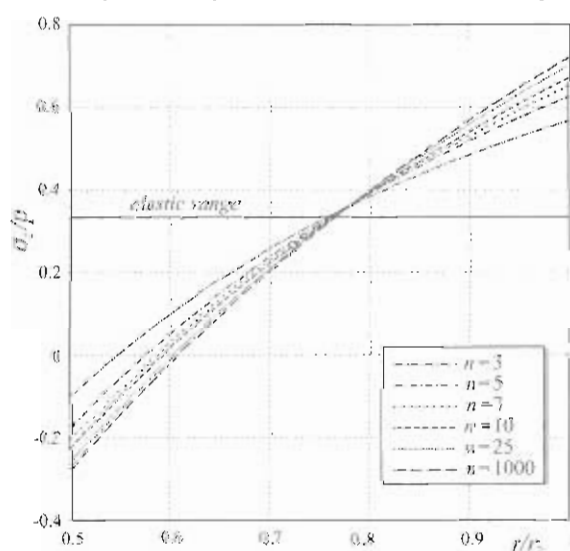


Figure 2. Distribution of axial stresses σ_z across the wall thickness of a tube / pressure vessel in the steady-state creep conditions and in the elastic range.

4. Concluding remarks

In this paper, stresses, strains, and displacements occurring in thick-walled pressure vessels and piping in the conditions of steady-state creep are analysed using analytical methods. The dependence of creep strain ϵ_c on stress and time at a given temperature can be written as a product of two functions according to expression (3). Exact analytical expressions (4) and (5) for stress distribution across the wall thickness of a pressure vessel or a tube have been derived. Stress distribution diagrams have been plotted and shown in Figures 1 and 2. Fundamental differences in stress distributions between the conditions of steady-state creep and in the elastic range are pointed out.

An exact analytical expression for the analysis of radial displacement u_c , in the conditions of steady-state creep of the material, has been derived in this study. Using this expression it is possible to analyse displacements by varying the pressure p , the geometry of pressure vessel / pipe (r_1 , r_2), and the material (by means of the parameter n which depends on the temperature T). Displacements occurring currently in the pressure vessel and/or pipe due to pressure and temperature can often be neglected in comparison with the displacements occurring in the conditions of steady-state creep, particularly in the case when the pressure vessel and/or tube is exposed to creep for a long period of time.

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LOADED ANNULAR PLATES

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1. Introduction

In engineering practice we often meet structural elements loaded in their own plane like rods or plates. Because of their importance stability problems of plates loaded in their own planes is an especially significant issue.

As regards the stability problems of annular plates, we mention that the first paper devoted to this question was published in 1890 (Bryan, 1890). At this point we also cite an article (Nádai, 1915) which is devoted to some fundamental stability issues. It is worth mentioning that a number of papers have been published since then but here we lay an emphasis only on those which deal with the influence of structural stiffening.

A structure can be stiffened in various ways. A solid circular plate (or an annular one) can be made to be more resistant to buckling by the use of a corrugation or by applying a stiffening along the diameter of the plate or by attaching a ring to the outer boundary of the plate or by making them more rigid with a cylindrical shell attached to the outer boundary of the plate.

Seeking in scientific literature for papers devoted to the stability problem of stiffened annular plates we have found some works dealing with this issue. A stiffened circular plate is investigated in the following papers: (Turvey and Der Avanessian, 1989; Turvey and Salehi, 2008).

The influence of a stiffening ring attached to solid and annular circular plates on the stability is investigated in (Frostig and Simitses, 1988).

Szilassy dealt with some stability problems of circular plates stiffened by a cylindrical shell on the outer boundary in his PhD. thesis (Szilassy, 1971) and in a further article (Szilassy, 1976).

This paper studies the stability problem of a

circular plate stiffened by a cylindrical shell on the outer diameter of the plate. It is assumed that (a) we can use the Kirchhoff theory of plates and shells, and (b) the deformations due to the load are non-axisymmetric. We have the following objectives: (1) to establish the equations that can be used to determine the critical load of a stiffened circular plate (solid or with a hole) (2) computation of the critical load for a stiffened plate with no hole.

As regards the solutions for the cylindrical shell we shall utilize some results of a thesis (Jezsó, 1980).

2. Problem formulation

The cross section of the shell-stiffened structure we are concerned with is shown in Fig. 1. The structure consists of either a solid circular plate or an annular one – the latter is shown in Fig. 1 – and a cylindrical shell, which stiffens the plate on its external boundary.

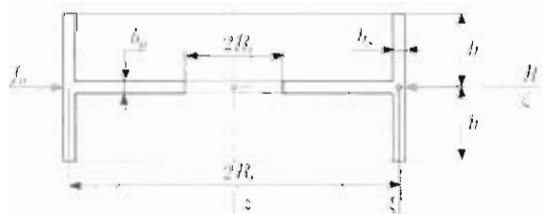


Figure 1. The structure and its load

The support arrangements applied are shown in Fig. 2.

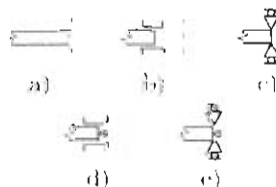


Figure 2. Support arrangements

Under the assumption of small, non-axisymmetric and linearly elastic deformations we shall determine (a) the critical load of the structure and (b) the effect of the stiffening shell on the critical load.

3. Line of thought

Let us expand the solution for the displacement field of the plate into a Fourier-series:

$$w = w_0 + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn}(r) \cos\left(n\varphi - m\frac{\pi}{2}\right) \quad (1)$$

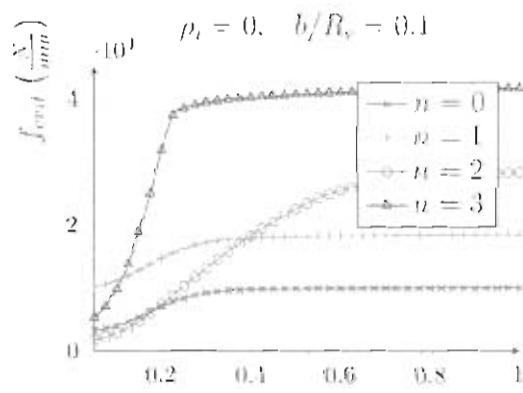
Every physical quantity will be written in the same form as series (1). The rotation, the bending moments, and the shear force can all be given in terms of the amplitudes of the displacement field.

The fundamental equations set up for the displacement coordinates of the shell will be fulfilled identically if we calculate the displacement coordinates from the Galerkin function (Jezsó, 1980). With regard to equation (1) we assume that the Galerkin function is expanded into a Fourier series. Every physical quantity can be written in a form similar to that of equation (1).

A solution for the amplitude of the displacement field on the middle surface of the plate contains 4, while a solution for the Galerkin function involves 8 integration constants. The boundary- and continuity conditions provide twelve homogenous equations for the twelve integration constants. These equations involve the load of the plate as a parameter. The critical value can be determined from the condition that the system determinant should vanish.

4. Results

The graphs in the diagram provide the critical load of a solid plate in terms of the height of the stiffening shell.



The stiffening significantly increases the critical load as the height is increased till it reaches a certain limit. The curves show the critical load for the axisymmetric deformation

and the first 3 members of the Fourier-series.

5. Concluding remarks

The paper is devoted to the determination of the critical load for a circular plate with a hole provided that the plate is stiffened by a cylindrical shell on its outer diameter. It is obvious from the results that the stiffening significantly increases the critical load.

6. Acknowledgement

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ANALYSIS OF BRIDGE AS A STIFFENED PLATE UNDER FULL VEHICLE MODEL

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1. Introduction

Structure-vehicle interaction is problem investigated very often for different purposes such as structure design, maintenance and driven persons safety and comfort. Structure is most frequently modelled as a beam or plate structure (Yagiz and Sakman, 2006.). This works for simpler vehicle models such as moving concentrated force or mass (Dehestani et. al, 2009.). In this work is influence of more realistic vehicle model on real spatial structure as stiffened plate analysed.

2. Problem formulation

In order to make easier to analyse real three-dimensional structures taking into account all important degrees of freedom of dynamic system moving over it, finite strip method is used for structure-vehicle interaction. It is based on dimensional reduction which is realised with separation of variables. This way number of the unknowns as well as the computational effort are significantly reduced. Flat shell strip is formed by combining the plate strip and the plane stress strip so it is capable of simulating both bending and in-plane deformations (Cheung and Tham, 2009.).

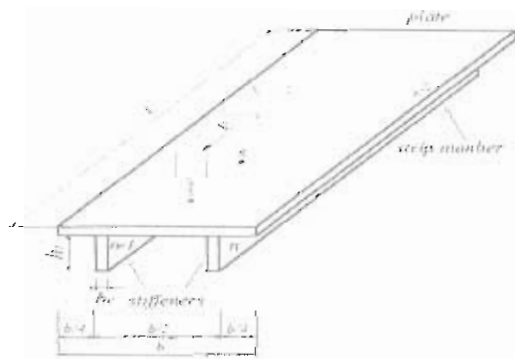


Figure 1. Stiffened plate discretised with finite strips

Discretisation is made in one direction only, usually transverse. In the longitudinal direction domain is discretised with only one strip. Both plate and stiffeners are modelled as shell elements, Figure 1. In the full vehicle model centre of mass has three degrees of freedom: bounce, roll and pitch; while the wheels have only one – bounce, Figure 2. Rolling is consequence of the front and back axle interference. Moment of inertia of the vehicle, with respect to imaginary transverse axis, excludes the possibility for front and back suspension loading to be equal and constant on uneven surfaces. In the purpose of simplification, some assumptions are introduced: there is no loss of contact between the wheel and the ground and profile of the road considered is a flat plane.

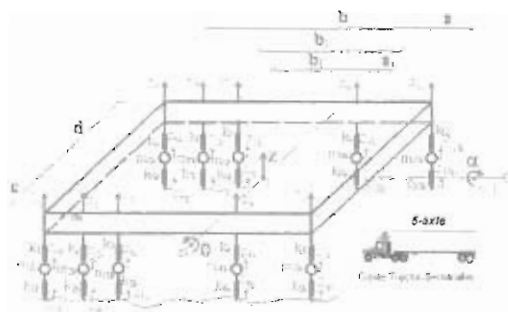


Figure 2. Full vehicle model

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3. Results and discussion

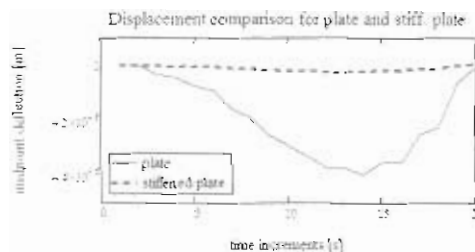


Figure 3. Plate and stiffened plate deflection under 3D vehicle model

Figure 3 presents displacement comparison for plate and stiffened plate. From this diagram dynamic magnification of plate is visible. Comparison for five- axle vehicle model and two series of five moving forces equivalent to loading of 3D vehicle model is made. 3D vehicle gives lower values than moving forces in the beginning, and higher when leaving the bridge.

4. Concluding remarks

In order to introduce more realistic structure and moving load model, stiffened plate under the influence of full vehicle model is analysed. Flat shell strips are used for space discretisation. Motion of full vehicle model is described with 12 differential equations, and together with equation of motion for plate are solved using impulse acceleration method (Kožar, 2009.). Results are compared with measurements taken from literature (Gonzales et.al, 2010) and they show good agreement. Finite strip method enabled to analyze 3D structure model together with 3D vehicle model. This way cross section influence can be observed and more realistic results obtained. Since plate gives significantly higher and unrealistic deflections, especially in case of heavy vehicles, such trailers, it is wise to model bridge structure as stiffened plate.

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HEAT CONDUCTION IN FRACTAL GEOMETRIE BY MIXED BOUNDARY ELEMENT METHOD

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1. Introduction

In the paper the numerical simulation of heat diffusion in the fractal geometry of Koch snowflake is presented using mixed Boundary Element Method (BEM). The idea and motivation of work is to improve the cooling of small electronic devices using fractal geometry of surface similar to cooling ribs. The heat diffusion is assumed as the only principle of heat transfer. The results are compared to the heat rate of flat surface.

The mathematical formulation of fractals originated in 17th century by Gottfried Leibniz. In 1975 Benoit Mandelbrot coined the word "fractal" to denote an object whose Hausdorff-Besicovitch dimension is greater than its topological dimension. He illustrated this mathematical definition with amazing computer visualizations.

There are many fields where fractals are used in engineering applications such as porous media modelling, nanofluids, fracture mechanics and many more. For example, Huang et al 2011 used fractals to model surface morphology in order to numerically compute thermal contact resistance in the field of heat transfer as in our case. His principle assumption is to neglect heat convection and radiation thus the heat conduction is the only mechanism for heat transfer in small air pockets between two surfaces.

The idea of using fractal to model cooling surface raise up from the fundamental question: "How long is the coast of Britain?" stated in paper by Mandelbrot. Using smaller and smaller ruler the length limits to infinite. The area is obvious finite. If this logic is applied to small electronic device cooler, large efficiencies must be obtained.

The BEM has a long tradition in our research group headed by prof. Škerget.

Presented numerical scheme is based on our work dated in 2004.

The full length article will be organized as follows. After Introduction in section 2 the problem definition is stated together with governing equations. In section 3, the numerical algorithm is explained briefly. The section 4 is dedicated to Results. First, the developed numerical algorithm will be verified using an analytical solved case of a flat surface cooler represented the first starting fractal. Different meshes are tested in detail. The second case is a Koch snowflake. The paper finished with conclusions.

2. Problem definition

The problem considered is a cut out of a small electronic device as shown in Figure 1.

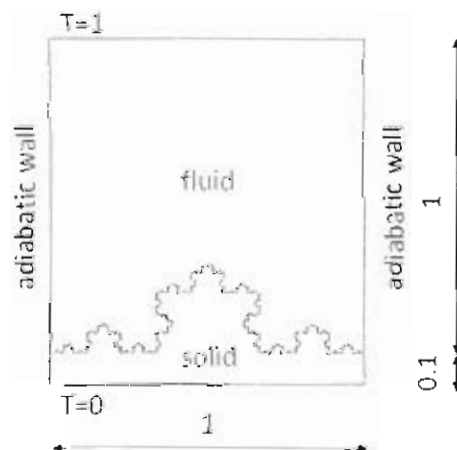


Figure 1: The geometry and boundary conditions of the air cooled fractal electronic device.

The physical model assumptions follow. First, the major assumption is that the heat conduction is the only mechanism of heat transfer thus neglecting heat convection in air and radiation. The basis of this assumption is that the dimension of the problem is small, for example 1 cm and still air around this device.

Another reason is small temperature difference causing small buoyant forces and causing minimal heat radiation. Second assumption is a steady state condition.

The problem has been non-dimensionalised as follows. Length quantities x and y are scaled with the length of the cut-off. Normalized temperature is obtained by scaling with the temperature difference between hot and cold walls. The material properties could be treated in two ways. Since only steady state is computed, the solution is only heat conductivity depended. If governing equation is non-dimensionalised using air conductivity, eg fluid non-dimensional conductivity becomes 1 and solid conductivity is equal to the ratio of solid and air conductivity. In the case of Cu, Al or steel cooler this ratio is 16000, 9000 and 2300 respectively. Such high values could cause numerical instabilities. The second possibility of non-dimensionalising is to use heat diffusivity a instead conductivity. In this manner the diffusivity ratio is only 5.4, 4.4 and 0.8. Anyway, interface boundary condition equals heat flux between solid and fluid resulting in high value heat conductivity ratio in system matrix in both cases of non-dimensionalising. This is the case, where complicated physic couldn't be overcome by numerical trick.

The non-dimensional governing equation is a Laplace equation

$$\frac{\partial}{\partial x_j} \left(a \frac{\partial T}{\partial x_j} \right) = 0$$

The boundary conditions are strait forward as shown in Figure 1.

3. Numerical algorithm

Multidomain mixed Boundary Element Method (BEM) numerical algorithm for solving 2D diffusion problems will be described in this section.

The developed solver has to fulfil following demands: accuracy, stability at high conductivity ratios, economy of the solution

and ability to solve high grid density with high longest/shortest element ratio.

The accuracy of BEM is a well-known quality of BEM. In comparison to the other numerical methods, the accuracy is more evident using low grid densities, see Ramšak et al 2004.

The stability is a common problem using BEM. The stability of our BEM numerical algorithm is the main topic in our previous work Ramšak and Škerget 2004, where considerable effort was done to increase the stability at a convection dominated laminar flows. The result was a stable algorithm solving a driven cavity case up to Reynolds number value 50 000. The usual maximal value is 10 000 to 15 000 using BEM.

The numerical method will be presented in detail in full length paper.

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ANALYSIS OF SMART PIEZOLAMINATED COMPOSITE PLATES USING FINITE ELEMENT METHOD

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Abstract

Piezolaminated smart structures are widely used as light weight structures to control structural response in various structural applications. Due to the application of piezoelectric materials to control structural response, stability and control of light weight structures tends to be the governing criterion which requires significant attention. A finite element methodology is developed for stability analysis of smart piezolaminated composite plates/shells subjected to combined action of electrical and mechanical loading. The degenerated shell element is formulated using total Lagrangian and higher-order shear deformation theory (HOST). Numerical discussion is made for vibration stability analysis of piezolaminated plates.

1. Introduction

Smart material systems and structures have been adopted by engineering and research community over the last two decades. The need of light weight structures in engineering applications has led to the gradual replacement of many isotropic materials with composites which provide both high stiffness and low weight. Dhonthireddy and Chandrashekhara [1996] carried out modeling and shape control of composite beams with embedded piezoelectric actuators. Eisenberger and Abramovich [1997] studied shape control of non-symmetric piezolaminated composite beams. Nguyen et al. [2007] presented evolutionary piezoelectric actuators design optimization for static shape control of smart plates. Mukherjee and Chaudhuri [2002] presented analysis of piezolaminated beams with large deformations. Kulkarni and Bajoria

[2007] carried out large deformation analysis of piezolaminated smart structures using higher-order shear deformation theory. Kulkarni [2004] performed geometric nonlinear analysis and active vibration control of piezolaminated plates/shells. Thus in the present work stability and control of piezolaminated plates/shells is studies subjected to combined action of electromechanical loading.

2. Finite Element Formulation

Higher order shear deformation theory assume realistic cross section deformation pattern through the shell thickness. Here a finite element formulation for stability analysis of piezolaminated smart plate / shell is based on higher order shear deformation theory.

To account for thickness of laminate, these displacement components are expressed in terms of thickness co-ordinate z (Khare et al, 2003). Hence the warping of the transverse cross-section is automatically incorporated. The displacement field is expressed as:

$$\begin{aligned} u &= u_0 + z\theta_x + z^2 u_0^* + z^3 \theta_x^* \\ v &= v_0 + z\theta_y + z^2 v_0^* + z^3 \theta_y^* \\ w &= w_0 \end{aligned} \quad (1)$$

Where, u , v and w are the displacement of any point in the plate domain in x , y and z direction respectively. u_0 , v_0 and w_0 are the displacement of midpoint of normal. θ_x , θ_y are the rotations of normal at the middle plane in x and y direction about y and x axis respectively. u_0^* , v_0^* , θ_x^* , θ_y^* and are higher order terms which accounts cubic variation of normal.

3. Electro-Mechanical Coupling

Due to the direct and converse piezoelectric effect there exists a coupling between electrical and mechanical loading in smart piezoelectric structures. Variation of temperature effect is neglected in formulation. For the piezoelectric layer polarized in the thickness direction, the dielectric displacement vector using direct piezoelectric equation is,

$$\begin{aligned} \{D\} &= [e]\{\varepsilon\} + [g]\{E^p\} \\ \{\sigma\} &= [C]\{\varepsilon\} - [e]^T\{E^p\} \end{aligned} \quad (2)$$

In all above equations, $\{D\}$ is electric displacement vector, $[e]$ is dielectric permittivity matrix, ε is the strain vector, $[g]$ is the dielectric matrix, $\{E^p\}$ is the electric field vector, $[\sigma]$ is the stress vector and $[C]$ is the elastic matrix for constant electric field.

4. Numerical Analysis

A smart composite simply supported plate of size $1.2\text{m} \times 1.2\text{m} \times 0.0012\text{m}$ with a piezolayer at top and bottom is analyzed. The plate consists of three layer of equal thickness. Natural frequency and buckling stress are given in table 1.

Frequency in rad/sec. Without piezolayer	Frequency in rad/sec. With piezolayer	% Variation	Critical buckling stress
6.8542	6.8859	0.4635	3.8165
18.136	18.157	0.1141	
18.326	18.412	0.4709	
37.339	37.428	0.2369	
39.527	39.557	0.0741	
39.937	40.058	0.3023	
70.977	71.018	0.0569	
72.517	72.531	0.0185	

Table 1 Natural Frequency and buckling stress

5. Concluding remarks

Piezoelectric actuators and sensors are being used increasingly in structural applications

involving shape/vibration/buckling control. With proper selection and placement of piezoelectric actuators, it is feasible to generate enough forces on a structure in order to control its response in vibration and buckling. It is observed that the maximum percentage variation in natural frequencies is 0.4709% for PZT layer. The critical buckling stress is 3.8176. Hence the increase in stiffness due to piezoeffect can be considered with sufficient accuracy to control its response.

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HOMOGENIZATION OF STEAM BOILER MEMBRANE WALL

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1. Introduction

Membrane wall of the steam boiler is geometrically determined as orthotropic plate. Boilers are normally supported or hanged on the collection/distribution headers. Due to specific furnace, some versions of the waste incinerators boilers are supported by the supports which are welded on the membrane walls. Because of orthotropic mechanical properties of the boiler construction (membrane walls), there is a problem of determining the reactions in supports, as well as deflection of membrane walls.

By existing analytical methods, which are based on the theory of bending the isotropic plate, obtained results have significant deviation in relation to the measured results. Furthermore, finite element method for the construction of boiler has significant limitations, because discretization by the shell elements requires a large number of finite elements.

This paper proposes a procedure for approximating the membrane wall by using orthotropic plate for approximating the stiffness of the real membrane wall.

2. Procedure to determine the stiffness matrix of orthotropic plate

Matrix equation of the relation between membrane forces and deformation of membrane wall is

$$\mathbf{F} = \mathbf{B} \cdot \boldsymbol{\varepsilon}, \quad (1)$$

where $\mathbf{F}$ is the vector of membrane forces, $\mathbf{B}$ is the stiffness matrix, and $\boldsymbol{\varepsilon}$ is deformation vector. Stiffness matrix $\mathbf{B}$ consists of members $\mathbf{A}$ related to the plane stress state and members $\mathbf{D}$ related to the bend stiffness of the membrane wall.

According to above mentioned, matrix equation (1) can be written as follows:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ & A_{22} & 0 & 0 & 0 & 0 \\ & & A_{33} & 0 & 0 & 0 \\ & & & D_{11} & D_{12} & 0 \\ & sym. & & & D_{22} & 0 \\ & & & & & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2)$$

2.1 Plane (membrane) stress state in the membrane wall of the boiler

Equations for the membrane forces can be derived if stresses equations of Hooke's law for plane stress state of orthotropic plate are multiplied with the thickness of the approximated plate h . From equations for the membrane forces, members of the stiffness matrix for the membrane stress state are:

$$\begin{aligned} A_{11} &= \frac{E_x \cdot h}{1 - \nu_{xy} \cdot \nu_{yx}}, & A_{12} &= \frac{E_y \cdot h \cdot \nu_{xy}}{1 - \nu_{xy} \cdot \nu_{yx}}, \\ A_{22} &= \frac{E_y \cdot h}{1 - \nu_{xy} \cdot \nu_{yx}} & \text{and} & \quad A_{33} = G_{xy} \cdot h. \end{aligned} \quad (3)$$

2.2 Bending load in the membrane wall of the boiler

Using Cauchy's equations for deformations of a thin plate and Hook's law for plane stress state of orthotropic plate, it is possible to derive equations for the stresses. With the integration of stresses through the thickness of approximated orthotropic plate, membrane forces reduced on the middle surface of the plate can be calculated.

According to (2) and obtained equations of membrane forces, members of stiffness matrix for the bending load of the approximated orthotropic plate are:

$$\begin{aligned} D_{11} &= \frac{E_x \cdot h^3}{12 \cdot (1 - \nu_{xy} \cdot \nu_{yx})}, & D_{12} &= \frac{E_x \cdot h^3 \cdot \nu_{xy}}{12 \cdot (1 - \nu_{xy} \cdot \nu_{yx})}, \\ D_{22} &= \frac{E_y \cdot h^3}{12 \cdot (1 - \nu_{xy} \cdot \nu_{yx})} & \text{and} & \quad D_{33} = \frac{G_{xy} \cdot h^3}{12} \end{aligned} \quad (4)$$

2.3 Calculation of wall thickness and elasticity constants of approximated plate

Wall thickness of the approximated plate h is calculated from the equality of moments of inertia for cross section of membrane wall and cross section of the plate.

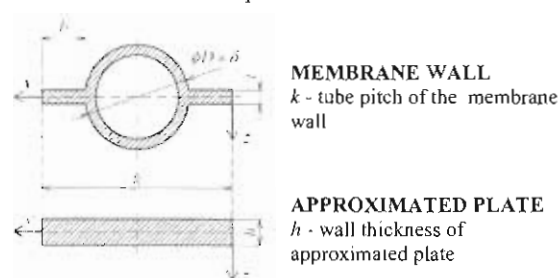


Figure 1. Geometry of cross section for the membrane wall and approximated plate

This paper proposes homogenization procedure for determining elastic constants of approximated orthotropic plate (analytically and numerically using finite element method).

3. Numerical results and discussion

In order to evaluate the accuracy of presented procedure, two models are numerically analysed, with clamped supports on all edges and simply supported on all edges. Dimensions of numerical models are 900 x 900 mm in both cases which are loaded with amount of pressure 0,025 MPa. Material of membrane wall is P235GH (calculating temperature is $\vartheta = 275$ °C). Figures 2 and 3 are showing numerical results for both cases.

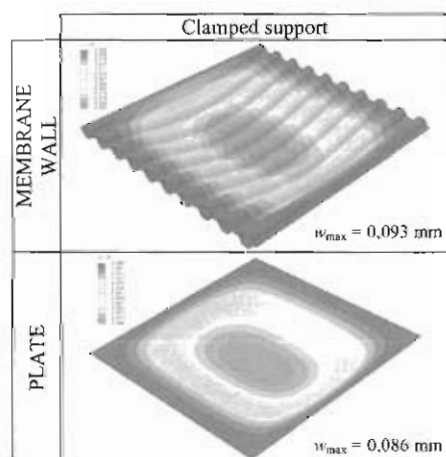


Figure 2. Comparison of numerical results, clamped support

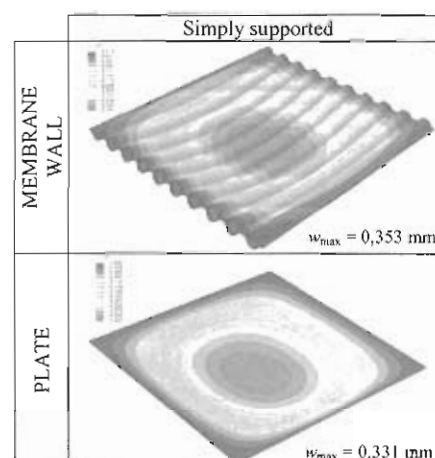


Figure 3. Comparison of numerical results, simply supported

4. Concluding remarks

This paper proposes a procedure for approximating the membrane wall of steam boiler by using orthotropic plate. The result of proposed method is more accurate calculation of the support reactions of steam boiler as well as the deflections of membrane walls. Further research could focus on the development of analytical procedure for calculating other elastic constants, which are in this paper calculated numerically.

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TO INVESTIGATION OF NONSTATIONARY PROCESSES OF COMPRESSION OF THICK-WALLED SPHERICAL AND CYLINDRICAL VISCOPLASTIC SHELLS

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1. Introduction

For testing new computational program systems and methods for numerical modelling problems of media mechanics it is necessary knowledge of analytical solutions. Particularly it is concern to numerical methods and computational programs systems which oriented on decision problems of dynamical thermoelastoviscoplasticity in view of their complication. It is known a few analytical solutions of their problems (Verney, 1968; Grigoriev, Dunin and Surkov, 1981; Golubev, 1983; Galiev, 1988; Kobyalkin, Selivanov, Soloviov and Syasoviev, 2004). Besides, obtained solutions for problems of compression and extension of pores in viscoplastic material (Grigoriev, Dunin and Surkov, 1981; Golubev, 1983; Galiev, 1988; Kobyalkin, Selivanov, Soloviov and Syasoviev, 2004) used for writing kinetic equations for parameter of volume damage of material, which describe known as viscous fracture (Kiselev and Yumashev, 1992; Kiselev, 1998; Kiselev, Lukyanov and Thiercelin, 2004; Kiselev and Zacharov, 2011; Kiselev, Zacharov and Nekhaeva, 2011).

2. Results

In this paper present original analytical solutions for one-dimensional problems of compression of elastoviscoplastic Perzyna type model (Perzyna, 1966) spherical and cylindrical shells. Solutions obtained at hypothesis that in initial moment distributions of radial velocities satisfy condition of incompressibility of shells materials. Obtained solutions may be used for calculation of stress-strain condition of constructions, their heating up, estimation of fracture beginning and contribution of viscosity

to energy dissipation, etc. Obtained solutions may be, in particular, used for testing computational programs and estimate of effectiveness of new numerical methods, as it is were made in paper (Howell and Bally, 2002) used results of paper (Verney, 1968).

2.1. Compression of spherical shell

Consider spherical shell having in initial moment $t=0$ internal radius R_0 and initial radius R_1 . In moment $t>0$ internal radius $r_0 = r(R_0, t)$, internal - $r_1 = r(R_1, t)$ (Figure). Make next simplifying assumptions.

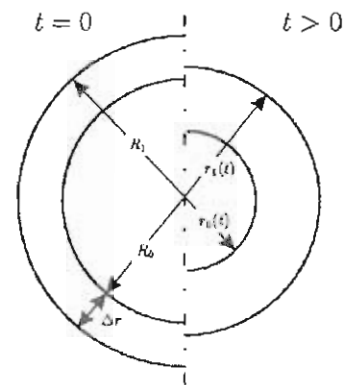


Figure.

1) Behavior of material describe of Perzyna type elastoviscoplastic model (Perzyna, 1966):

$$\dot{\epsilon}_{ij} = \frac{\dot{S}_{ij}}{2\mu} + \frac{S_{ij}}{2\eta} \frac{\sqrt{S_{ij}S_{ij}} - \sqrt{\frac{2}{3}}Y_0}}{\sqrt{S_{ij}S_{ij}}} \times$$

$$\times H(\sqrt{S_{ij}S_{ij}} - \sqrt{\frac{2}{3}}Y_0). \quad (1)$$

Here $\dot{\epsilon}_{ij}, S_{ij}$ – deviators of strain rates and stresses; Y_0 – yield limit under simple tension; $H(x)$ – Heviside function; μ, η – shear module and dynamic viscosity.

2) Elastic deformations can be neglected: $\dot{\epsilon}_{ij}^e = 0$, $\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^p$ ($\dot{\epsilon}_{ij}^e, \dot{\epsilon}_{ij}^p, \dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p$ – elastic, plastic and total strain rates respectively). Plastic flow is incompressible: $\dot{\epsilon}_{kk}^p = \dot{\epsilon}_{kk} = 0$.

3) Problem solve in one-dimensional assumption: all parameters depend on radial Lagrange coordinate R and time t .

4) Process of shells deformation is adiabatic.

Therefore equations (1) reduced to one equation

$$\sigma_R - \sigma_\theta = Y_0 + 2\eta(\dot{\epsilon}_R - \dot{\epsilon}_\theta). \quad (2)$$

Here v – is radial velocity; $\dot{\epsilon}_R = \partial v / \partial R$, $\dot{\epsilon}_\theta = v / R$ – radial and ring strain rates; σ_R , σ_θ – radial and ring stresses.

Hypothesis of material incompressibility $\dot{\epsilon}_R + 2\dot{\epsilon}_\theta = 0$ give equation for determination of velocity distribution in shell:

$$\partial v / \partial R + 2v / R = 0. \quad (3)$$

Solution of equation (3) has next form:

$$v = C(t) / R^2. \quad (4)$$

$C(t) \leq 0$, because take place compression of shell and velocity $v \geq 0$. In initial moment $t = 0$ $C(0) = V_0 R_0^2$, where V_0 – initial velocity of internal surface of shell.

Introduce pressure $p = -(\sigma_R + 2\sigma_\theta) / 3$. Then, using formulas (2) – (4), we obtained

$$\begin{aligned} \sigma_R &= \frac{2}{3} Y_0 - 4\eta \frac{C(t)}{R^3} - p, \\ \sigma_\theta &= -\frac{1}{3} Y_0 + 2\eta \frac{C(t)}{R^3} - p. \end{aligned} \quad (5)$$

The equation of momentum for spherical shell is

$$\rho \dot{v} = \frac{\partial \sigma_R}{\partial R} + 2 \frac{\sigma_R - \sigma_\theta}{R}. \quad (6)$$

Here ρ – density of shell material; the dot over symbols indicates the material derivative with respect to time.

Equation for density of internal energy U^{sph}

$$\rho \dot{U}^{sph} = \sigma_R \frac{\partial v}{\partial R} + 2\sigma_\theta \frac{v}{R},$$

taking into account (3) and (5), received next form:

$$\rho \dot{U}^{sph} = -2Y_0 \frac{v}{R} + 12\eta \left(\frac{v}{R} \right)^2.$$

Rate of change of total internal energy for spherical shell is

$$\dot{U}_\Sigma^{sph} = \int_{R_0}^{R_1} 4\pi R^2 \rho \dot{U}^{sph} dR.$$

Taking into account formula (4), we received

$$\begin{aligned} \dot{U}_\Sigma^s &= 16\pi\eta C^2(t) \left(\frac{1}{R_0^3} - \frac{1}{R_1^3} \right) - \\ &- 8\pi Y_0 C(t) \ln \frac{R_1}{R_0}. \end{aligned} \quad (7)$$

Substitutes expressions (2), (4), (5) into equation of momentum, we received next equation:

$$\frac{\partial p}{\partial R} = -\frac{\rho}{R^2} \dot{C}(t) + 2 \frac{Y_0}{R}. \quad (8)$$

Kinetic energy of spherical shell determine as

$$\begin{aligned} K^{sph} &= \frac{1}{2} \int_{R_0}^{R_1} 4\pi R^2 \rho V^2 dR = \\ &= 2\pi \rho C^2(t) \left(\frac{1}{R_0} - \frac{1}{R_1} \right). \end{aligned} \quad (9)$$

Initial kinetic energy of spherical shell is

$$K_0^{sph} = 2\pi \rho V_0^2 R_0^4 (1/R_0^4 - 1/R_1^4). \quad (10)$$

In process of compression of spherical shell its kinetic energy pass into internal energy and in moment of stopping compression of shell $t = t_s^{sph}$ all kinetic energy pass into internal energy. In moment t ($0 \leq t \leq t_s^{sph}$) realized principle of conservation of total energy:

$$\begin{aligned} K^{sph}(t) + U_\Sigma^{sph}(t) &= K_0^{sph} \text{ or} \\ \dot{K}^{sph}(t) + \dot{U}_\Sigma^{sph}(t) &= 0. \end{aligned} \quad (11)$$

Used formulas (7) and (9), we received from (11) ordinary differential equation for function $C(t)$:

$$\begin{aligned} \dot{C}(t) + \alpha C(t) + \beta &= 0; \\ \alpha &= \frac{4\eta}{\rho} \frac{R_0^2 + R_0 R_1 + R_1^2}{R_0^2 R_1^2}, \\ \beta &= -\frac{2Y_0}{\rho} \frac{R_0 R_1}{R_1 - R_0} \ln \frac{R_1}{R_0}. \end{aligned} \quad (12)$$

Decision of equation (11) with initial condition $C(0) = V_0 R_0^2$ is

$$C(t) = (V_0 R_0^2 + \gamma) \cdot e^{-\alpha t} - \gamma; \quad (13)$$

$$\gamma = \frac{\beta}{\alpha} = -\frac{Y_0}{2\eta} \frac{R_0^3 R_1^3}{R_1^3 - R_0^3} \ln \frac{R_1}{R_0}.$$

Moment of stopping of shell compression $t = t_s^{sph}$ define from condition $C(t_s^{sph}) = 0$ and is decision of equation $(V_0 R_0^2 + \gamma) e^{-\alpha t_s^{sph}} = \gamma$:

$$t_s^{sph} = \frac{1}{\alpha} \ln \left(1 + \frac{V_0 R_0^2}{\gamma} \right). \quad (14)$$

Gradient of pressure we can find from formula (8):

$$\frac{\partial p}{\partial R} = 2 \frac{Y_0}{R} + \alpha \frac{\rho_0}{R^2} (V_0 R_0^2 + \gamma) e^{-\alpha t}. \quad (15)$$

In initial $t=0$ and in final $t = t_s^{sph}$ moment gradient of pressure are:

$$\frac{\partial p}{\partial R} \Big|_{t=t_s^{sph}} = 2 \frac{Y_0}{R} - 2 \frac{Y_0}{R^2} \frac{R_0 R_1}{R_1 - R_0} \ln \frac{R_1}{R_0}, \quad (16)$$

$$\frac{\partial p}{\partial R} \Big|_{t=0} = \frac{\partial p}{\partial R} \Big|_{t=t_s^{sph}} + \frac{4\eta}{R^2} \frac{R_0^2 + R_0 R_1 + R_1^2}{R_1^2} V_0.$$

With respect to internal energy in moment then compressing of shell stop $U_\Sigma^{sph}(t_s^{sph})$, we find mean value of temperature differential in shell:

$$\Delta T^{sph}(t_s^{sph}) = \frac{3}{2} \frac{V_0^2 R_0^3}{R_1 (R_0^2 + R_0 R_1 + R_1^2) c}. \quad (17)$$

Here c – specific heat of shell material.

Maximum of initial kinetic energy of spherical shell under fixed external radius R_1 and initial velocity V_0 reached under $R_0 = 0,75 R_1$: $(K_0^{sph})_{\max} = 0,5 \pi \rho V_0^2 R_0^3$.

Count deformations from initial condition under $t=0$, we find deformations of shell in final moment $t = t_s^{sph}$:

$$\begin{aligned} \varepsilon_\theta^{sph} &= \frac{1}{\alpha R^3} (V_0 R_0^2 - \gamma \ln(1 + \frac{V_0 R_0^2}{\gamma})), \\ \varepsilon_R^{sph} &= -2 \varepsilon_\theta^{sph}. \end{aligned} \quad (18)$$

Radiuses of internal and external surfaces of shell in moment of stop motion are:

$$\begin{aligned} r(R_0, t_s^{sph}) &= R_0 [1 + \frac{1}{\alpha R_0^3} (V_0 R_0^2 - \gamma \ln(1 + \frac{V_0 R_0^2}{\gamma}))], \\ r(R_1, t_s^{sph}) &= R_1 [1 + \frac{1}{\alpha R_1^3} (V_0 R_0^2 - \gamma \ln(1 + \frac{V_0 R_0^2}{\gamma}))]. \end{aligned} \quad (19)$$

2.2. Compression of cylindrical shell

In case of cylindrical shell under made above assumptions and since condition that displacements along axis z are lacking, i.e. strain rates $\dot{\varepsilon}_z = 0$, equation (1) result in form

$$\begin{aligned} \sigma_R &= \frac{1}{\sqrt{3}} Y_0 + 2\eta \dot{\varepsilon}_R - p, \\ \sigma_\theta &= -\frac{1}{\sqrt{3}} Y_0 + 2\eta \dot{\varepsilon}_\theta - p. \end{aligned} \quad (20)$$

Taking into account hypothesis of incompressibility of shell material $\dot{\epsilon}_r + \dot{\epsilon}_\theta = 0$, then similarly as in case of spherical shell instead of formulas (3) – (19) we received

$$\frac{\partial v}{\partial R} + \frac{v}{R} = 0. \quad (21)$$

$$v = B(t) / R, \quad B(t) \leq 0. \quad (22)$$

$$\sigma_R = \frac{1}{\sqrt{3}} Y_0 - 2\eta \frac{B(t)}{R^2} - p, \quad (23)$$

$$\sigma_\theta = -\frac{1}{\sqrt{3}} Y_0 + 2\eta \frac{B(t)}{R^2} - p.$$

$$\rho \dot{v} = \frac{\partial \sigma_R}{\partial R} + \frac{\sigma_R - \sigma_\theta}{R}. \quad (24)$$

$$\rho \dot{U}^{sph} = \sigma_R \frac{\partial v}{\partial R} + \sigma_\theta \frac{v}{R}. \quad (25)$$

$$\rho \dot{U}^{sph} = -\frac{2Y_0}{\sqrt{3}} \frac{B(t)}{R^2} + 4\eta \frac{B^2(t)}{R^4}. \quad (26)$$

$$\begin{aligned} \dot{U}_\Sigma^{cyl} = & -\frac{4\pi}{\sqrt{3}} Y_0 B(t) \ln \frac{R_1}{R_0} + \\ & + 4\pi\eta B^2(t) \left(\frac{1}{R_0^2} - \frac{1}{R_1^2} \right). \end{aligned} \quad (27)$$

$$\frac{\partial p}{\partial R} = -\frac{\rho}{R} \dot{B}(t) + \frac{2}{\sqrt{3}} \frac{Y_0}{R}. \quad (28)$$

$$\begin{aligned} K^{cyl} = & \frac{1}{2} \int_{R_0}^{R_1} 2\pi R \rho V^2 dR = \pi \rho \int_{R_0}^{R_1} R \frac{B^2(t)}{R^2} dR = \\ = & \pi \rho B^2(t) \ln \frac{R_1}{R_2}. \end{aligned} \quad (29)$$

$$K_0^{cyl} = \pi \rho V_0^2 R_0^2 \ln \frac{R_1}{R_0}. \quad (30)$$

$$\dot{K}^{sph}(t) + \dot{U}_\Sigma^{sph}(t) = 0. \quad (31)$$

$$\dot{B}(t) + \bar{\alpha} B(t) + \bar{\beta} = 0; \quad (32)$$

$$\bar{\alpha} = \frac{2\eta}{\rho} \frac{R_1^2 - R_0^2}{R_0^2 R_1^2} \left(\ln \frac{R_1}{R_0} \right)^{-1}, \quad \bar{\beta} = -\frac{2Y_0}{\sqrt{3}\rho}.$$

$$B(t) = (V_0 R_0 + \bar{\gamma}) \cdot e^{-\bar{\alpha} t} - \bar{\gamma}; \quad (33)$$

$$\bar{\gamma} = \frac{\bar{\beta}}{\bar{\alpha}} = -\frac{Y_0}{\sqrt{3}\eta} \frac{R_0^2 R_1^2}{R_1^2 - R_0^2} \ln \frac{R_1}{R_0}.$$

$$t_s^{cyl} = \frac{1}{\bar{\alpha}} \ln \left(1 + \frac{V_0 R_0}{\bar{\gamma}} \right). \quad (34)$$

$$\frac{\partial p}{\partial R} = \frac{2Y_0}{\sqrt{3}R} + \bar{\alpha} \frac{\rho}{R} (V_0 R_0 + \bar{\gamma}) e^{-\bar{\alpha} t}. \quad (35)$$

$$\frac{\partial p}{\partial R} \Big|_{t=0} = 2 \frac{\eta}{R} \frac{R_1^2 - R_0^2}{R_0 R_1^2} \left(\ln \frac{R_1}{R_0} \right)^{-1} V_0, \quad (36)$$

$$\frac{\partial p}{\partial R} \Big|_{t=t_s^{cyl}} = 0.$$

$$\Delta T^{cyl}(t_s^{cyl}) = \frac{V_0^2 R_0^2}{(R_1^2 - R_0^2)c} \ln \frac{R_1}{R_0}. \quad (37)$$

$$\epsilon_\theta^{cyl} \Big|_{t=t_s^{cyl}} = \frac{1}{\bar{\alpha} R^2} \left(V_0 R_0 - \bar{\gamma} \ln \left(1 + \frac{V_0 R_0}{\bar{\gamma}} \right) \right), \quad (38)$$

$$\epsilon_R^{cyl} \Big|_{t=t_s^{cyl}} = -\epsilon_\theta^{cyl} \Big|_{t=t_s^{cyl}}.$$

$$r(R_0, t_s^{cyl}) = R_0 \left[1 + \frac{1}{\bar{\alpha} R_0^2} \left(V_0 R_0 - \bar{\gamma} \ln \left(1 + \frac{V_0 R_0}{\bar{\gamma}} \right) \right) \right],$$

$$r(R_1, t_s^{cyl}) = R_1 \left[1 + \frac{1}{\bar{\alpha} R_1^2} \left(V_0 R_0 - \bar{\gamma} \ln \left(1 + \frac{V_0 R_0}{\bar{\gamma}} \right) \right) \right].$$

3. Concluding remarks

Obtained solutions easily transform for cases of extension of shells then $V_0 > 0$. For that sufficiently in all formulas to substitute Y_0 on $(-Y_0)$, where Y_0 – yield limit under simple tension in Perzyna model (1).

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WIND ENGINEERING

DESIGN AND VALIDATION OF EXPERIMENTAL RIG FOR WIND TUNNEL TESTS ON OFFSHORE WIND TURBINES

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1. Introduction

The main purpose of the following work is to propose an experimental methodology to investigate the dynamics of scale offshore floating wind turbines within a wind tunnel facility, with the ability to measure forces at the base of the tower and at the nacelle in order to correlate a wind state and sea state (imposed motion at the base of tower) with the loads acting on the structures (blades, tower, nacelle, etc...). This work has been carried out at Politecnico di Milano wind tunnel 13.84 x 3.84 m boundary layer test section, where a 2 degrees-of-freedom mechanism for wave dynamics simulations has been designed, realized and validated. A 1/25 rigidly-scaled wind turbine model has been mounted on the above mentioned simulator platform and tested during the experimental campaign. The motion of the mechanism at issue is provided by two hydraulic actuators, one for each degree-of-freedom. Different combinations in frequency and amplitude, concerning the sinusoidal input of the imposed motion, have been tested and the forces acting on rotor and tower have been measured.

Although the interest in offshore floating wind turbines is relatively recent, relevant efforts have been already made in terms of numerical aero-hydro-elastic codes [2,3,4,7] used for preliminary phases in the design of floating platform prototypes and mooring lines systems [9]. In literature not many experimental rigs dedicated to this topic have been already developed and the ones most similar (see for example [1]) have not been tested in wind tunnel test sections as big as Politecnico di Milano tunnel, so that the reliability in the quality of wind inflow is not guaranteed.

2. Experimental Setup

The 2 DOF mechanism provides the possibility of investigating different experimental configurations (Fig.1). The coupled motions of surge/pitch and sway/roll can be analysed; moreover, the mechanism is mounted on a rotating platform, so that different yaw angles, with respect to wind direction, can be studied. The translational motion is provided by a slider and rotations are ensured by a slider-crank mechanism rigidly jointed to the translational slider. The loads acting on the scale model and the displacements related to the 2 degrees of freedom are measured respectively by two balances and two laser displacements transducer. The balances are positioned at the base of the tower of the wind turbine and in the nacelle.



Figure 1. Experimental rig for testing offshore wind turbines within a wind tunnel facility

3. Results and discussion

The experimental campaign has been carried out in different configurations (Tab.1) and for various combinations in frequency and amplitude of the imposed motion. Representing the results with respect to displacements and apparent wind, hysteresis cycles have emerged in the measured forces (Fig.2). The rotation of the blades is ensured by an electrical motor speed-controlled that has been set to 16 rad/s (Fig.2), consistently with the properly scaled nominal operating conditions, as the parameters of wind in Tab.1, indeed. The wind turbine prototype considered during the model realization is Vestas V52-850kW. The hysteresis area increase with the augmentation of frequency or amplitude of the imposed motion. The variation in aerodynamic efficiency of wind turbine blades has been analysed and correlated to the variation of the angle of attack seen by blades during the functioning of the experimental rig (Fig.3,4). What can be easily captured in the Fig.3 is that the highest variation in the aerodynamic efficiency (which is strictly correlated to the power generation) is related to the highest dynamic conditions of imposed motion at the base of the tower.

Parameter	Unit	Value			
Frequency (f)	[Hz]	0.2	0.4	0.6	1
Amplitude (A)	[mm]	10	20	40	80
Wind Speed (W)	[m/s]	4.5	5	5.5	6

Table 1. Characteristic parameters of the experimental campaign.

The aerodynamic effects of such experimental conditions have been studied and linked to the torque motor and consequently to the generation of power (Fig.2). Considerations upon the development of possible blade pitch

control systems have been reported.

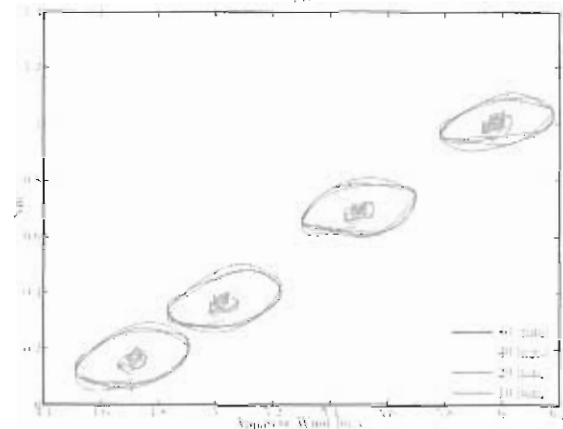


Figure 2. Measured motor torque as function of apparent wind; frequency of the imposed motion: 0.4 Hz.

4. Concluding remarks

The proposed experimental methodology is suitable for testing scale models of wind turbines for offshore installations, within a wind tunnel facility; so that a high control in terms of wind profile generation and turbulence index is ensured compared to other facilities as water tanks where only big fans are used (even if the hydro-platform interaction is more realistic). Sophisticated numerical aero and hydro-elastic models can be experimentally validated. The aerodynamic effects of such floating wind turbines, operating in harsh environmental conditions, can be studied and linked to the power generation; consequently the concerned blade pitch control systems can be tested and improved using the reported rig used in a real-time mode, in order to develop dedicated algorithms to compensate the dynamical variation in aerodynamic efficiency that consequently leads to variation in the power generation.

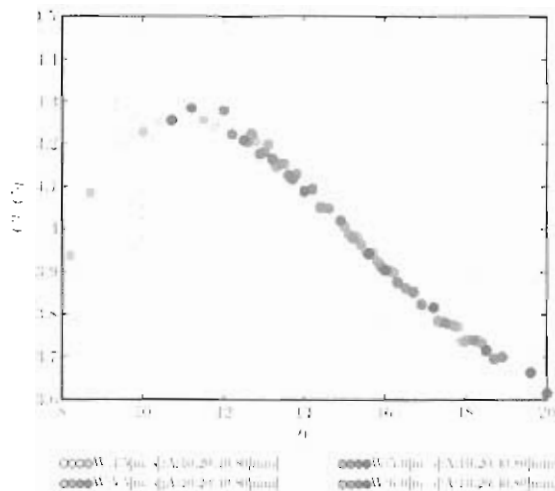


Figure 3. Lift to drag ratio (aerodynamic efficiency) as function of the angle of attack varying within the testing range.

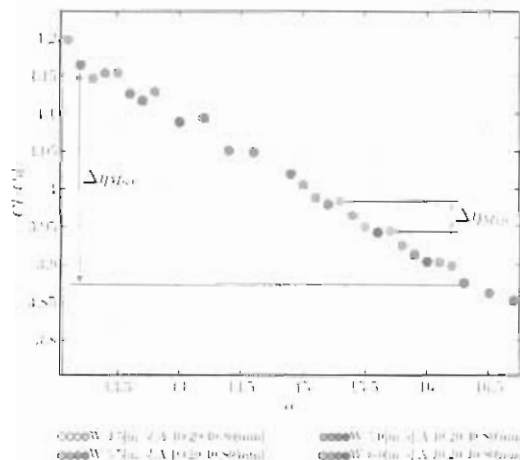


Figure 4. Detail of Fig.3

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NUMERICAL SIMULATION AND WIND TUNNEL STUDIES OF THE WIND ENVIRONMENTS AFFECTED BY THE WIND TURBINES

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1. Introduction

Renewable energy production and demand are gaining momentum in many ways across the world. There is a booming demand of the wind power today. Wind turbines are used to generate electricity from the kinetic power of the wind. To understand the effects on wind environment by the wind turbines are essential. Computational fluid dynamics (CFD) is one of the branches of the fluid mechanics that uses numerical methods and algorithms to solve and analyze problems that involve the fluid flows. Advanced technology makes computers faster and more powerful, which allows the CFD procedures to be applied to many experimental flow problems [1],[2].

This paper presents a combined experimental and computational study into the wind environments affected by the wind turbine. The wind tunnel tests were carried out to obtain the downstream wind speed of the area of the wind turbines by using the hot film probes. Three dimensional unsteady CFD models were generated to predict the same data as the wind tunnel test. Considering the dynamics similarity between the wind tunnel test and the field measurement, the wind tunnel test performed with the lower rotating speed of the wind turbine than the full scale wind turbine. If a CFD model can be established for predicting the dynamic behaviors of the wind turbine, it will be very helpful for the designer to get more information about the effects of the wind turbines at downstream area for the wind environments during the design and the planning stage of the project.

2. Experimental Setup

Fluid modeling

The experiments were performed in a low-

speed environmental wind tunnel located at Tamkang University in Tamsui, Taiwan. The test section of the wind tunnel has dimensions of $3.5 \times 18 \times 2.0$ m and can generate an average wind speed of up to 16 m/sec. To avoid having the walls of the tunnel interfere with the flow rates and streamlines, the cross-sectional area of an obstacle should be less than 5% of the tunnel crosssectional area. The boundary layer thickness from the tunnel walls showed less than a 5% change (from 14 to 14.5 cm thick) with the model present. Therefore, we can safely assume for our experiments that the tunnel wall has a negligible effect on the bulk of the flow field within the test section

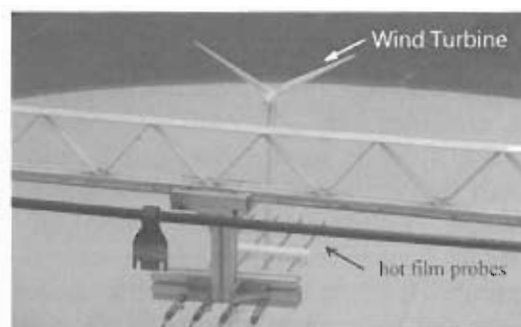


Figure 1. Measurements of wind speed at downstream of the wind turbine by hot film probes

Numerical modeling

The numerical simulation tool used in this study was the computational fluid dynamics software, Fluent [3]. The Fluent CFD software was based on a finite volume discretization of the equations of motion. In this study, steady and unsteady with the RNG kappa-epsilon ($\kappa - \epsilon$) and Large Eddy Simulation turbulence models were adopted to calculate the flow fields around the wind turbine and examine the results of the wind velocity and turbulence intensity profiles downstream of the wind

turbines. Both of the sliding mesh model and the dynamic mesh model were also used.

Sliding Mesh Model

When a time-accurate solution for rotor-stator interaction is desired, the sliding mesh model is used to compute the unsteady flow field. The sliding mesh model is not only the most accurate method for simulating flows in multiple moving reference frames, but also the most computationally demanding. Most often, the unsteady solution that is sought in a sliding mesh simulation is time-periodic. That is, the unsteady solution repeats with a period related to the speeds of the moving domains. However, the other types of transients including translating sliding mesh zones can also be simulated. For example, two cars or trains pass in a tunnel.

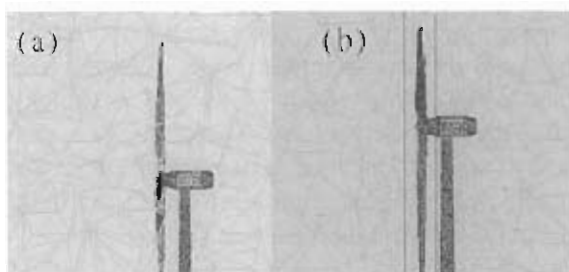


Fig.2 Grid point setup image with (a) Dynamic Mesh Model. And (b) Sliding Mesh Model

Dynamic Mesh Model

The dynamic mesh model can be used to model flows where the shape of the domain is changing with time due to motion on the domain boundaries. The motion can be a prescribed motion or an unprescribed motion where the subsequent motion is determined based on the solution at the current time. The update of the volume mesh is handled automatically by software at each time step based on the new positions of the boundaries. To use the dynamic mesh model, the user needs to provide a starting volume mesh and the description of the motion of any moving zones in the model. Software allows the user to describe the motion using either boundary profiles, user-defined functions (UDFs), or the Six Degree of Freedom solver. Figure 2 showed the geometry and grid point setup for

numerical model. The minimum face grid resolution on the wind turbine blade is 8.0×10^{-7} M2. For the resolution of the turbulent flow scales, and the correct representation of the wake. That meshes are acceptable.

3. Results and discussion

This research is expected to find the correlation of the wind turbine and the wind environment by using the wind tunnel experiment and computational fluid dynamics to set up an assessment method to predict wind environments. Both the numerical dynamic mesh and sliding mesh models were selected for the numerical simulations. Since the wind tunnel experiment can't (reflect) satisfy the real flow field of the wind turbine, the study is attempted to find the best mode and parameter by comparing the numerical simulation with the flow characteristic of wind tunnel experiment. The mode and parameter are set up to imitate the real dimensions of the wind turbine. The result is then utilized to cooperate with wind tunnel experiment to assess the influence on wind environment. Slide mesh is selected for use on the technology of the numerical simulation for there is not much difference after comparing the function test of dynamic mesh and slide mesh. However, in the result of the numerical prediction of the wind turbine, the wind velocity profiles have the same trends with the wind tunnel experiment results and the results of the turbulent intensity profile are close to wind tunnel data.

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AIR VENTILATION ASSESSMENT (AVA) FOR BUILDING DEVELOPMENTS

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1. Introduction

Air quality is becoming an important design consideration for major world cities. Air ventilation assessment (AVA) is an initiative introduced in 2006 by the Government of the Hong Kong Special Administrative Region (HKSAR) to address the alarming deterioration in the past decades of air quality in Hong Kong. This paper outlines the contributions of wind engineering in the implementation of AVA, particularly in the process of expert evaluation, and in developing wind tunnel test techniques and analysis methodologies for site wind availability study and detailed AVA study specified in the technical guide for AVA. A case study is used to highlight the role and importance of each of the suggested critical elements and each step of the proposed analysis procedure in the wind tunnel simulation. The case study also provides a relevant benchmarking to verify the achievability of the proposed air ventilation guideline that the mean wind speed of the development site should be at least or more than 1.5 m/s at pedestrian level in an existing urban area. The impact of the interaction between the prevailing wind climate and the urban fabric on air ventilation performance is discussed.

2. Implementation of Air Ventilation Assessment (AVA)

In view of developing understandings of wind-structure interaction that governs wind flow within urban areas populated with high-rise buildings, air ventilation assessment (AVA)

methodology has been developed to evaluate the wind environment at locations within heavily built-up areas with complex topography, such as those commonly found in Hong Kong. AVA is usually conducted in the following stages:

- a) Expert evaluation;
- b) Site wind availability study; and
- c) Detailed AVA study, employing either computation fluid dynamics (CFD) or wind tunnel testing.

The assessment methodologies adopted for expert evaluation, site wind availability study and detailed AVA using wind tunnel test techniques are briefly outlined. The wind tunnel test techniques adopted are expected to demonstrate compliance with current international best practice techniques (e.g. AWES-QAM-1-2001, ASCE, 2004).

3. A Case Study of Detailed AVA

A case study, which focuses on pedestrian level wind environment in an existing built-up area in Hong Kong, is presented here to highlight the critical elements and steps taken in the model simulation in the wind tunnel. This case study is a practical example to demonstrate the application of the proposed AVA guideline and to assess the likelihood of achieving the AVA target mean wind speed of 1.5 m/s at pedestrian level in and around a development in an existing built-up area in Hong Kong.

The eastern region of the Central district of Hong Kong was chosen as the study area. Central is a typical commercial district in Hong Kong, where the urban fabric is characterised

by high-rise buildings with unique and irregular shapes. The street pattern is also irregularly oriented due to geographical features and the arrangement of built-up areas. The pedestrian level wind environment study was conducted in the high-speed section of the CLP Power Wind/Wave Tunnel Facility (WWTF) at the Hong Kong University of Science and Technology (HKUST) using a 1:400 scale model of the region within a radius of 600 m around the study area, as shown in Figure 1.

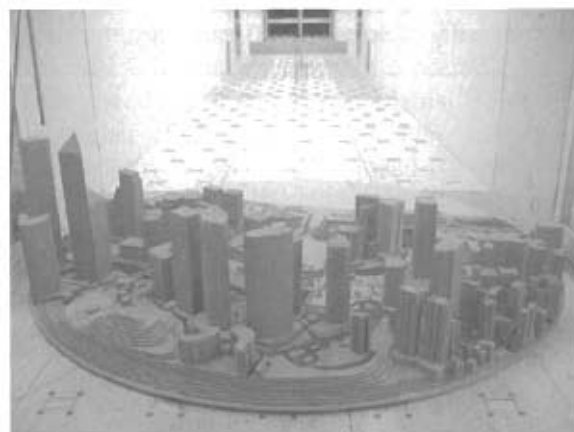


Figure 1. A 1:400 scale model of the study area in HKUST WWTF's high-speed test section

Velocity ratios and mean wind speed distributions were determined and compared at 16 measurement positions. Directional velocity ratios, which were normalised by a reference wind speed, of the measurement positions, were used for detailed wind environment analysis. A typical directional velocity ratio plot is shown in Figure 2. By examining the directional velocity ratio plot at each position, the impacts of surrounding buildings and exposure to prevailing winds on the local wind flow characteristics can be evaluated reliably. Although spatially averaged velocity ratios provide a general idea of the wind environment of the study area, it is more comprehensive and more beneficial in terms of mitigation strategy to assess directional wind speed or velocity ratio distribution with respect to relevant guidelines or acceptance criteria for AVA and/or pedestrian environment wind.

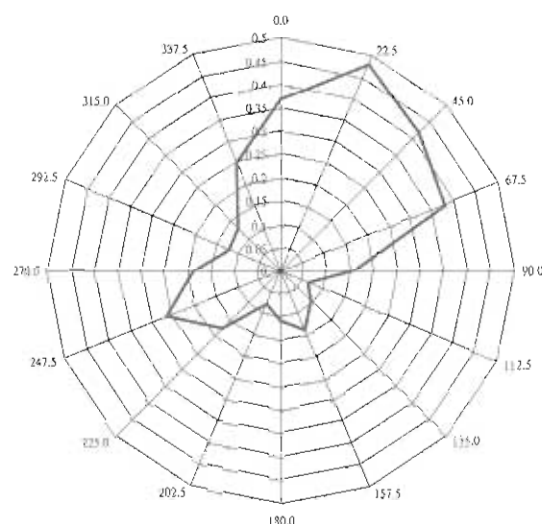


Figure 2. Directional velocity ratios for Position 9

4. Concluding remarks

This paper outlined the contributions of wind engineering in the implementation of air ventilation assessment (AVA), particularly in the process of expert evaluation, and in developing wind tunnel test techniques and analysis methodologies for site wind availability study and detailed AVA study. The wind tunnel modelling requirements and analysis procedures for detailed AVA study in a complex terrain such as Hong Kong has also been highlighted.

Acknowledgements

The authors acknowledge the contributions by Dr Peter Hitchcock, staff and students of HKUST's CLP Power Wind/Wave Tunnel Facility and the Planning Department of HKSAR. The research is supported by the Research Grants Council of Hong Kong (Project HKUST6301/04E & 9041338).

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DISPLACEMENT EVALUATION OF ANTENNA COLUMN BASED ON THE DEFORMATION RECORDS

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1. Introduction

The objective of this paper is monitoring local winds characteristics in the coastal area of Croatia and slender structure response when recorded wind load is applied. Slender structures are very sensitive to wind loads, having significant horizontal displacements, which results in serviceability limit state being crucial part of the design. As an example, antenna columns have specified orientation directions and displacement limitations of 1° corresponding to vertical rotation. In order to have designed displacements match those recorded on site adequate wind load is as equally important segment as structural modelling. The region is known for its unstable regime of air circulation with the dominant winds of Bora and Jugo. Usually wind is observed as a global phenomenon which results in neglecting some areas of highly variable local wind characteristics.

2. Experimental Setup

Perennial field measurements were carried out (Fig. 1) on an antenna column (40 m high) at the Bobani location ($\varphi = 43^\circ 36'$, $\lambda = 16^\circ 27'$). There are no barriers or obstacles around the mast. The measurements of the wind direction and velocity are performed at three levels: 10m, 20m and 35 m above the ground level, recording in second intervals, with a cup anemometer. Installed measurement systems were calibrated and adjusted according to the Meteorological and Hydrological Service. Structure response is monitored at 4 levels, strain gages have been positioned and deformations have been measured and recorded. Strain gages used are Vishay Micro-Measurements N2A-06-S150R, T-Rosette in full – bridge configuration. Data is continuously sampled at frequency of 33 Hz. Strain gages are calibrated on site using system of cables and load cells.

To extrapolate deformations and inner forces from recorded data, numerical model of tower is used.



Figure 1. Photograph of the column at Bobani

3. Results and discussion

From recorded data, analysis is performed in respect to major local winds: bora and jugo. For each wind relation is determined between wind speed and deflection of characteristic points on construction. Further, comparison is performed between new data obtained in this paper with data given in our past papers and norms. From this analysis the new insight in differences and similarities between major local winds will be given.

Acknowledgements

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the IAWC. This experiment is made possible thanks to HRVATSKI TELEKOM d.d. which provided us right to use antenna tower at test location.

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WIND-INDUCED FATIGUE CYCLE COUNTS – SENSITIVITY TO EXTREME WIND CLIMATE AND DYNAMIC RESPONSE

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1. Introduction

Fatigue failures produced by turbulent wind forces are relatively common, and a number of authors have predicted damage rates and fatigue lives based on assumed relationships between stress range and cycles to failure (s-N curves), and known wind climates (e.g. Holmes [1], Robertson *et al.* [2], Repetto and Solari [3], Barle *et al.* [4]).

An alternative, and earlier, approach is to specify the number of load repetitions, or exceedences of a certain load, or stress threshold, and not to prescribe the material s-N relationship. This approach was proposed by Davenport as early as 1966 [5]. Such an approach is suitable for a loading code or standard, and has in fact been adopted by the Eurocode for Wind Actions [6].

This paper will study the sensitivity of normalized stress exceedence count curves, such as that provided in the Eurocode, to the Weibull wind climate parameters, and to the exponent relating the standard deviation of fluctuating stress to the mean wind speed.

2. Stress cycle count

The Weibull Distribution is commonly used to describe average wind speeds (averaged over 10 minutes to 1 hour) in synoptic wind climates. The distribution which takes the form of Eq. (1).

$$P(> \bar{U}) = \exp \left\{ - \left(\frac{\bar{U}}{c} \right)^k \right\} \quad (1)$$

where c is the scale factor and k a shape factor.

The further assumption is made that the standard deviation of the fluctuating stress is related to the mean wind speed by the power law function of Eq. (2).

$$\sigma = A \bar{U}^n \quad (2)$$

The exponent n has a value of 2.0 when the structure reacts quasi-statically to wind loads but can take a higher value if there is some resonant dynamic response.

At a site with a wind speed climate described by Eq. (1), the total number of stress cycles with a range exceeding s can be written:

$$N(s) = \frac{k+1}{c^k} \int_0^\infty \bar{U}^{k+1} \exp \left[- \left(\frac{\bar{U}}{c} \right)^k - \left(\frac{s^2}{EA^2 \bar{U}^{2n}} \right) \right] d\bar{U} \quad (3)$$

(Note that the original version of Eq. (3) in Ref. [1] was based on stress amplitude (i.e. half range) instead of stress range).

Eq. (3) is based on Rice's level crossing formula [7] which is strictly applicable to narrow-band random processes, and overestimates the cycle count for wide-band processes.

In Eq. (3), the average cycling rate of the stress cycles has been assumed to vary with mean wind speed according to Eq.(4):

$$v_c = v_1 \bar{U}^b \quad (4)$$

where v_1 is notionally the cycling rate when the mean wind speed is 1 m/s.

T is the total time (in seconds) over which the stress cycles are counted.

3. Eurocode

Figure B.3 in the Eurocode [6] gives a curve showing the number of times, N_s , that the value ΔS of an effect (e.g. stress) produced by wind action is reached, or exceeded, during a period of 50 years. ΔS is given as a percentage of the

maximum load effect, S_k , in the period of 50 years.

The relationship between $\Delta S/S_k$ and N_s in the Eurocode is the following:

$$(\Delta S/S_k) = 1 - 0.174 \log_{10} N_s + 0.007 (\log_{10} N_s)^2 \quad (5)$$

Note that $(\Delta S/S_k)$ is expressed as a fraction in Eq. (5).

4. Sensitivity and application

An example of the numerical evaluation of Eq. (3) and comparison with the normalized form in the Eurocode is shown in Figure 1. It has been shown that the cycle count is independent of the parameter A in Eq. (2), and is insensitive to the parameters b , c , n and T . There is a greater sensitivity to the Weibull shape factor, k , for low values of 1.5 or less, but such values are unusual in wind climates.

It has also been found that the Eurocode function (Eq. (5)) is a good representation of the cycle count for a broad range of parameters.

When the normalized fatigue cycle count is applied with a family of three-segment fatigue damage lines (s - N relationships for steel), a simple relationship between fatigue damage and the ratio of maximum stress to the 'detail category' stress is obtained. This can potentially lead to a simple design approach to steel design for wind-induced fatigue loading

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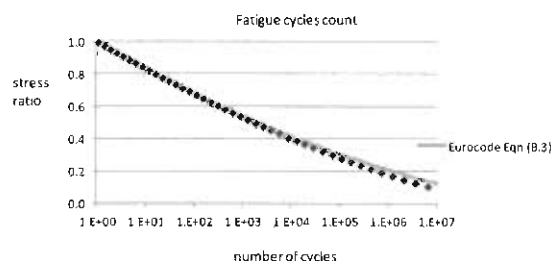


Figure 1. Number of cycles a stress level is exceeded under wind loading

($b = 0.5$, $c = 6.0$ m/s, $k = 2.0$ and $n = 2.2$)

FLOW-INDUCED VIBRATIONS OF STRUCTURAL ELEMENTS

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1. Introduction

Steel masts for carrying flood lights or signal lamps are sometimes constructed by using structural elements with Π -like or semi-circular cross-sections. A paper presented at the IACWE 2011 (Snaebjörnsson et al) dealt with wind induced vibrations of a beam with Π -like cross section with mounted flight control lamps. Soon after installation large cross-wind oscillations were observed. An extensive computational simulation resulted in the conclusion, that vortex excitation was causing the vibrations.

2. Experiments and Results

It was supposed that the assumption of galloping excitation should not be disregarded totally in this case.

Simple experiments in a small tunnel, normally used for demonstration of flow induced vibrations, gave the first hints. It is an open circuit wind tunnel with a closed test section. The rigid models were mounted on helical springs, so allowing vertical vibrations due to flow forces. The vibrations of the several models with Π -like cross sections started at distinct flow speeds but were not limited to a speed range. The amplitudes increased with increasing flow speed. This effect is an indication for galloping.

The next experimental steps were measurements of the cross-force coefficients in the wind tunnel for building aerodynamics at the Institute for Fluid Mechanics and Heat Transfer of the University of Technology, Vienna. The six-component balance of the tunnel is fitted with piezo-electric transducers essentially used for measuring fluctuating forces. In this case it was not necessary to measure unsteady forces. Experience has proved, that the quasi-static measurement of the cross-force coefficient as a function of angle of attack suffices as a basis for the description of the galloping phenomenon (Simiu 1986).

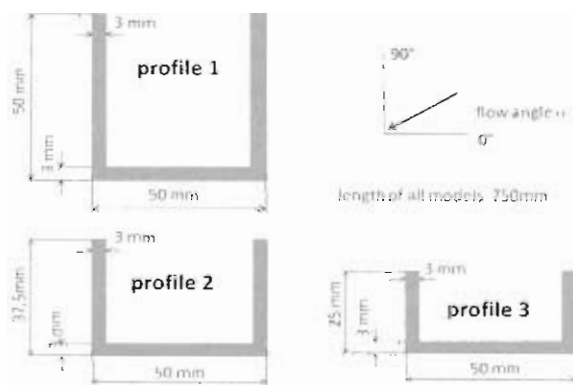


Figure 1: Cross sections of Π -models

Fig. 1 shows the cross-sections of the three investigated models with different Π -shapes. Cylinders with semi-circular shape and D-shape (both with a diameter of 50 mm and a wall thickness of 2 mm) were also investigated. The height of all models was 750 mm. Since all models were used for vibration tests too, they were made of light-weight plastics, because a low mass-damping parameter is essential in these tests. Parkinson (1974) mentions, that a cylinder with semi-circular or D cross-section will exhibit both vortex-induced oscillation and galloping, when the open, respectively the flat face is upstream and the semicircular cylinder forms the afterbody. Beams with these cross section shapes are also sometimes used for carrying flood lights.

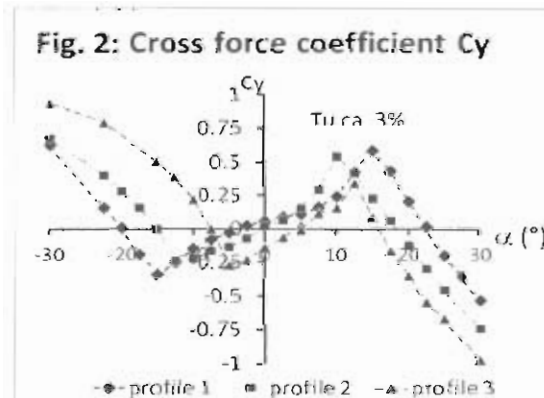


Fig. 2 shows for the Π -shapes the cross force coefficient C_y as a function of the angle of

attack α in a low turbulence flow. The results for higher turbulence intensity of 10% are similar. The shapes of the curves imply that these cross-sections are sensitive to galloping excitation.

The third part of the tests was accomplished also in the wind tunnel for building aerodynamics. The upright standing cylinders were mounted on a base platform carried by two bearings consisting of two hard spikes resting in conical deepening thus minimizing bearing friction. Since the bearings were aligned with the flow, only cross-flow deflections were possible. The model base was connected to adjustable helical springs which generated the restoring force to the aerodynamic interaction (fig.3).



Figure 3: Test rig

All cylinders with Π -like cross sections showed a similar behaviour. The cross-flow vibrations started at a certain flow speed and the amplitudes increased with increasing flow speed which is typical for galloping. The critical speed was approximately the critical speed for vortex excitation. Wawzonek et al 1979 have shown, that there is an interference of vortex shedding and galloping if the critical speeds of both effects are nearly the same. In that case vibrations start always at the critical speed of vortex shedding but the amplitudes increase with increasing flow speed.

The models with D-shape and semi-circular shape showed vibrations typical for vortex excitation.

3. Concluding remarks

Experiments in the wind tunnel at the Institute for Fluid Mechanics and Heat Transfer of the University of Technology in Vienna were

performed to investigate the sensibility of structural elements used for carrying flood lights to wind-excited vibrations. The Π -like cylinders are sensible to galloping, the galloping started approximately at the critical wind speed for vortex excitation. If the critical speed for vortex excitation and galloping are not so close together as in these experiments, it can be expected, that the two vibration effects can be observed separately. But anyway beams with Π -like cross-section seem sensible to wind excited vibrations and it is recommendable to undertake investigations for each special case, because the occurrence of vibrations depends on the special configuration, especially on the mass-damping parameter.

A cylinder of semi-circular or D-shaped cross-section exhibits vortex-excited vibrations and galloping if the flat face is upstream and the semi-circular face forms the after-body, as already mentioned by Parkinson 1979.

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REYNOLDS STRESS MODELING IN COMPUTATIONAL SIMULATIONS OF THE URBAN ATMOSPHERIC BOUNDARY LAYER FLOW

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1. Introduction

An appropriate modeling of the atmospheric boundary layer (ABL) flow is a major prerequisite for a successful computational study on wind loading of structures, air pollutant dispersion, wind farm design, pedestrian comfort. As wind-tunnel ABL simulations are already well established, they increasingly tend to be complemented with the less expensive and time consuming computational methods.

In this study, it was attempted to reproduce the urban ABL flow conditions using the $k-\varepsilon$ turbulence model without adding additional terms to transport equations, but simply by replicating the wind-tunnel test section in the computational domain and regulating the pressure drop along the domain to achieve the correct velocity at the reference height.

2. Numerical Setup

The computational simulations were performed using the $k-\varepsilon$ turbulence model to reproduce characteristics of the urban ABL flow, as obtained in wind-tunnel experiments reported in Kozmar (2011). The computational domain was a 2D cut through the symmetry plane of the wind tunnel test section, as presented in Figure 1, and calculations were performed at the wind-tunnel scale.

The slope of the ceiling in the computational domain was set to 0.8° to comply with a value used in the wind-tunnel experiments. The value of pressure is set at inlet and outlet boundaries, and the normal gradient of the velocity u , turbulence kinetic energy k , and turbulence dissipation rate ε fields is set to zero at both inlet and outlet boundaries.

An approach developed in this study uses the $k-\varepsilon$ turbulence model, Jones and Launder (1972), which has five constants, i.e., C_v , σ_k , σ_ε , C_1 , and C_2 . The values used in this study are $C_v = 0.044$, $C_1 = 1.44$, $C_2 = 1.92$, $\sigma_k = 1$, $\sigma_\varepsilon = 1.67$. Therefore, C_v was calculated from experimental results to fit the ratio between the measured Reynolds stress and the turbulence kinetic energy. Wall functions using the aerodynamic surface roughness length were implemented at the bottom and top walls of the computational domain. The applied aerodynamic surface roughness length z_0 at the bottom wall is $1.55 \cdot 10^{-2}$, and at the top wall it is $5 \cdot 10^{-4}$.

The computational results were validated against the experimental results obtained in the low-speed boundary layer wind tunnel at the Technische Universität München (TUM). The urban ABL wind-tunnel simulation has following characteristic parameters: power-law exponent α is 0.37, z_0 is 0.009 m, reference height z_{ref} is 0.202 m, reference velocity $\bar{u}_{\text{ref}}$ is 10.14 m/s, friction velocity u_τ is 1.43 m/s.

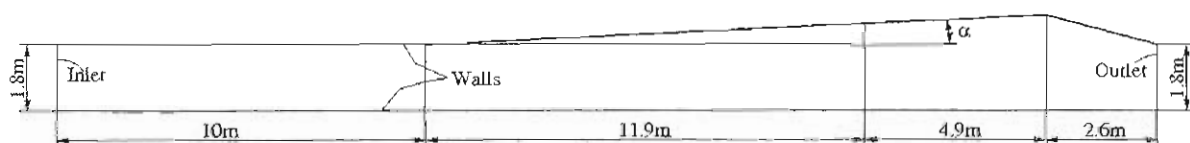


Figure 1. Geometry and dimensions of the computational domain

3. Results

Figures 2, 3, and 4 show a comparison of measured and calculated mean velocity, Reynolds stress, and turbulence kinetic energy profiles, respectively. As commonly the lower 100 m to 200 m of the ABL flow are in the primary focus of interest for a wind engineer (Holmes, 2007), the results are presented up to 1 m wind-tunnel scale that corresponds to 269 m full-scale when scaling-up the results of the urban ABL wind-tunnel simulation.

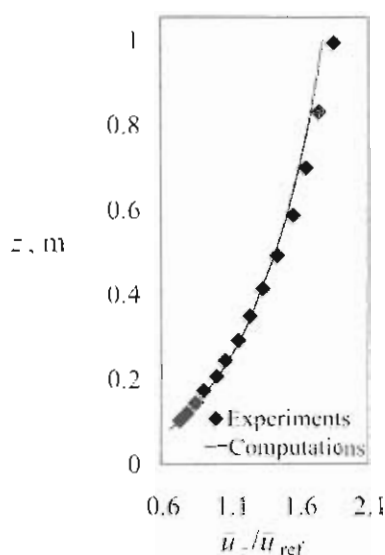


Figure 2. Comparison of measured and calculated mean velocity profiles

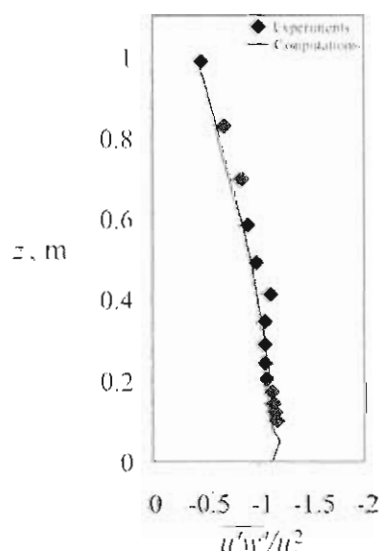


Figure 3. Comparison of measured and calculated Reynolds stress profiles

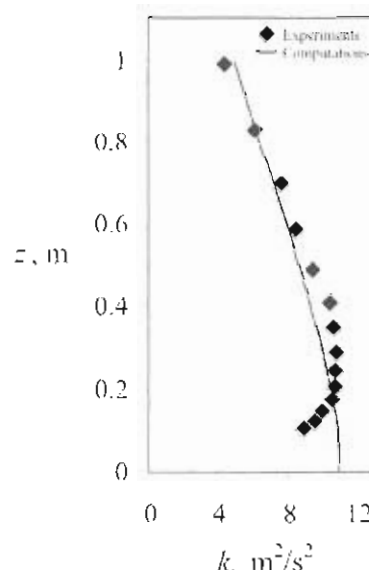


Figure 4. Comparison of measured and calculated turbulence kinetic energy profiles

Computational results reported in Figures 2, 3, and 4 are within $\pm 10\%$ of the wind-tunnel experiments.

4. Concluding remarks

A new approach for determining the physically valid boundary conditions has been developed to allow for an appropriate treatment of Reynolds stress modeling in computational simulations of the urban ABL flow using the $k-\varepsilon$ turbulence model.

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VERIFICATION OF NUMBER OF CYCLES FOR FATIGUE LIFE ESTIMATION OF WIND-SENSITIVE STRUCTURES

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1. Introduction

Fatigue life evaluation can be carried out using representative service stress spectra. The fatigue strengths related to constant and variable stress amplitudes are given by the Woehler curve or Gassner curve respectively. The fatigue strength under variable service stresses, Gassner-curve, depends on the spectrum of the service stresses $S_a = f(N)$, which is defined by the maximum value S_a^* , shape β and total number of cycles N_0 . The three parameters of the load spectrum just mentioned have a clear intuitive meaning that is helpful for setting simplifying assumptions needed for modelling. Namely, N_0 is directly related to the duration of loads and dynamic properties of the structure. Parameter β captures the stochastic nature of wind loads filtered through the dynamic properties inherent to the structure. The maximum load cycle S_a^* is a consequence of the load level, shape of the structure and foregoing parameters N_0 and β . As presented in (Barle et al., 2011), parameters S_a^* and β can be established by a short-time in-field tests. The main objective of this paper is the verification of the estimate of the N_0 based on spectral analysis.

2. Experimental Results

The results presented in Figures 1 and 2 were taken from two specific wind load conditions (northerly and southerly winds, Bura and Jugo) on a free standing 40m high lattice tower, placed in the Croatian coastal area, (Barle et al., 2010). The load condition Jugo has 3h 16min record with maximum wind speed $v_{max} = 27.0$ m/s, and for Bura : 6h 27min record with $v_{max} = 23.4$ m/s. Bending moment near the base is chosen because it excludes small directional variations within respective epoch

that could be encountered with stress spectra. The load case epochs are segmented in series of one-hour intervals. Frequency and load spectra were calculated for every data segment.

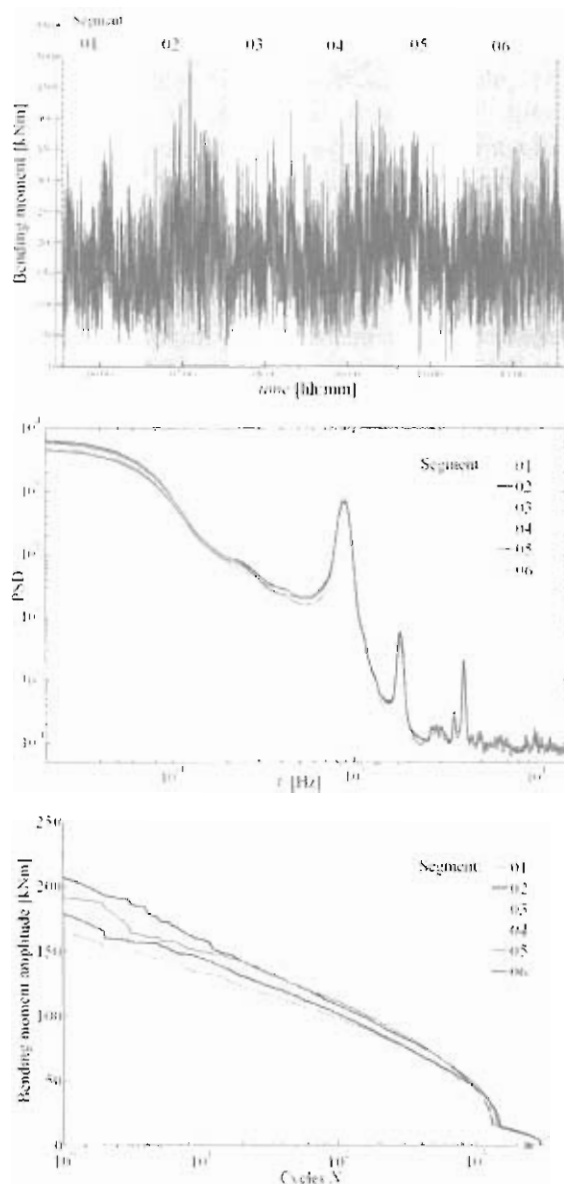


Figure 1. Six one-hour segments of the load condition Bura: Time-trace of the bending moment (top), frequency spectra (middle), load spectra (bottom).

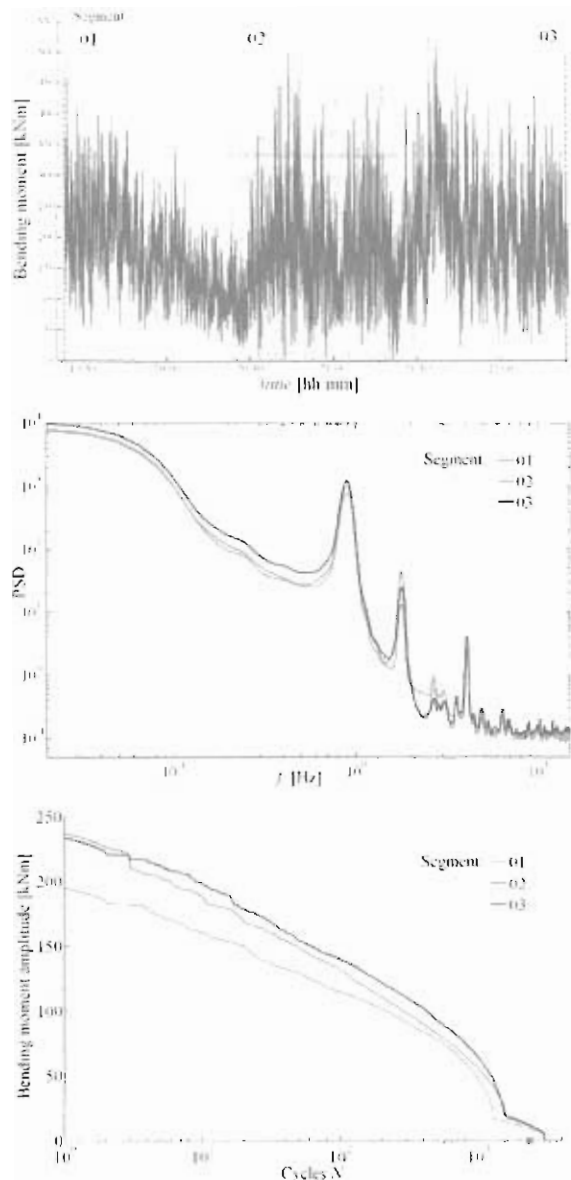


Figure 2. Three one-hour segments of the load condition Jugo: Time-trace of the bending moment (top), frequency spectra (middle), load spectra (bottom).

3. Results and discussion

Table 1 shows detailed analysis of the frequency spectra with calculated total number of cycles N_0 . After Rice, 1944 or Holmes 2002, the total number of cycles in a time period T is $N_0 = T/2\pi \cdot \sqrt{m_2/m_0}$, where m_k is the k^{th} moment of the spectral density.

The calculated values of cycle counts N_0 are plotted on the respective load spectra abscissas, see Figs. 1 and 2.

Segment	Moments of frequency spectra					N_0
	m_0	m_1	m_2	m_3	m_4	
Load condition Bura						
01	2.6977e3	4.1564e3	4.6310e4	2.2449e6	1.7516e8	2374
02	3.4993e3	4.8666e3	5.3917e4	2.6308e6	2.0580e8	2249
03	2.8717e3	4.2478e3	4.6997e4	2.2774e6	1.7799e8	2318
04	3.3140e3	4.4816e3	4.8153e4	2.3234e6	1.8178e8	2184
05	3.3584e3	5.2710e3	5.8756e4	2.8372e6	2.2161e8	2397
06	2.6600e3	4.5005e3	5.0851e4	2.4597e6	1.9248e8	2505
Load condition Jugo						
01	4.3159e3	5.9045e3	6.9128e4	3.4522e6	2.7246e8	2293
02	4.3800e3	7.4145e3	7.9888e4	3.5826e6	2.7767e8	2447
03	5.6589e3	8.9248e3	9.3679e4	4.2437e6	3.2985e8	2331

Table 1. Results of frequency spectra analysis; one line per segment.

4. Concluding remarks

It has been shown that the total cycle count N_0 can be accurately estimated using frequency parameters of the load record. Because of the structure simetry, cycle count does not depend on the load case.

5. References

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DESIGN WIND LOADS AND DIRECTIONALITY

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1. Introduction

The wind climate of Western Europe is mainly governed by strong frontal depressions which form over the Northern Atlantic and travel westwards. Therefore, the extreme wind speeds induced by this storm type show a distinct directionality, i.e. hourly wind speeds at storm level are only to be expected for a confined sector. The Eurocode allows specifying directional factors to exploit resulting favourable effects; however, there is no generally agreed method available for the specification of the directional factors. Basically, directional factors may be specified in the National Annexes. For Germany, the design engineer has to assume that the design wind load may occur for any wind direction.

Strictly speaking, the problem of directionality can not be treated by exclusively analyzing the wind climate. At least the directionality of the aerodynamic coefficient has to be considered as well. An enveloping solution for all types of buildings and structures has been proposed by Cook (1983), demanding that the exceedance probability of the directional design wind speed is the target value p_{target} divided by the number of sectors considered for directional design. This approach inevitably leads to a directional factor larger than unity, which has provoked at lot of unfortunately fruitless discussions over the recent decades.

A consistent extreme value analysis of the strong wind climate only becomes possible when the physically different and independent storm phenomena are separated (Gomes & Vickery, 1978). For Germany, at least two storm types have to be distinguished: storms induced by strong frontal depressions, as the mainly occur in the winter season, and thunderstorms, as they mainly occur in the summer. An extreme value analysis based on yearly extremes sampled for 12 different 30°-wide sectors violates the above basic demand. There-

fore, it remains unclear how far the produced results differ from the 'true' design values. Nevertheless, this strictly speaking wrong approach is widely used simply because it is so easy.

2. Basic variables

A more consistent approach requires treating the maximum hourly mean wind speed during an independent storm as basic variable, and considering all additional storm level hours during the respective event as dependent variables. Then, the following list of basic variables has to be considered:

- the number of independent storms per 12-month season
- the maximum hourly mean wind speed per independent storm
- the corresponding mean wind direction
- the duration of an independent storm in terms of the number of hours above the storm threshold value
- the relative intensity of the additional storm hours
- the probability of a direction change in the different storm hours

Additionally, the directional characteristics of the aerodynamic coefficients have to be considered. Basically, the aerodynamic coefficient may correspond to a local pressure, or to a global wind action like drag, uplift and overturning moment, or to a wind action effect in terms of a structural response like a bending moment or an internal stress. The required statistical information has to relate to the extremes with reference to a single hour under design wind speed conditions. In a first and conservative approach, the extreme value distribution type I can be used which is given as follows:

$$F(x) = \exp \left[-\exp \left(\left[\frac{\pi}{\sqrt{6}} \frac{x-m}{\sigma} \right] \right) \right] \quad (1)$$

m - mean value, σ - standard deviation

3. Convolution of the basic variables

The design value of the aerodynamic coefficient c can be obtained from solving the following basic convolution integral:

$$P_{\text{target}} = \sum_{\Phi} \int_{v=0}^{\infty} 1 - F_c(v, \Phi) \cdot (dv \cdot c_{\text{lim}}) \quad (2)$$

$f_v(v, \Phi)$ - joint probability of the extreme wind speed v and the wind direction

Φ - discrete sector for wind direction

$F_c(c, \Phi)$ - cumulative probability of c for wind direction Φ

The limiting value c_{lim} depends on the design wind load and the actual wind speed as follows:

$$c_{\text{lim}} = \frac{w_{\text{des}}}{0.5 \rho v^2} = \frac{v_{\text{des}}^2 \cdot c_{\text{des}}}{v^2} \quad (3)$$

w_{des} - design value of the wind load or wind action effect

c_{des} - design value of c

The probability density of v has to consider the number of storms per 12-month season, the probability density of the independent maximum hourly means, the probability density of the duration and the probability of a direction change. The accumulation of the exceedance probability over the duration of individual storms is obtained as 1 minus the product of the non-exceedance probabilities of all individual hours per storm.

4. Example of application

The consistent approach is applied to the wind induced actions and action effects of a simple box-shaped building (fig. 1). Along four cross-sections, the local pressures are measured in a boundary layer wind tunnel experiment for six positions on the wall and eleven positions on the roof. Beside local pressures, the global wind actions and the bending moments in the two corner of the frame are analyzed.

For the wind climate, the meteorological observations at Düsseldorf airport are used. Storm

level winds with hourly mean wind speeds above 14.1 m/s only occur for the sector from 180° to 330°. The corresponding cumulative probability distribution of the affected wind direction is shown in figure 2 for the strongest storm hour during independent storm events.

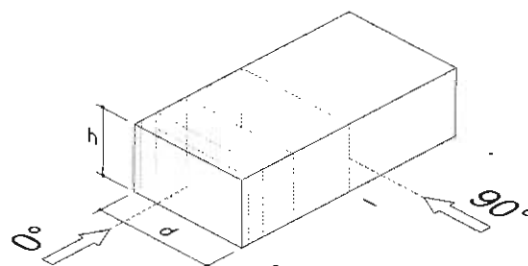


Figure 1. Basic geometry of the building ($h/d/l = 0.6/1/2$).

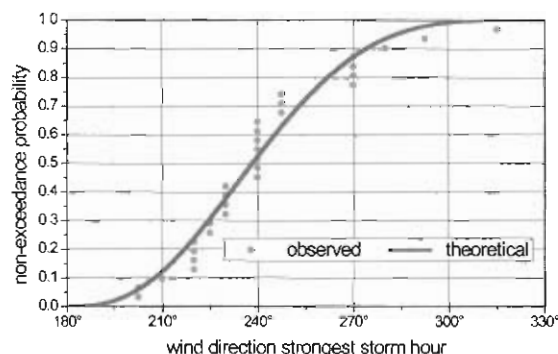


Figure 2. Cumulative probability distribution of the wind direction during the strongest storm hour.

The full paper will discuss the basic directional features of the wind climate in regard to all basic variables and will present design values for local pressures, global wind actions and wind actions effects varying the orientation of the building.

Acknowledgements

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WIND RESOURCE ASSESSMENT IN COMPLEX TERRAIN WITH LIMITED DATA

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1. Introduction

In the present study conventional techniques, such as statistical methods and linear model of WAsP computer code (Mortensen et al., 2004), will be combined by computational fluid dynamics (CFD) simulation in order to estimate potential wind energy in complex terrain of Ivan Sedlo region, 40 km west from city of Sarajevo, based on 18 months long measurements of wind characteristics. The conventional techniques cannot give reliable estimation of wind energy when the flow field differs significantly from the assumptions based on which statistical and linear models are developed. In addition, limited experimental data imposes further restrictions in providing a global view of the wind flow over the complex terrain and prevents the identification of separated flow regions, clearly unsuitable for installation of wind turbines.

The present study will explore possibility of combining results of linear (WAsP) and non-linear (CFD) models in order to obtain more reliable estimation of wind energy in complex terrain and in situation where limited measurement data exists. In addition, a comparative analysis will be performed, to illustrate how different the results obtained by two model types can be.

2. Methodology: experimental techniques and mathematical models

The methodology combined field (point) measurements by cup anemometers, followed by computer simulation of the wind flow using the WAsP model and CFD. The measurements were conducted by Mechanical Engineering Faculty, University of Sarajevo within the EU

project Advanced Decentralised Energy Generation. A single measurement unit was located at region of Ivan Sedlo. Two anemometers were installed on 10 m and 30 m above ground level while wind direction was measured on 30 m above ground level. Temperature and pressure were also measured. The frequency of measured data was 0.0016 Hz or each 10 minutes.

2.1 Terrain and wind characteristics

The terrain has characteristic saddle shape in the central part of the domain. Elevation of the measuring station is 1027 m above sea level. The position of measuring station is indicated in Fig. 1.



Figure 1. Three-dimensional view of the Ivan sedlo region showing region of interest, with contour levels every 20m

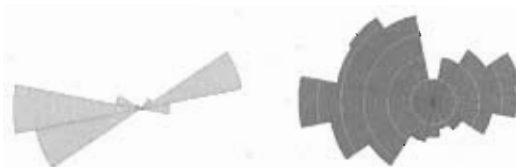


Figure 2. Wind rose of wind frequency (left) and mean wind velocity (right)

The wind roses shown in Fig. 2 indicate existence of dominant wind direction. This

direction will be used for setting up boundary conditions for numerical simulations.

2.2 Linear model (WASP)

The WASP is widely used computer code by the wind energy community. Its well-known limitations arise from the fact that code is based on linearised forms of the fluid flow equations. The effect of turbulence is represented through some empirical and theoretical correlations valid only for limited number of flow situations, mainly flow field that does not differ much from flow over a flat plate. These kind of models under-predict the wind speed reduction and cannot predict flow separation (Ayotte and Hughes, 2004; Castro et al., 2003) in the lee-side of the hills (not to mention mountains), and are clearly unsuitable for complex terrain.

2.3 Non-Linear model (CFD)

Contrary to the linear models, CFD models solve transport equations for velocity components and pressure equation, while turbulence is modelled by one or more turbulence quantities for which additional transport equations need to be solved. CFD simulations in this study are done using the commercial software STAR-CCM+. Beside efficient utilities for generation of the geometric model with computational mesh, the software also offers the use of a variety of physical models. For the best computational trade-off in the near-ground modelling of the wind flow, turbulence models with compound wall treatment will be applied.

3. Results and discussion

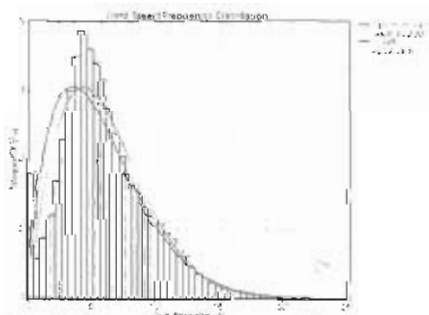


Figure 3. Wind speed frequency distribution

At the moment only preliminary results exist. Fig. 3 shows wind speed frequency distribution predicted by several different

methods. The predicted distribution is close to the measured one. The 3D model with the numerical mesh used in CFD simulations is illustrated in Fig. 4. The computational domain size is 3.3km x 2.3km x 2 km. Fig.5 shows streamlines of mean velocity.

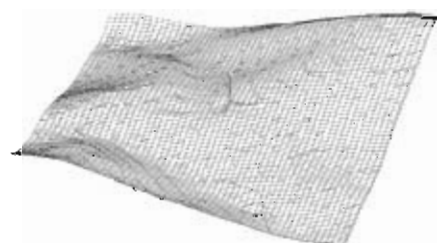


Figure 4. Three-dimensional view of the ground surface model with numerical mesh

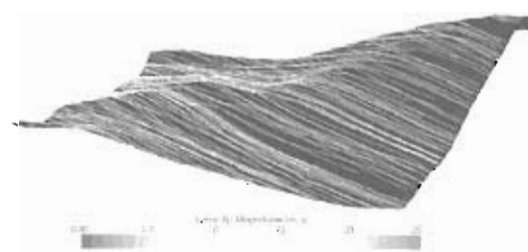


Figure 5. Streamlines of the air flow past the saddle.

4. Concluding remarks

The undergoing research will provide with more qualitative data for comparison of different models and possible conclusions regarding the adopted approach.

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EFFECTS OF WIND FORCES ON THE BEHAVIOR OF A CABLE-STAYED BRIDGE DURING CONSTRUCTION

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1. Introduction

The great development of new monitoring devices and new processing tools has allowed the conception of innovative monitoring strategies during the construction or service stage of an engineered structure. The construction of special bridges, as cable-stayed bridges, involves numerous activities. This is the case for the construction of the most important cable-stayed bridge in Mexico, which is located in a mountain area in the Northwest of Mexico. Due to the topographic features of the site of construction of the bridge, wind effects are of paramount importance.

Effects of wind forces derived from monitoring devices on different structural members of the cable-stayed bridge during construction will be evaluated. The implications of the analyses results on the structural behaviour of the bridge are discussed extensively.

2. Recording of wind velocities

The monitoring of wind velocities and directions started in June 2008 and continues up to date. An ultrasonic anemometer placed at the launching beam started to register wind velocity and direction in May 2010, about the time when the construction of the superstructure was initiated. Recording rate of weather stations can be as small as one sample per minute, whereas the ultrasonic anemometer can be activated manually to register 10 minute records or wind velocity and direction with a sampling rate of 10 Hz. The ultrasonic anemometer measures the three components of wind velocity (two horizontal and one vertical) and the azimuth. Records from the weather station placed at the launching beam are employed to compare the magnitude of the wind velocity to that obtained with the ultrasonic anemometer. It is observed that the results are comparable, except that the

ultrasonic anemometer captures the energy content related to the inertial range (where civil structures stand).

With the purpose of verifying the wind power spectrum (turbulence spectrum) associated to each of the orthogonal components of the wind velocity recorded by the ultrasonic anemometer, the normalized power spectral density function of selected records of wind speed was estimated by Welch's method (Welch, 1967). Davenport's spectrum of turbulence (Davenport, 1964) for the longitudinal component were also derived and compared.

3. Mathematical model

A finite element model was developed in the program SAP2000 using standard beam elements to model the superstructure and substructure. To account for the geometrical nonlinear behavior due to the cable sag, cables were modeled as truss elements and characterized with the modulus of Ernst (Ernst, 1965). Piers and towers were fixed at the base. The deck was modeled with beam elements and characterized with the properties of the actual dowels/segments (i.e., area, inertia and torsional constant).

4. Structural behaviour of the bridge during construction stages under wind forces

To study the structural behavior of the bridge under wind loads during construction, a parametric analysis was carried out.

In order to define wind actions acting on the cantilevers, turbulent wind speeds were simulated along the cantilever length. Each of these wind speeds were characterized, in terms of energy content, using the wind records obtained with the ultrasonic anemometer during the monitoring plan. Wind forces were then

estimated with the use of the turbulent wind speeds and applied to the mathematical model along the cantilever length. The model was employed to determine the lateral tip displacement of the cantilever due to wind forces at different construction stages.

Preliminary results are presented in this paper. Several construction stages (cantilever lengths) and mean wind speeds at deck level were considered.

5. Conclusions

Monitoring of wind velocities at the site of construction of the cable-stayed bridge were recorded and used to study the wind-induced structural behavior of the bridge during construction stages.

The monitoring program implemented was useful to estimate wind-induced responses of the bridge during construction stages with actual data.

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AERODYNAMIC LOADS ON VEHICLES DUE TO THE GUSTY BORA WIND

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1. Introduction

A major threat to traffic in coastal Croatia is a strong, cold and gusty north-easterly bora wind blowing over the Dinaric Alps along the eastern Adriatic. Hence, major freeways connecting the mainland with the coast have had to be closed to traffic very often in the past.

The bora's most prominent feature is strong gustiness. Mean hourly wind speeds rarely exceed 17 m/s (Koračin, 1984), while gusts may reach values to 69 m/s (Bajić and Peroš, 2005). Spectral analyses of bora gusts (Petkovšek, 1982; Belušić and Bencetić Klaić, 2004) reveal spectral peaks between 3 and 11 minutes, often with doubling of periods (Petkovšek, 1984), and a possibility of cessation and even reappearance of the pulsations in the course of a single bora episode (Watanabe, 1976). The gusts usually start rather suddenly, then weaken followed by gaps or weak wind periods which last about a third of the whole period, Petkovšek (1987). The main bora gusts are a consequence of cold air cylinders rolling down the slope on the warm side of the ridge, Petkovšek (1987). Main rolls together with atmospheric turbulence and partly sliding cold air develop a general flow with observed bora characteristics.

While there are numerous meteorological studies on the bora wind characteristics, reports on engineering simulation of that wind and its interaction with structures and vehicles are still rare. In this study, wind-tunnel experiments were carried out to simulate the bora wind loading of vehicles placed on a bridge deck.

2. Experimental setup

Experiments were carried out in the Transient Flow Field Simulator (TFFS) in the NatHaz Modeling Laboratory at the University of Notre Dame. The TFFS is an open-circuit

blowing type wind tunnel with an open test section. A battery of four fans allows a precise control and generation of transient events. At the inlet of the test section a maximal velocity of about 5 m/s can be obtained. The 114 mm long, 57 mm high and 57 mm wide prism, made of Plexiglas, represents a generic van model at the scale of 1:40. The vehicle model is placed on a generic bridge deck, as presented in Figure 1.



Figure 1. The vehicle model placed on a bridge deck in the TFFS at the University of Notre Dame

Two tests were reported in this study: a) aerodynamic loads on a vehicle model exposed to steady wind without gusts, b) aerodynamic loads on a vehicle model exposed to unsteady wind characterized with bora-like gusts. In the test without wind gusting, an average velocity was 4.6 m/s. In the test with wind gusting, the velocity time record consisted of an intermittent switching between 'regular' wind (smaller velocity) and wind gusts (larger velocity) each 3 s. Velocity during 'regular' wind events was fluctuating around 3.7 m/s, during gust events around 4.8 m/s, while an average wind velocity through the entire time record is 4.2 m/s. Horizontally and vertically the wind-tunnel model is oriented normal to the oncoming air flow. Velocity in the undisturbed flow was measured using a single hot-wire probe, while aerodynamic loads on a vehicle were determined by integrating the static surface pressures, as obtained from the tapings, over the respective model surface. The sampling rate in both tests is 500 Hz.

3. Results

Figures 2, 3, and 4 show the power spectral density of side force, lift force, and overturning moment fluctuations experienced by a vehicle, respectively. Vortex shedding from the bridge and wind gusting determine transient aerodynamic loads on a vehicle, as two peaks can be observed. Low frequency peak correspond to wind gusting of the wind flow each 3 s, while high frequency peak is due to shed vortices from the bridge, and their interactions with the van model.

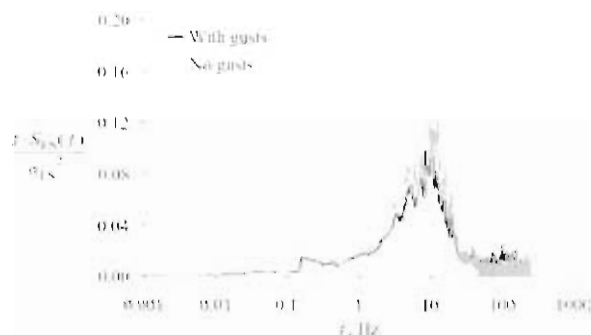


Figure 2. Power spectral density of side force fluctuations

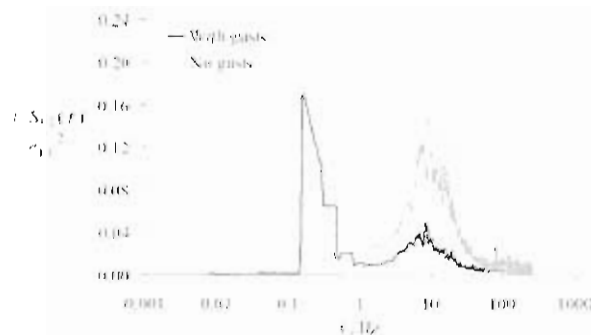


Figure 3. Power spectral density of lift force fluctuations

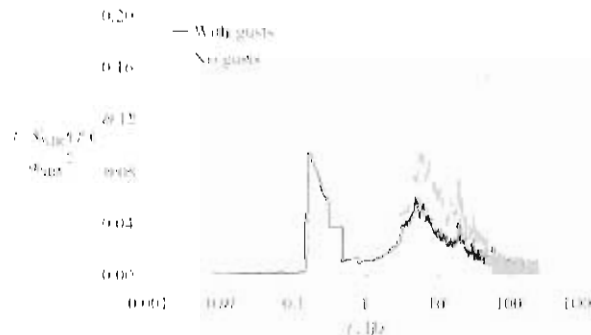


Figure 4. Power spectral density of overturning moment fluctuations

4. Concluding remarks

Wind gusting similar to the bora wind causes the turbulent wind energy to shift toward lower frequencies that can significantly deteriorate the vehicle stability, as just 2 s to 3 s are required to blow over a vehicle (Baker and Robinson, 1990).

Acknowledgements

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VORTEX SHEDDING WITH LONG PERIOD IN LEE OF ROTATING CIRCULAR CYLINDER

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1. Introduction

Numerous studies on flow phenomenon around stationary circular cylinder in free stream have been conducted experimentally and computationally for decades; however, very little is found for the case of rotating cylinder except for inviscid fluid regime. The flow around rotating circular cylinder in viscous free stream was studied by Ingham et al (1990) in the laminar flow. Kang et al (1999) and Stojkovic et al (2003) numerically investigated the vortex shedding around a rotating circular cylinder, and found a critical rotating speed at which the shedding diminishes in laminar flow. They also found out that the critical speed increases as Re grows where $Re = UD/\nu$, U , D and ν are the free stream speed, the diameter of the cylinder and the kinematic viscosity of the fluid, respectively. In addition, it was found out that vortex sheds on one side with very long period for the certain range of rotating speed, which accompanies the drag dramatically increases. However, the subjacent physics regarding the occurrence of the vortex shedding with long period at high the rotating speed is not clear, which leads the authors' further investigation for a circular cylinder with high rotating speed.

2. Numerical Method

This study employs so-called immersed boundary method (IBM) to solve the unsteady Navier-Stokes equation and continuity equation for 2D laminar flow of incompressible and viscous fluid. In IBM inspired by Peskin (1982), no-slip condition on the body surface is

satisfied by momentum forcing in Cartesian grid, and mass source is added for continuity (Kim et al, 2001). Governing equations non-dimensionalized by U , D , density ρ are listed as follows.

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i$$

$$\frac{\partial u_i}{\partial x_i} - q = 0$$

x_i and u_i denote Cartesian coordinate and velocity component, respectively. p represents pressure, q and f_i denotes mass source and momentum forcing, respectively. In order to evaluate the momentum, approximation is made by using a 3rd order Runge-Kutta method for convection term and a forward Euler method for diffusion terms.

Figure 1 shows the 2D computational domain of $45D$ in length and $50D$ in width. Uniform flow is modelled as the inlet condition while a convective condition and free slips are imposed on the outlet boundary and the upper/lower boundaries, respectively. Total number of grid employed in the computations is 30720 where the smallest width of a cell is $0.03D$.

The rotation of speed ω in counter-clock wise direction is defined as positive and the nondimensional speed of rotation is introduced as the rotation rate, $\alpha = \omega D / 2U$ shown in Figure 2.

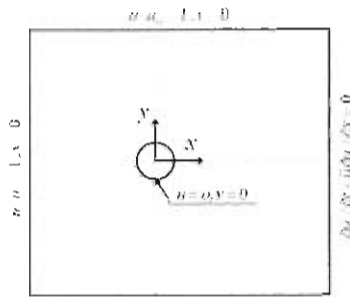


Figure 1. Computational domain and boundary condition.

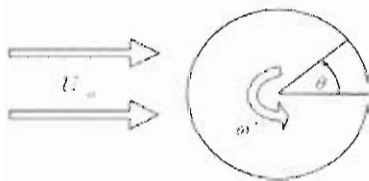


Figure 2. Rotating circular cylinder

3. Results and discussion

Figure 3 Shows the change of lift coefficient for $Re=60$ with variation of α from 1.0 to 6.0. Periodic shedding of Karman vortex sustains in the range of α from 1.0 to 1.4 and disappears at α greater than 1.6. Beyond $\alpha=1.6$, the flow becomes nearly steady until α reaches 5.2. In the range of α between 5.2 and 5.4, there occurs secondary vortex shedding with relatively long period as found in Stojkovic et al (2003).

Unlike a fixed cylinder, the rotation results in non-zero mean lift force which is well known Magnus effect. Since the rotation is in counter-clock wise, the mean lift tends to be downward in negative direction defined in Figure 2, which grows as the rotation speeds up.

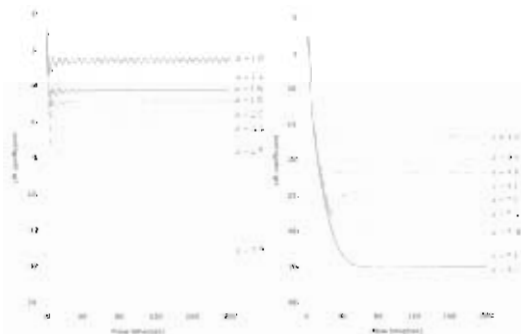


Figure 3. Lift coefficients for $Re=60$ for rotation rate between 1 and 6

4. Concluding remarks

This paper presents numerical investigations on aerodynamic forces acting on a rotating circular cylinder in 2D laminar flow. IBM is employed to solve the unsteady Navier-Stokes equations and the flow fields of $Re=60$ with nondimensional rotating speed $\alpha=1.0$ to 6.0. The appearance of secondary vortex shedding with long period at higher rotational speed are also well reproduced, agreeing with the previous studies of Stojkovic et al (2002, 2003)

Acknowledgements

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IMMERSED BOUNDARY APPLICATION TO AERODYNAMIC FORCES ON SHARP-EDGED SECTIONS

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1. Introduction

Numerical studies on flow around bridge decks pose troublesome difficulties in mesh generation and numerical stability due to existence of sharp edges. Immersed Boundary Method (IBM) also experiences similar problems in application since it uses extrapolated momentum forcing inside the solid boundary in order to satisfy the no-slip on the boundary. In this paper, interpolation method is implemented into IBM in such a way that the momentum forcing is applied within the fluid domain. The developed method is applied to 2 dimensional flow around equilateral triangular shape, and the results are verified with previous study.

2. Numerical Method

This study employs so-called immersed boundary method (IBM) to solve the unsteady Navier-Stokes equation and continuity equation for 2D laminar flow of incompressible and viscous fluid. In IBM inspired by Peskin (1982), no-slip condition on the body surface is satisfied by momentum forcing in Cartesian grid, and mass source often employed in IBM with extrapolation scheme for forcing is not applied. Governing equations non-dimensionalized by U, D, density ρ are listed as follows.

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

x_i and u_i denote Cartesian coordinate and velocity component, respectively. p represents pressure, q and f_i denotes mass source and momentum forcing, respectively. In order to evaluate the momentum, approximation is made by using a 3rd order Runge-Kutta method for convection term and a forward Euler method for diffusion terms (Kim and Moin, 1985).

Figure 1 shows the 2D computational domain of 100H in length and 50D in width, which H represents dimension of equilateral triangle in the direction vertical to inflow as shown in Figure 2. The smallest grid near the wedge was determined to H/64 after preliminary computations were performed. Uniform flow is modelled as the inlet condition while a convective condition and free slips are imposed on the outlet boundary and the upper/lower boundaries, respectively.

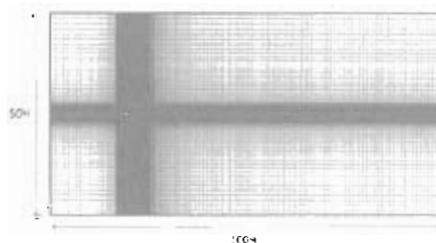


Figure 1. Computational domain.

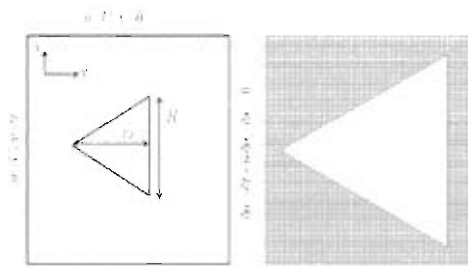


Figure 2. Boundary condition and magnified grid near wedge

3. Results and discussion

Figure 3 Shows the vorticity contours at $t=T/4$, $T/2$, $3T/4$ and T where T is shedding period for $Re=100$. The present results yields 0.188 as Strouhal number defined as $St=fH/U$ where f is the shedding frequency, which agrees well with 0.19 predicted by De and Dalal (2006).

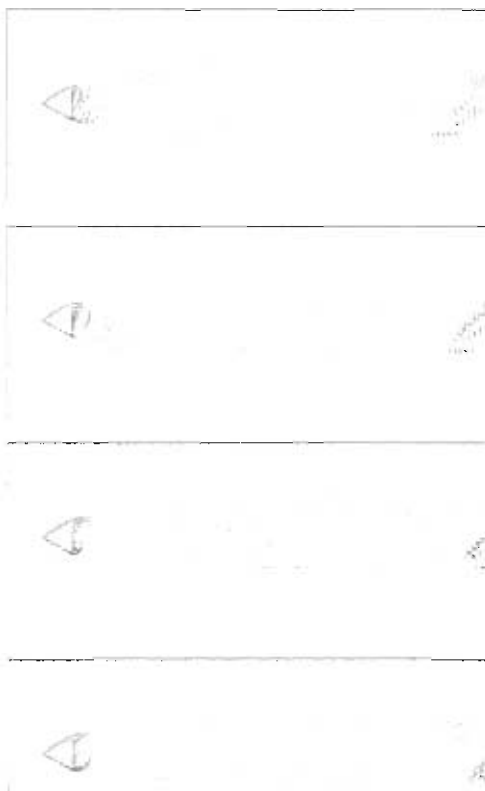


Figure 3. Vorticity contours at $t=T/4$, $T/2$, $3T/4$ and T where T is shedding period.

4. Concluding remarks

This paper presents the application of immersed boundary method with interpolation scheme for momentum forcing to the flow around sectional shapes with sharp edges. While conventional IBM with extrapolation scheme often leads to numerical instability due to unrealistic behavior around the sharp edges, the present method results in stable and accurate estimation on flow characteristics around equilateral triangular section in laminar flow.

Acknowledgements

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MOUNTAIN METEOROLOGY FOR WIND ENERGY

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1. Introduction

Wind energy resources rely heavily on the underlying airflow characteristics; in particular, on the mean wind speed and its spatio-temporal variability (e.g., Horvath et al. 2011). Since there is hardly any horizontally homogeneous natural land-surface, over which a homogeneous wind field exists (not to mention stratification effects), mountain meteorology (MM) becomes an essential ingredient in wind engineering. Here we skim over three chapters of MM and show in passing our contribution to some of these MM chapters: valley flows, mountain waves and downslope windstorms. Among other features, we tackle rotors and Bora-wind pulsations (e.g., Grisogono and Belušić, 2009). The related atmospheric processes greatly affect wind energy resource assessments in Croatia and elsewhere. For example, the nature of airflow over a narrow hill is quite different than that over a broad plateau, even if the terrain height and other factors are the same.

2. Observations, theory and simulations

Since there is no unifying theory for MM, for all the spatial and temporal scales involved, our knowledge and understanding of overall mesoscale meteorology still remains fragmented and incomplete. Furthermore, we live in our ever changing laboratory, called the Earth's atmosphere; hence, each measurement is unique and not repeatable. To restate, long-lasting, dedicated field measurements are simply irreplaceable and very precious. Like in the whole of geophysics, one needs 1) high-quality measurements and field campaign data, 2) numerical simulations and 3) development of theory. Only in this way, we may firmly proceed further substantially in the understanding and eventual applications of MM

for e.g., wind energy purposes. The authors contribute to all these three essential scientific components.

Figure 1 shows vertical cross-section of the cross-mountain wind component for a highly-nonlinear flow having its vertical Froude number, $Fr = U/(NH) = 0.6$, where N and H are the buoyancy frequency and the maximum terrain height, respectively. Even for constant input parameters (U , N , surface roughness, etc.), the flow develops internal critical level(s) under which the mountain wave breaks yielding high wind speeds in the immediate lee of the mountain where rotors or flow pulsations may occur.

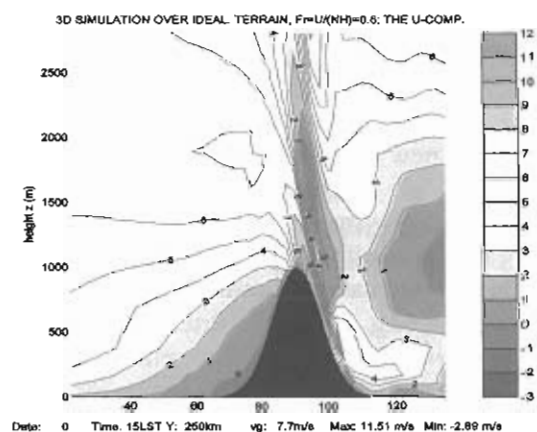


Figure 1. 3D simulation of the wind U-component, from left to right, over idealized terrain (black), for constant input parameters; vertical Froude number $Fr = 0.6$, which is also the inverse of dimensionless mountain height.

Various nonlinear models may simulate such and more realistic, e.g., input-changing flows. For Figure 1, MIUU model from Uppsala University, Sweden, is deployed (e.g., Grisogono and Enger, 2004). From a larger perspective, weather- and wind-systems in the broader region of Croatia are more variable and complicated (e.g., Horvath et al. 2008) than, for example, in the Scandinavia where wind

engineering is probably the strongest in the world.

3. Valley flows, mountain waves and downslope windstorms

Systematic valley flows, or even more simplified - slope flows, may easily reach sustainable wind speeds over 6 ms^{-1} ; however, these almost never go over 12 ms^{-1} . Gap flows and low-level jets, on the contrary, may approach mean wind speeds over $20\text{-}25 \text{ ms}^{-1}$. While linear or weakly nonlinear mountain waves can produce lee-side wind speeds up to $\sim 8 \text{ ms}^{-1}$, their fully nonlinear, large-amplitude forms may give much higher wind speeds. Namely, severe downslope windstorms that are related to mountain-wave breaking, including strong-to-severe Bora, reach wind gusts that are over 70 ms^{-1} (e.g., Grisogono and Belušić, 2009).

Our observations (e.g., Večenaj et al. 2010) show that Bora-related turbulent kinetic energy (TKE) follows closely its dissipation rate. This perhaps means that various turbulence transport terms have systematic contributions to TKE that do not vary greatly over a few hours. Moreover, we have just found out observed and simulated $T \approx 8 \text{ min.}$ periodicity, like pulsations, in a Bora case upstream from Split; this is a new result of this study. As implied by Horvath et al. (2008), large variability in basically all mesoscale processes occur in Croatia, thus affecting many coastal mountain-induced circulations, and the related airflow interplays (e.g., Telišman Prtenjak et al. 2010). It follows then naturally that setting up a wind energy farm in the broader region of Croatia should be primarily governed by the local and sub-regional airflow dynamics and its climatic future scenarios.

4. Concluding remarks

Several aspects of mountain meteorology are discussed in terms of data, theory and numerical simulations. These aspects, pertaining to airflow dynamics ranging from turbulence, slope and valley flows, low-level jets to mountain waves and their nonlinear

forms leading to severe downslope windstorms, should be carefully assessed when planning for wind engineering. Bora pulsations, $T \approx 8 \text{ min.}$, are discovered at the mid-NE Adriatic coast.

Our future work will explore vertical turbulent fluxes and their divergence during Bora flows and the related detailed simulations of the wind pulsations at the middle Adriatic coast.

Acknowledgements

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DESIGN SNOW LOADS FOR GABLE ROOFS

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1. Introduction

The snow load on gable roofs is influenced by a complex process of accumulation, decay and re-distribution. Especially, the accumulation and the re-distribution of snow are influenced by the wind. Therefore, a consistent load model for snow has to consider the wind speeds during and after the snow fall events. However, a generally agreed method is still missing on how to specify the amplitudes of the non-uniform distributed snow load on gable roofs. Therefore, the actual codal provisions in the Eurocode have been obtained rather by engineering judgement and calibration to traditional rules than by a consistent scientific approach.

Beside the question how much snow may accumulate on the downwind part of the roof, the question on how much snow remains on the upwind part is heavily discussed. This question is critical for collar beam roofs, since the snow load on the upwind roof reduces the bending effects in the downwind roof structure. For a consistent probabilistic approach, at least three basic variables have to be considered: the snow load amplitude on the ground, the shape factor to obtain the total amount of snow on the roof and the re-distribution coefficient to describe the unbalanced snow load. The probability distribution of the snow load on the ground can be obtained from an appropriate extreme value analysis of meteorological data. The JCSS probabilistic model code provides some basic information on the probability densities of the shape factor and the re-distribution coefficient, considering the influence of the wind during and after the snow fall event. Generally, the amount of snow which accumulates on the roof decreases when the wind speed increases. Considerable re-distribution occurs for roof slopes larger than 15°. Especially large re-distributions are obtained for roof slopes between 30° and 45°.

For the specification of the design values it is important to note that the interaction between snow and wind has to be analysed on the design level, i.e. for very rare events. Hence, the triggering wind conditions can be not be obtained from the average over the winter period but have to be specified for the extreme snow fall event.

2. Basic probability models

The JCSS model code recommends the Beta distribution as basic model for the shape factor and the re-distribution coefficient. The probability density of the variable x is given as:

$$f(x) = \frac{(x-a)^{r-1} \cdot (b-x)^{t-1} \cdot \Gamma(r+t)}{(b-a)^{r+t-1} \cdot \Gamma(r) \cdot \Gamma(t)} \quad (1)$$

a - minimum value

b - maximum value

Γ - Gamma function

The snow load on the ground s_{gr} is assumed to follow an extreme value distribution type I, which has the following cumulative probability distribution:

$$F(s_{gr}) = \exp \left[-\exp \left\{ - \left[\frac{\pi}{\sqrt{6}} \frac{s_{gr}-m}{\sigma} \right] \right\} \right] \quad (2)$$

m - mean value

σ - standard deviation

3. Target exceedance probability values

The Eurocode - as many other codes worldwide - generally specifies the design values of life loads as the product of the characteristic value, corresponding to an annual exceedance probability of 1/50, and a partial factor. Unfortunately, the Eurocode fails to specify the target values of the annual exceedance probability of the design snow load. A more consistent approach specifies the target exceedance values

with reference to the projected or design working life. This approach origins from earthquake engineering and nowadays is slowly adopted in wind engineering. Typically, four classes of structures are distinguished:

- A - structures with a special post disaster function (hospitals, schools, transmission lines, bridges)
- B - buildings which as a whole contain people in crowds (high-rise buildings, stadia, concert halls)
- C - normal structures (office buildings, commercial buildings, factories, residential buildings)
- D - structures presenting a low degree of hazard to life and other properties (farm buildings)

Tentative values for the ultimate limit state are summarized in table 1. These target values can be translated to corresponding yearly exceedance probabilities as follows:

$$p_L = 1 - (1 - p_L)^{1/L} \quad (3)$$

p_L - target exceedance probability in the design working life

p_L - annual exceedance probability

L - design projected working life in years

Table 1: Target values of the exceedance probability of the design wind load for the ultimate limit state - reference period: design working life

structural class	A	B	C	D
$p_{L, target}$	0.025	0.05	0.1	0.2

For sake of consistency, the target exceedance probability for the design value of the snow load on the ground should equal the basic target value of the exceedance probability. Then, the shape factor and re-distribution coefficient of each slope can be treated individually on this common basis.

The specification of the design values of the shape factor and the re-distribution coefficient requires some side conditions. For the shape

factor, it seems to be reasonable to demand that the design value of the total amount of snow on the roof should meet the target exceedance probability. For the re-distribution, the bending moment in the roof structure can be used as command variable.

4. Design value for the shape factor and the re-distribution coefficient

The design value for the shape factor is obtained from convolving the probability densities of the snow load on the ground and the probability density of the shape factor. The final equation for the specification of the design value η_{des} then becomes:

$$p(\eta > \eta_{des}) = \int_0^{\infty} f(s_{gr}) \cdot [1 - F(\eta_{lim})] ds_{gr} \quad (4)$$

$$\text{with } \eta_{lim} = \frac{s_{gr, des} \cdot c_r}{s_{gr}}$$

For the specification of the design value of the re-distribution coefficient c_r , the above convolution integral has to be extended by the corresponding probability density.

If the roof structure is a pitched free roof, instead of the bending moment the sum of η and c_r can be used as command variable. If, however, the roof is designed as a collar beam roof, the two contributions from η and c_r occur as a weighted sum. The respective weighting coefficients are obtained from a simple static analysis. For a roof slope of 45° and the collar beam at mid height of the roof, the weighting coefficients are 0.25 for the shape factor and 1 for the re-distribution coefficients.

The full paper will present the design values for the shape factor and the re-distribution coefficient for roof slopes from 0° to 60° varying the wind speed from 0 to 10 m/s.

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CYBERBASED ANALYSIS, MODELING AND SIMULATION OF WIND LOAD EFFECTS

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1. Introduction

Wind-related catastrophes (e.g., hurricanes, tornadoes, thunderstorms/downbursts) inflict enormous devastation on the built environment and result in a staggering number of fatalities. To better manage the impact of extreme wind events, a new culture of resilience needs to be developed based on innovative design solutions. By harnessing new technologies, quality of life and economic strength can be improved. The concept of shared resources is at the center of any plan and includes the exchanging of databases, computational and experimental resources, and full-scale data, as well as participating in collaborative research. Current examples of networked facilities for hazard reduction are the Network for Earthquake Engineering Simulation (NEES) (Van Den Eijnde et al., 2007). These facilities have made high-end resources accessible to groups otherwise limited by their personal research tools allowing them to expand the scope of their research to address complex problems. A similar model in the area of wind effects that shares resources and includes global engagement with centers of research, education, and design would result in similar productive advances.

The interdisciplinary nature of wind effects on structures requires a knowledge base from a number of subject areas, including structural engineering, engineering mechanics, probabilistic methods, fluid dynamics, turbulence, structural dynamics, experimental methods and risk and reliability, to better quantify the load effects (e.g., Simiu and Scanlan, 1996; Kareem, 2005). Under this umbrella, reliance is on analytical (Chen and Kareem, 2003, 2005), computational (Kareem, 1987; Yu and Kareem, 1998; Fluent, 2006; Kareem, 2008; OpenFOAM, 2011), and

experimental tools (Denoon et al., 2001; Bi and Smith, 2005; Burton et al., 2006; Sarkar et al., 2006; Butler and Kareem, 2007), full-scale measurements (Kareem, 1985, 1986; Main and Jones, 1999; Tamura et al., 2002; Masters et al., 2003; Jain and Smith, 2003; Satake et al., 2003; Campbell et al., 2004; Gurley et al., 2005; Smith et al., 2005; Kijewski-Correa et al., 2006a, 2006b; Kwon et al., 2010), codes and standards (SA/SNZ, 2002; AIJ, 2004; BSI, 2004; ASCE, 2005; NRC, 2005; Tamura et al., 2005; ISO, 2009), and databases (Simiu et al., 2003; Zhou et al., 2003; Cheng and Wang, 2004; Cheng et al., 2007; Kwon et al., 2005, 2008; NIST, 2006; TPU, 2011; FCMP, 2011; NIST, 2011). This presents a large set of subject areas and topics worthy of an Engineering Virtual Organization (EVO) to help assimilate information and resources into a publicly-accessible collaboratory. The resulting EVO would serve as an end-to-end system that integrates domestic and international community resources to facilitate an effective, transformative, and conveniently accessible venue for the acceleration of advances in research and development, as well as teaching and learning, in the area of wind effects.

Despite many advances in the area of wind effects on structures in recent decades, research has been conducted with limited resources scattered physically throughout universities, government, and private research laboratories as well as industry and trade organizations. With the trend toward increasingly complex designs such as free form architectures and the escalating potential for losses in coastal communities, the old paradigm is no longer optimal and requires the pooling of resources through a virtual organization reliant on cyberinfrastructure (CI). By centralizing tools and services within a flexible CI architecture to support research and education objectives in

real-time, this synergistic, integrative approach offers efficacious tools that the community can use to minimize windstorm damage and meet the challenges posed by burgeoning emergence of wind sensitive structures in expanding urban and suburban locales.

Interestingly, other technology fields are fast recognizing the potential impact of virtual organizations, as evidenced by digital airports and, more recently, digital oil fields, which involve effective gathering, analyzing, and visualization of data in real-time during drilling to quickly react to problem spots as they are detected (Holland and Campbell, 2007). This is serving as a catalyst to gain competitive advantage over others in a business with astronomical stakes. Similarly, the structural engineering field is currently at the dawn of a new information technology (IT) known as Building Information Modeling (BIM), which promises to revolutionize the design and construction of buildings. Building Information Models are 3-D, smart, parametric e-models of buildings that are shared by a team of designers and builders to facilitate the exchange and interoperability of information in a digital format. While BIM addresses the visualization and information exchange associated with structural projects, it possesses limited if any advanced analytical resources requisite for the design process. Certainly, the enormous reported losses from wind-related events and the increased sensitivity of freeform and super tall buildings, long-span bridges, and deep water offshore platforms to wind make it an ideal hazard for an EVO providing these much needed design tools.

An engineering virtual organization (EVO) would enable such a paradigm shift by offering real-time shared access to geographically dispersed resources for more effective research and education to achieve improved understanding and modeling of wind effects on structures. This paper summarizes such a program recently launched by the authors to develop a prototype EVO, **Virtual Organization for Reducing the Toll of EXtreme Winds (VORTEX-Winds)**. The goals of this initiative are (i) to establish and sustain such a virtual

community for wind hazard mitigation; (ii) to enhance this community's analysis and design capabilities to address next generation challenges posed by wind; and (iii) to facilitate education and training of the future workforce in the field. The steps toward the former goal have already been initiated through the collection of the field's leading universities, organizations, firms and government agencies to form this prototype collaboratory and their commitments to contribute their resources. The latter two goals will be accomplished through the formation of a virtual organization utilizing cyberinfrastructure technologies to stitch these geographically dispersed e-analysis and design modules encompassing Database-Assisted Design, Full-Scale Data, Stochastic Tools, Tele-Experimentation, Uncertainty Modeling, Damage Assessment, and Computational Platforms. The prototype EVO will allow access to the modules, while the fully functional EVO will also have the capability for automated, integrated analysis and design using multiple modules. In addition, both the prototype and full version of the EVO will offer an interactive knowledge base intended to aggregate and centralize the shared knowledge of the collaboratory, including a wind-wiki, damage database, help desk/FAQ, bulletin boards and curriculum tools to facilitate dissemination and education.

2. Overview of VORTEX-Winds

In order to mitigate escalating damage to property, loss of lives and disruption of local economies (AAWE, 2004; Burton et al., 2006; Prietula et al., 2007), a new research, teaching and design paradigm is proposed addressing wind effects on structures through the formation of a virtual organization utilizing an integrated cyberinfrastructure technologies, **VORTEX-Winds** (Figure 1). Wind hazards would particularly benefit from this paradigm given the reliance on experimental and empirical data in the design process.



Figure 1. Front page of VORTEX-Winds portal

Vision and goals

The basic vision of VORTEX-Winds is the development a comprehensive gateway for research and education to achieve improved understanding and modeling of wind effects on structures to counter the escalating loss of property and associated indirect losses and the increase in the sensitivity of emerging structural systems to winds. In response to this vision, the authors have established a virtual organization employing integrated cyberinfrastructure-based system that facilitates real-time, shared access to integrated design aids and services using geographically dispersed databases, specialized design/analysis tools, experimental facilities and full-scale monitoring networks, as well as providing a knowledge-base, with the following goals:

- (i) To establish and sustain a community contributing to and employing the resources integrated by cyberinfrastructure technologies to facilitate the mitigation of escalating damage, loss of lives and disruption of local economies posed by wind;
- (ii) To enhance analysis and design capabilities to address the challenges of innovative structural systems needed to realize, in a cost effective manner, buildings with ever increasing heights, bridges that span oceans, and offshore platforms tapping hydrocarbons in deeper waters exposed to weather extremes like hurricanes;

- (iii) To facilitate education and training of the future workforce in the field so that the growing competition in the global market is met through a cadre of well trained professionals and educators.

Structure & Shared Resources

The EVO structure is conceptually defined in Figure 1 as having two branches: the e-analysis and design modules and the Knowledge Base. The e-analysis and design modules are harvested from the independent work of a number of universities (largely supported by federal funding) and NIST and will be classified into seven divisions. These seven divisions are *Database-Assisted Design*, *Full-Scale/Field Site Data Repository*, *Statistical/Stochastic Toolboxes*, *Tele-Experimentation Services*, *Uncertainty Modeling*, *Damage Assessment*, and *Computational Platforms*. Examples of the modules offered within each of the divisions are shown in Figure 2. As discussed later in this paper, the modules can be interrogated independently or automatically queried and input into an integrated analysis and design approach. This integrated approach mimics the traditional off-line analysis and design processes, pooling contributions from a number of different disciplines. The second branch of VORTEX-Winds is the knowledge base intended to aggregate and centralize the shared knowledge of the collaboratory. Services in this area include the virtual encyclopedia or *wind-wiki* encompassing basic terminology and concepts pertaining to wind-structure interaction, a *damage database* (curated archives of post-disaster reconnaissance, e.g., Kareem, 1985, 1986), a *help desk*, where users can submit a question to the collaboratory and where past responses are archived as FAQs, *bulletin boards* hosting open discussions, *email list servers* for rapidly circulating announcements and other information, and *curriculum tools* to provide educators a means to formally integrate EVO services into their teaching. The knowledge base material will be broadly classified into 4 major areas: *Engineering Micrometeorology*; *Aerodynamics and Aeroelasticity*; *Structural Dynamics*;

Experimental Methods; Performance Evaluation (encompassing Risk/Reliability and Codes/Standards).

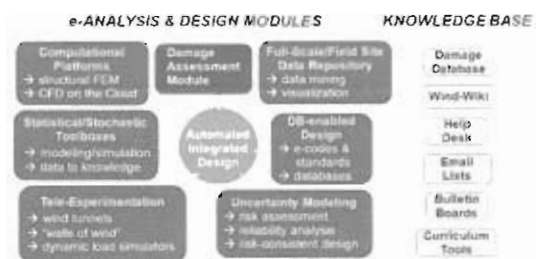


Figure 2. Schematic of VORTEX-Winds capabilities

Participants

Members: *Members* of the EVO will explicitly provide some shared resource in support of the stated mission. This diverse collection of resources includes aerodynamic and damping databases, wind load and response simulators and tunnels, computational fluid dynamics (CFD) platforms and real-time streaming meteorological (met) data. Most of these resources will come from leading universities in the area of wind effects on structures and generally have been the result of federal or state funding. A number of federal agencies are also poised to contribute to the EVO, including National Institute of Standards and Technology (NIST) and National Oceanic and Atmospheric Agency (NOAA). Members obtain this status after successfully contributing to the EVO. As a result, they will take a primary role in the governance of the organization.

An effective strategy for self-organization and governance is required to insure a collaboratory that is reputable, sustainable and technically reliable. This aspect of the EVO is particularly important due to the unique challenges that arise when pooling diverse resources that span institutional, cultural and professional boundaries. As such, VORTEX-Winds has adopted many of the proven organization and governing strategies that have already been accepted in professional circles around the world. The initial membership of the EVO has already been populated by many of the world's leading university research centers, laboratories and NIST specializing in wind

effects on structures. These researchers agree to provide modules in support of the prototype, to contribute to the knowledge base, and serve as end users of the services in their educational endeavors.

End Users: While the collaboratory members plan on using the shared resources to further their own research and educational missions, a number of other end users have also been identified within the community. *End users* are defined as those accessing the various services offered by the EVO, but not directly contributing to them. This class of participants is primarily associated with designers of structures with particular sensitivity to wind effects, such as tall buildings; however, the EVO will also reach out to educators as another category of end users. Other private sector entities, such as wind tunnels and risk analysis firms, may also become end users of various modules within VORTEX-Winds. All persons accessing the EVO (as end users or members) will be required to register, which will not only streamline operations, but will also gather data important to the evaluation and assessment of the EVO's impacts.

A selective group of end users has also been identified and will help to drive the services and features of the EVO and beta test the prototype developed. These private sector end users are leading design firms: Skidmore Owings and Merrill LLP (Chicago, USA), Samsung Corporation (Seoul, Korea), LERA (New York, USA), Shimizu Corporation (Tokyo, Japan), McNamara-Salvia (Boston, USA) and Weidlinger (Los Angeles, USA).

Stakeholders: *Stakeholders* indirectly benefit from the EVO through the actions of members and end users using the VORTEX-Winds' services to improve wind-resistant design. These stakeholders represent all who benefit from safer civil infrastructure. Obvious examples of stakeholders include risk management firms like Aon, Institute of Business and Home Safety (IBHS) and Risk Management Services (RMS), the wider professional community representing end users, such as Council on Tall Buildings and Urban Habitat (CTBUH), American Association for

Wind Engineering (AAWE), and American Society of Civil Engineers' (ASCE) committee on Tall Buildings and Technical Council on Wind Engineering, and organizations dealing with codes and standards, such as the International Code Council (ICC). However, all of society, as users of civil infrastructure, would benefit from more wind-resistant structures and thus also become indirect stakeholders.

Conceptual CI Design

VORTEX-Winds envisions to include all aspects of cyberinfrastructure support capabilities for the wind effects on structures research and engineering community, including: 1) communication tools to support collaboration, 2) high performance computational infrastructure, 3) data storage, data mining, visualizations, and data warehousing, and 4) access to remote sensors and tele-experimentation. As a result, VORTEX-Winds will ultimately offer three levels of functionality:

- Level 1: Knowledge Base Queries – these would be handled within the Knowledge Base's resources such as the help desk, Wind-Wiki or bulletin boards.
- Level 2: Module Stitching/Integration – utilization of specific e-analysis and design modules, e.g., retrieval of wind field data from the data repository for a specific region.
- Level 3: Integrated Analysis and Design – integrated analysis and design using multiple modules, e.g., online execution of basic wind-resistant design with computation hosted by the EVO and automated queries to multiple modules as shown in Figure 3.

These services will be hosted on the front-side by a single gateway server, with computation and analysis conducted on backside parallel servers. With the organizational and governance structure of VORTEX-Winds already defined, the primary technical hurdle is in achieving the three levels of functionality introduced in the previous section. This will require significant effort and coordination, particularly with respect to the integrated design feature. As a result, the prototype deployment that will be pursued in

this study will establish the basic components of the Knowledge Base and stitch together a number of geographically distributed resources representative of the seven module divisions identified in Figure 2. At this stage, VORTEX-Winds has been focused on the achievement of the Level 2 functionality. The next section will discuss more detailed information about a highlight of CI modules for analysis, modeling and simulation modules of wind load effects on structures.

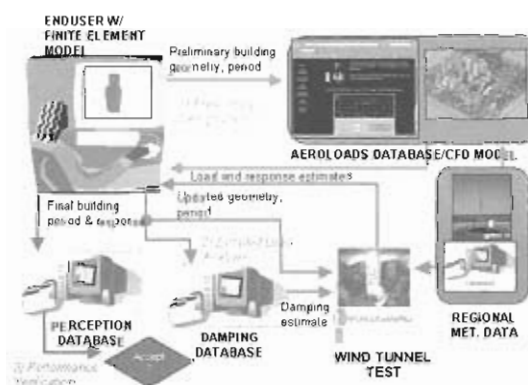


Figure 3. Schematic of integrated analysis and design concept

3. Cyberbased analysis, modeling and simulation modules of wind load effects in the VORTEX-Winds

Damage Database in the Knowledge Base Division

The Damage Database is a digital repository of reconnaissance documenting damage to structures during wind events such as hurricane/typhoon/cyclone, tornado, thunderstorm/downburst, extratropical winds etc. This has been established based on a typical database-query method and powered by Google Map/Earth API, which allows users to geographically view submissions of fellow VORTEX-Winds members, sorting by location, event classification and damage attributes (Figure 4). User can search damage archives either by pointing a spot in the Google map or by using query-criteria in the right side of the map. Each damage report is comprised of wind event on damage, maximum wind speed, damage location in either address or GPS coordinates, structural properties, damage

photos etc. which registered members can upload their own damage reports to the database via a user interface (Figure 5). It is envisioned that the database will be enriched more with members' contributions.

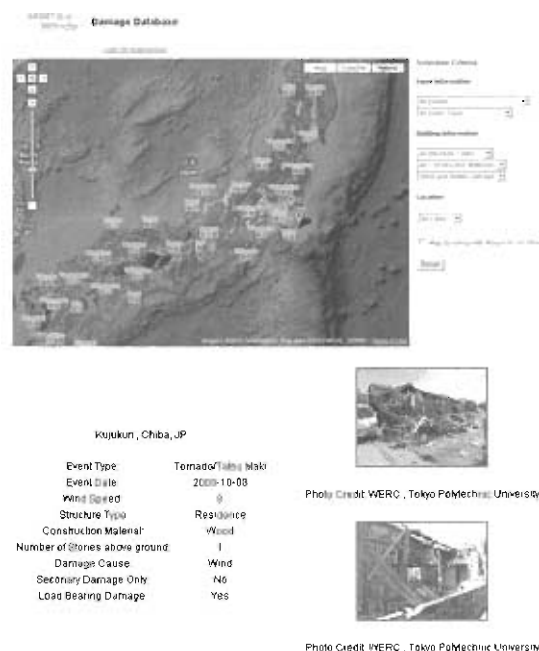


Figure 4. User interface of Damage Database: damage archive in the Google map and query-based database search (top); damage information with photos (bottom).

NatHaz Aerodynamic Loads Database in the Database-Enabled Design Division

Most international wind codes and standards traditionally have relied on reductive formats and simplifications, which often lead to tables and plots that describe wind loads on structures. The level of accuracy inherent in codification information in this format and the uncertainty associated with interpolation or extrapolation of information may compromise the overall accuracy in code-specified load effects. This has led to database-enabled design procedures, which offer convenient meshing with existing analysis software. Primarily, such databases rely on wind tunnel-derived data, which may be couched in analysis portals to provide desired load effects.



Figure 5. User interface for uploading damage report in the Damage Database

The NatHaz Aerodynamic Loads Database (NALD) introduced in 2000 has served an important first step in establishing an on-line experimental archive of high frequency base balance (HFBB) data for use in the preliminary design of high-rise buildings subjected to wind loads. As a result, NALD was recently introduced in the Commentary of ASCE 7-05 (C6.5.8) as an alternative means of assessing the dynamic wind load effects on high-rise buildings. The NALD ver. 2.0 has integrated the latest advances in data management and mining for interactive queries of aerodynamic load data and an integrated on-line analysis framework for determining the resulting base moments and equivalent static wind loads (ESWL) for survivability and accelerations for serviceability (habitability). The key feature of NALD ver. 2.0 is the flexibility its analysis module offers: users may select not only the data from the on-line NatHaz aerodynamic loads database, but also may input desired power spectral density (PSD) expression or wind tunnel-derived PSD data set obtained from HFBB experiment for the evaluation of wind load effects on high-rise buildings. Thus it

serves as a stand-alone analysis engine. The NALD ver. 2.0 provides a platform that can be readily expanded and supplemented to yield a comprehensive, simplified and efficient avenue for e-analysis of high-rise buildings. The architecture of NALD ver. 2.0 and the role of various web-based tools are summarized in Figure 6 and more detailed information can be found in Kwon et al. (2005, 2008).

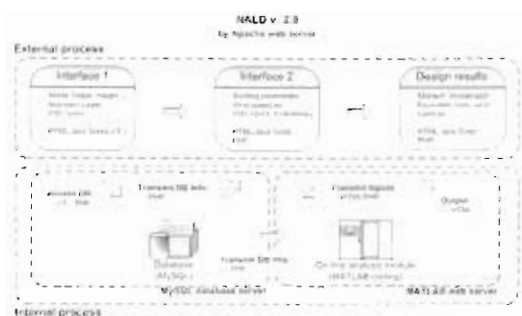


Figure 6. Diagram of NALD ver. 2.0 architecture

NatHaz Gust-front factor in the Database-Enabled Design Division

In comparison with atmospheric boundary layer winds, which are customarily treated as stationary, winds associated with gust-fronts originating from a thunderstorm/downburst exhibit rapid changes during a short time period which may be accompanied by changes in direction. This introduces nonstationarity both in the mean and the standard deviation of wind fluctuations. In order to realistically capture characteristics of gust-front winds and their attendant load effects, a new analysis framework is presented which is named here as the gust-front factor approach. This is akin to the gust loading factor format used in codes and standards world-wide for the treatment of conventional boundary layer winds. The gust-front factor expresses a generalized description of the genesis of the overall wind load effects on structures under both gust-front and boundary layer winds and it reduces simply to the gust loading factor for the case of conventional boundary layer winds. This approach encapsulates both the kinematic and dynamic features of gust-front induced wind effects on structures which distinguish

themselves from those experienced in conventional boundary layer flows, i.e., variation in the kinematics of the velocity profile and its effects on the associated aerodynamics; dynamic effects induced by the sudden rise in wind speed; non-stationarity of turbulence in gust-front winds; transient aerodynamics (Figure 7).

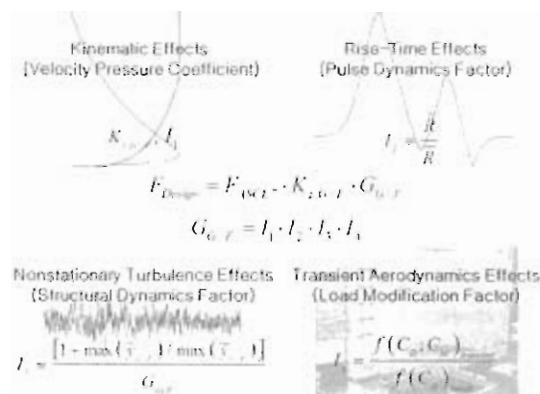


Figure 7. Schematic diagram of gust-front factor framework

To facilitate expeditious utilization of this framework in design practice and inclusion in codes and standards, the analysis framework and its workflow is introduced within a web-based portal. This eliminates the need for an in-depth understanding of the background within the framework and the need for associated computational effort. The portal has a user-friendly interface, permitting convenient analysis of several design scenarios with a host of potential loading conditions including the current ASCE 7-05 procedure in boundary layer winds for immediate comparison. More detailed information can be found in Kwon and Kareem (2007, 2009).

NatHaz on-line wind simulator in the Statistical/Stochastic Toolboxes Division

The NatHaz On-line Wind Simulator (NOWS) provides user with on-line simulation of stationary Gaussian multivariate wind fields for the longitudinal direction on-the-fly. With the help of rapid development of Internet/information technologies, the NOWS enables user to simulate stationary random

wind fields at any time/space through network-enabled computer system and a general web browser such as Internet Explorer, Firefox etc. In addition, intuitive user-friendly interface and result interface makes user easy to input any terms and to check simulation results such as display of simulated wind histories. This web framework features supporting both Metric (SI) and English units as input/output with on-line unit converter, location inputs for vertical, horizontal or arbitrary 2-D coordinates, ASCE 7-based mean wind calculation in terms of terrain roughness to minimize user's input, offering to download simulation results providing user with further off-line analysis such as structural dynamic analysis under wind loads.



Figure 8. Web-based on-line gust-front factor framework : user interface

In particular, one unique feature of this on-line analysis module is the flexibility for user to choose one of four simulation schemes such as Discrete frequency function with FFT (Wittig and Sinha, 1975), Schur decomposition approach with autoregressive (Di Paola, 1998; Di Paola and Gullo, 2001), Ergodic spectral representation method (Deodatis, 1996; Ding et al., 2006) and Conventional spectral

representation method (Shinozuka and Deodatis, 1991). Modified Kaimal spectrum is adopted for the PSD of longitudinal wind velocity fluctuations (Kaimal et al., 1972; Simiu, 1974), and Davenport coherence function (Davenport, 1967) is utilized to describe spatial correlation in the frequency domain. The user and results interfaces are given in Figure 9.



Figure 9. Nathaz on-line wind simulator (NOWS): user (top) and result (bottom) interfaces

VORTEX-Winds on-line wind simulator in the Statistical/Stochastic Toolboxes Division

The VORTEX-Winds on-line wind simulator (VOWS) offers wind simulations for three directions such as the longitudinal (u), lateral (v) or vertical (w) atmospheric turbulence components of winds. The wind profile is described by logarithmic law, which this framework accepts one of four different inputs such as friction velocity & roughness length, reference velocity at 10 m & roughness length, Eurocode (BSI, 2004) based inputs, and mean wind velocity & standard deviation/integral length scale of wind components. In addition, users can select one of three wind turbulence components (u , v or w) to be simulated (Carassale and Solari, 2006). Similar to NOWS, VOWS also offers to download simulation results as a file in the result interface, in addition to a comparison between target and simulated PSD for verification (Figure 10).

The VOWS is established in collaboration between Dr. Luigi Carassale, Univ. of Genoa (Italy) and NatHaz Modeling Laboratory, Univ. of Notre Dame (USA), which is the first international collaboratory to develop a cyberbased module in the VORTEX-Winds.

Damping database in the Data Repository Division

Despite the advancements that have been made in structural engineering in the last century, one critical parameter, damping, remains the complexity which is in part due to the diversity of sources contributing to the overall energy dissipation capability. The shortcomings of assuming viscous damping levels became apparent with the transition to light and flexible structures, as assumed levels of damping, on the order of 1% critical damping for steel structures and 2% for concrete, were often not realized in the constructed building. For this class of tall, wind-sensitive structures, the diminished levels of damping realized in practice led to a host of serviceability and especially habitability problems that were not anticipated in design.

The resounding difficulty in engineering known levels of damping in design was partly responsible for the flurry of auxiliary damping devices in recent years, which were found to provide measurable and controllable levels of this critical parameter (Kareem et al., 1999).

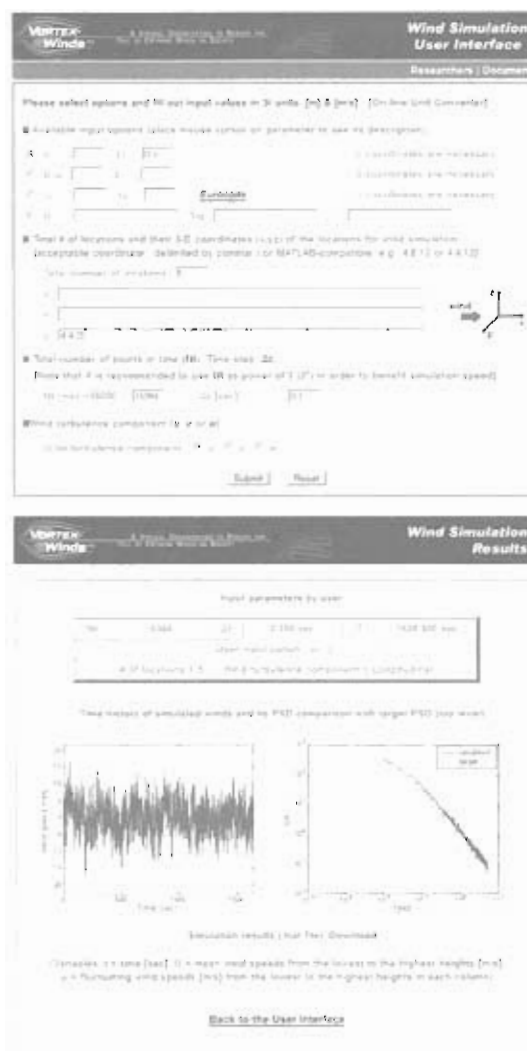


Figure 10. VORTEX-Winds on-line wind simulator (VOWS): user (top) and result (bottom) interfaces

The Damping database is an initiative project to provide users with on-line damping information for various building structures in terms of query-based module. The query parameters include not only building information such as cross-sectional shapes, widths, depths and heights but also test excitations, damping estimation methods,

ranges of damping ratios etc. (Figure 11). This database is currently established with the Japanese damping data sets (AIJ, 2000; Satake et al., 2003), however, it is expected that the contents will be expanded with other damping data in the future.



Figure 11. Damping Database: database-query (top) and result (bottom) interfaces

Database-enabled design module – high-rise in the Database-Enabled Design Division

The Database-enabled design module – high-rise (DEDM-HR) provides a web-based on-line tool for the preliminary design of high-rise buildings subjected to wind loads utilizing experimental archives of HFBB results, similar to NALD ver. 2.0 described earlier. It offers

building response estimates, i.e., base moments, maximum displacements & accelerations, and equivalent static wind loads (ESWL) etc. (Fig. 10). This e-module offers a unique feature, accommodating multiple data sets in collaboration with other research groups to overcome a limitation of database-enabled design procedure being a lack of data sets. Note that the DEDM-HR utilizes an advanced approach, distributed databases, which a central server communicates with multiple databases in terms of advanced IT solutions. This is more efficient way not to modify pre-existing web servers located in other regional/international groups, especially in view of disparate web server environments each other. Currently, the DEDM-HR is associated with two databases: one is NatHaz database from NatHaz modeling laboratory, University of Notre Dame, USA, which is the same with the one used in the NALD (Zhou et al., 2003; Kwon et al., 2005, 2008), and the other is Tamkang database from Wind Engineering Research Center (WERC), Tamkang University, Taiwan (Cheng and Wang, 2004; Cheng et al., 2007). Accordingly, the DEDM-HR offers data that pools information from both sources, thus expanding the range of building configurations and exposures. The examples of user and result interfaces are shown in Figures 12 and 13.

A bank of database involving synchronously measured pressure field around building surface is being developed to supplement the HFBB DEDM-HR. A schematic diagram is shown in Figure 14.

Database-enabled design module – low-rise in the Database-Enabled Design Division

The Database-enabled design module – low-rise (DEDM-LR) provides a web-based on-line tool for the preliminary design of low-rise buildings subjected to wind loads utilizing experimental archives of synchronous wind pressure measurements for a variety of models (Figure 15). The wind load associated with each of the pressure taps is distributed to the primary structural system at a number of discrete locations, i.e., the wind loads are distributed to

the structural frames at the attachment points of girts and purlins which support the cladding panels. Time series of structural loads are obtained from the measured time series of wind pressures for each wind direction. Then, these pressure time series are utilized to compute peak wind effects of the structural frames corresponding to winds with various speeds and directions.

Design Inputs (1/2)

Step 1: Select cross-sectional shape of interest. Values are side ratios (D/B).

0.20	0.25	0.33	0.50	0.67
1.00	1.50	2.00	3.00	4.00

Step 2: Wind exposure category (ASCE 7), building dimensions and 3-sec gust wind speed.

Input units (Default: Metric). Check for English and output (Default: Metric).

Exposure Category (ASCE 7):

Building width (B), Depth (D) and height (H):

3-sec gust wind speed (U_{ref}) e.g. ASCE 7 windmap:

Design Inputs (2/2)

User has selected Side Ratio (D/B) = 1.00 with Building dimensions of B = 40 m, D = 40 m and H = 200 m under Exposure A, which lead to Repeat Ratio ($H/\sqrt{B D}$) = 5.

In addition, User has also selected Metric (SI) units as input units and Metric (SI) units as Output units.

Step 4: Additional User Inputs

Recommended data set (R = Repeat Ratio). Please select one if multiple options are available:

Natural frequencies of building for three directions: alongwind (f_x), acrosswind (f_y), and torsional (f_t):

Mode shape expansion(s) for three directions (C/H² default: linear mode shape $\Phi=1.0$):

Peak-to-peak height of building, H_{pp} , all directions (along wind coefficient C_{pe}):

Figure 12. User interfaces of the DEDM-HR

The analysis codes are similar to WindPRESSURE software, which is an off-line

codes provided by NIST (Main and Fritz 2006). The wind pressure data sets used in this DEDM-LR are TPU (Tokyo Polytechnic University, Japan) aerodynamic database which is available at <http://wind.arch.t-kougei.ac.jp/system/eng/contents/code/tpu> (TPU 2011). The TPU database contains numerous results of low-rise models for three roof types such as Gable, Hip and Flat and each model was tested in various wind directions. This project is in the progress of a beta testing and it is expected to be finalized in the near future. The user interface is shown in Figure 16.

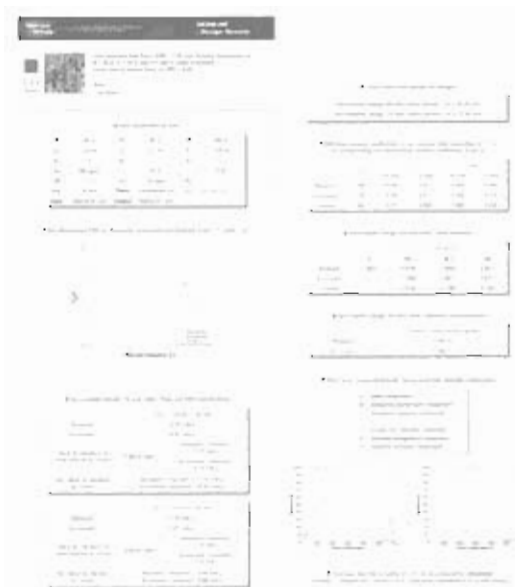


Figure 13. Result interface of the DEDM-HR

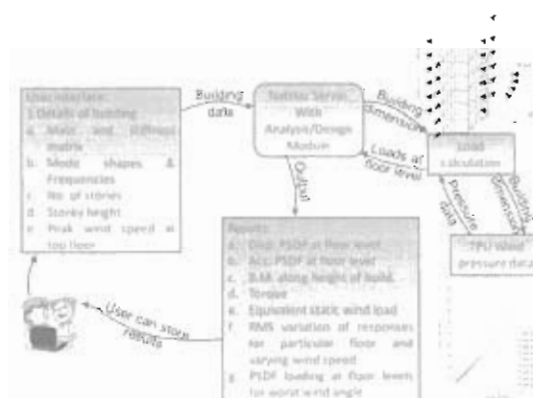


Figure 14. A schematic diagram of analysis of high-rise building using synchronous pressure data

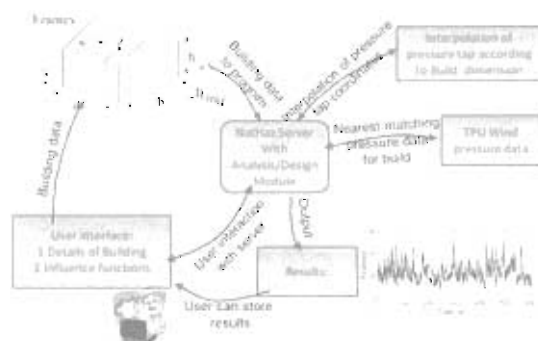


Figure 15. A schematic diagram of DEDM-LR

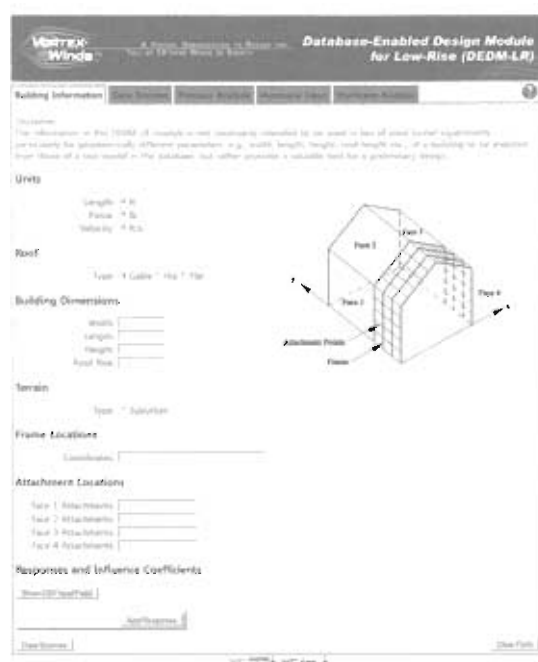


Figure 16. User interface of the DEDM-LR

CloudCFD in the Computational Platforms Division

Computational Fluid Dynamics (CFD) has been extensively developed over the last three decades and widely used in many fields in both academic research and industrial applications. From the basic discretization scheme, mesh generation and treatment, turbulence modeling, matrix solver, to parallel computing, all have advanced over the last thirty years. But what has not improved is the steep learning curve needed for using CFD despite availability of

numerous CFD documentations and textbooks in the market. A beginner may easily get lost while deciding which turbulence model to choose, or what parameters to set and how to control the solver, not even mentioning mesh generation and complex post-processing. From the programming point of view, when a code is written to solve a practical problem, the complexity of Navier-Stokes equation will soon make any good code unwieldy, and the final software is usually so difficult to handle (definitely needs a team to maintain) that even some basic verification and validation would require months to finish. Another fact is that CFD by nature needs more powerful computational resources than most other disciplines. For medium to large simulation cases, even if the code is tailored to perform effectively in a parallel mode, Infiniband support is still crucial to get a good scalability. And beginners usually have no understanding of these details.

What we see here are two road blocks; one is the lack of a user-friendly access platform that can encourage "trial and error", another is the availability of computational resources. The CloudCFD project is designed to remove these barriers; it aims to provide a platform for a broader user's community with various levels of skill, including amateurs, beginners, intermediate and even advanced CFD users. It also aims to be user-friendly and easily accessed and operated from a remote interface as well as offers resources needed to run the CFD solver with dynamic allocation (including server, cloud and computing nodes). Our philosophy in this endeavor is: to provide easy to use hands on experiences to beginners so that they are more enthused by the capabilities of CFD so that they start developing their own codes and on the other hand offer a versatile robust platform to those who are interested in application only. To accomplish this we need to design a task management system in such a way that it can dynamically allocate resources to the end-users, so that the end-users can focus more on their applications and not be distracted by the hardware issues. And just as in new field, our platform is not without guidelines; it is created with many useful "templates", so that

users can start from a template most fitting to their needs. Additionally, the default numerical settings for categories such as boundary conditions, discretization schemes, and turbulence modeling are preset, so that users can input minimal parameters and their simulations do not blow-up.

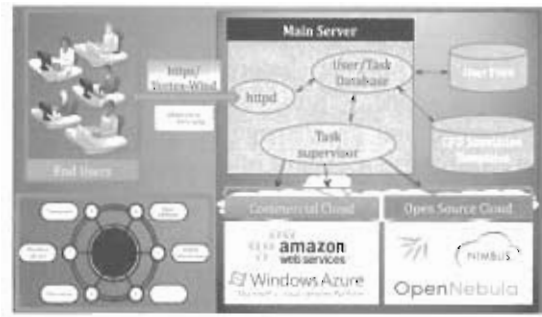


Figure 17. Architecture of CloudCFD

Figure 17 shows the vision and architecture of CloudCFD platform. It will have the following features (many of which have been completed.): (i) basically, it is a Software as a Service (SaaS) platform and is developed from dozens of open-source codes; (ii) it moves the CFD operation from a desktop-based to a web-based system. By doing this, it easily allows geographically dispersed users to access the platform; (iii) the computational resources burden is moved from the local desktop to a remote server and handled by a newly designed task management system so that the server, cloud, computing nodes or even a user defined IP node, can be dynamically allocated; (iv) the central pillar of the scheme is based on a well trusted CFD code (OpenFOAM); (v) by virtue of the back-end and front-end, modules and templates, it allows independent development and contribution.

Currently, the CloudCFD platform has been tested on two different groups of people and the results are promising (Figure 18). The back-end task management is stable. Four templates have been implemented (One 3D channel flow, three 2D cylinder external flows). Monitoring of the Courant number as well as visualization of mesh, scalar and vector fields and basic post processing tools have been provided. In the next stage, besides adding more templates, we will also: (i) give more flexibility to the users to

perform advanced tasks; (ii) allow users to operate on the template directly; (iii) generate mesh for arbitrary 2D shapes automatically; (iv) enhance the post-processing utility and visualization tools; (v) provide an even more user-friendly interface.

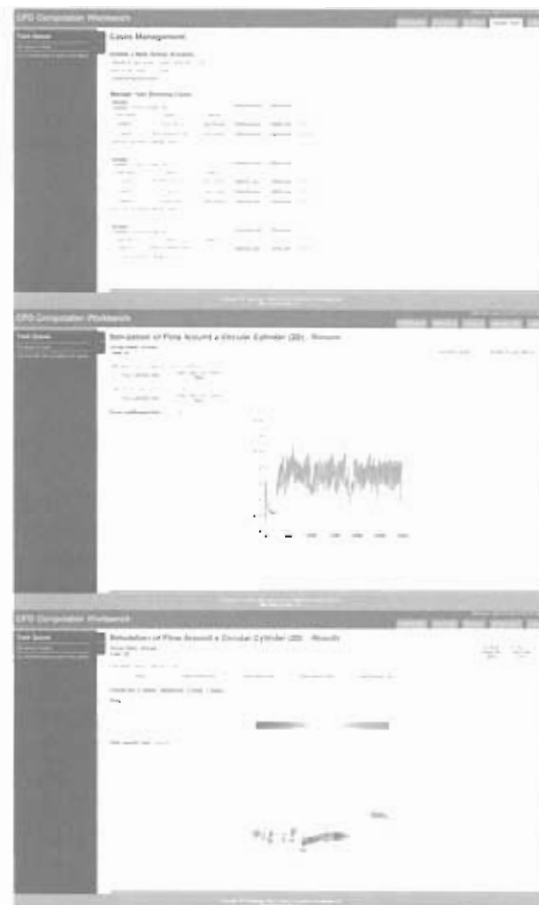


Figure 18. User and result interfaces of CloudCFD

NatHaz Interactive Wind Tunnel (NH-GUST) in the Tele-Experimentation Services Division

Given the interconnectedness of research, and the lack of available experimental hardware at every research location, developing a system that can bring the greatest amount of technology to as many people as possible is certainly warranted. To that end, a prototype system for an interactive wind tunnel, *NH-GUST*, has been developed. This system, conceptualized in Figure 19, consists of various components arranged within a local network. The end user does not interact with all of the

local network components, only with the front end system displayed on a webpage or other similar form. A dedicated web interface connects to the networked components. The prototype interface was originally designed to be controlled from a series of LabView (National Instruments) based control panels, however this is highly cumbersome and does not work with many operating systems.

Creating a cross-platform simulation framework, similar to that of the Network for Earthquake Engineering Simulation (NEES), for severe wind experiments can be a method for improving both the range and quantity of experimental data and for enhancing the wind engineering field as a whole. Instead of only a few schools sharing a limited number of wind tunnel facilities, a customizable (although limited) wind tunnel framework can be developed for anyone to input a wind field scenario to see how it will impact a physical model, either through the measurement of surface pressures or through a high frequency force balance (HFFB). Modification of existing wind tunnels and the development of research portals can have the effect of quickly building the body of knowledge for the effect of severe storm events, matching the body of knowledge that has been produced over several decades regarding the effects of boundary layer flows on prismatic structures.

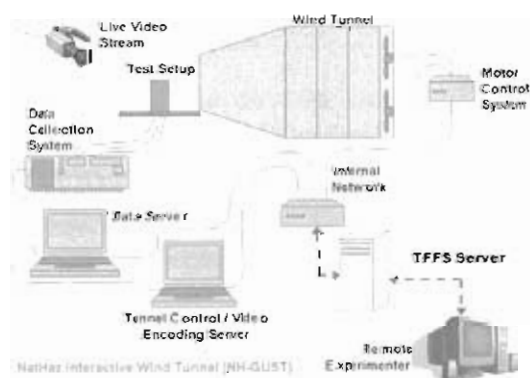


Figure 19. Prototype system for an interactive wind tunnel capable of generating gust-like events for remote experimentation

Damage Assessment from Remote Sensing Imagery in the Damage Assessment Division

This module is focused on automating damage assessment scheme based on before-and-after storm satellite/aerial imagery. The module aims at providing an accurate per-building assessment of damage. The framework for damage estimation consists of three steps. First, objects of interests such as buildings are detected automatically from pre-storm images. Next, correction of photometric and geometric differences between before and after image-pairs is done. Third, change detection is performed and the damage is classified based on quantitative measures of change.

In registering before and after images, the interest point detector finds the points which are repeated in both the images irrespective of variations due to damage, lighting conditions or camera viewpoint. For feature extraction we adopted a scale- and rotation-invariant interest point detector and descriptor termed SURF (Speeded Up Robust Feature). The choice of SURF features for this application is inspired by the fact that they are more robust and faster than other state-of-the-art detectors and descriptors while maintaining high accuracy in matching. An approximate k-Nearest Neighbor algorithm is used for feature matching. A speed-up of 10x was achieved without compromising robustness through randomized kd-trees and a constrained RANSAC algorithm.

Automatic color balancing approaches for different applications have been studied by different research communities in the past decade. However, in our work we addressed color balancing for the purpose of change detection. For multitemporal images, an ideal color correction approach should be effective at transferring the color palette of the source image to the target image for the unchanged areas while being able to transfer the global color characteristics for the changed area without creating visual artifacts. Towards this goal, we proposed a new local color balancing approach that uses adaptive windowing. We evaluated the proposed method against other state-of-the-art ones using a database consisting of aerial image pairs. The test image pairs were taken at different times, under different lighting conditions, and with different scene geometries

and camera positions. On this database, our proposed approach outperformed other state-of-the-art algorithms.

A segmentation-based scheme that uses a maximum likelihood classifier along with k-means clustering was adopted for building detection. We use MEPS features (Measure of Estimated and Predicted Shadows) along with shape-based features to combine segments together. We proposed a fusion of color and edge based features to classify rooftops into damage states. The evaluation was performed on a database of post-hurricane rooftops images that is larger than the ones used in previous studies. The classifier was found to perform with 80% accuracy for a 3-scale damage metric and 72% accuracy for a fine grained 4-scale damage metric. To validate results of building segmentation and damage classification, preparation of ground truth datasets is being done and is hoped to be made publicly available for research communities.



Figure 20. Source images (left), Target images (middle) that have been corrected geometrically. New target image transformed using the proposed local color transfer technique (right)



Figure 21. Example of a satellite image (left) and extracted buildings in white (right)

The first row in Figure 22 shows a collapsed building and the corresponding false color images for edge density, V Histogram (edge based features) and H means (color based feature). Notice that the edge density values are significantly higher in this case. The second row corresponds to a partially damaged

building with a cavity in the rooftop. V histogram indicates a significant change in the cavity area and H means shows a minor damage on the roof.

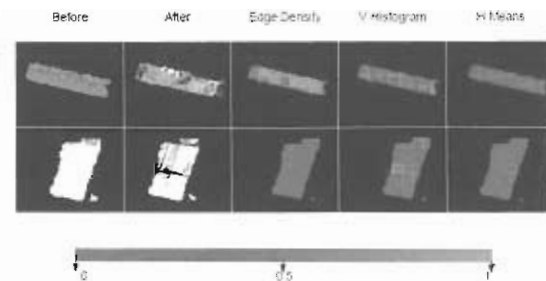


Figure 22. Damage assessment: extracted features for a collapsed building (top) and a partially damaged building (bottom)

This automated damage assessment scheme based on satellite imagery is being transported to a web-based portal for inclusion in VORTEX-Winds.

4. Concluding remarks

Despite many advances in the area of wind effects on structures in recent decades, research has been conducted with limited resources scattered physically throughout universities, government, and private research laboratories as well as industry and trade organizations. This has not permitted the community to fully benefit from the collective physical, computational and intellectual resources dedicated to this topic. The enormous reported losses from wind-related events and the increased sensitivity of freeform and super tall buildings, long-span bridges, and deep water offshore platforms to wind make it an ideal hazard for a virtual organization and associated cyberbased modules providing these much needed design tools. Through a collection of tools and services networked with a flexible architecture and interfaces to support research and education objectives in real-time, these cyberbased analysis, modelling and simulation tools in the VORTEX-Winds EVO promise to enhance the capability of each individual beyond one's current resources through a synergistic, integrative approach to understanding and modeling the complex wind-structure interactions. The result will be a

community as a whole better positioned to address the next frontiers in the field. Accordingly, VORTEX-Winds and associated cyberbased modules would serve as an end-to-end system that integrates domestic and international community resources related to wind effects on structures. It would facilitate an effective, transformative, and conveniently accessible venue for the acceleration of advances in research and development, as well as teaching and learning, in this area and would have a revolutionary impact on this field due to its unprecedented dissemination of knowledge and resources.

Acknowledgements

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APPLICATION OF THE LUMPED CAPACITANCE METHOD FOR THE COOLING OF VERTICAL PLATE IMMersed IN WATER

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We consider a cooling process of resting vertical plate at initial uniform temperature $T_{p,0} = 90^\circ\text{C}$ immersed in unbounded water at uniform initial temperature $T_0 = 20^\circ\text{C}$. The water is initially at rest and due to temperature difference between the plate and water natural convection develops. Since the intensity of natural convection changes over time, in this paper the heat transfer coefficient is considered variable.

The problem of cooling of vertical plate has been solved by using the following two approaches:

1) Full approach with heat conduction in the plate and laminar natural convection in the water. For this purpose AVL FIRE software was used.

2) Approach using lumped capacitance method taking into account the time value of a variable heat transfer coefficient.

In the first approach the governing equations for the water are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \quad (1)$$

$$\frac{\partial (\rho \bar{v})}{\partial t} + \nabla \cdot (\rho \bar{v} \bar{v}) = \nabla \cdot [\mu (\nabla \bar{v} + (\nabla \bar{v})^T)] - \nabla p + \rho \bar{g} \quad (2)$$

$$\frac{\partial (\rho c T)}{\partial t} + \nabla \cdot (\rho c \bar{v} T) = \nabla \cdot (\lambda \nabla T) \quad (3)$$

and for the plate the energy equation is

$$\frac{\partial (\rho_p c_p T)}{\partial t} = \nabla \cdot (\lambda_p \nabla T) \quad (4)$$

where, t is the time, $\bar{v}$ is the velocity of water, ρ is the temperature dependent water density, μ is the viscosity of water, c and c_p are the specific heat capacities of water and of the plate, respectively, λ and λ_p are the thermal conductivities of the water and of the plate, T is the temperature, p is the pressure, $\bar{g}$ is the gravity vector, ∇ is the vector differential operator and $(\nabla \bar{v})^T$ denotes the transpose of velocity gradient.

In the second approach the energy equation for the plate is

$$-\rho_p V_p c_p dT_p = \alpha_m A_p (T_p - T_0) dt \quad (5)$$

where α_m is the heat transfer coefficient, A_p is the plate surface area and V_p is the plate volume while T_p is the average plate temperature. The heat transfer coefficient is defined in [Leinhard J. H. IV, Leinhard J. H. V.] and it can be calculated from the following expression:

$$Nu = \frac{\alpha_m H}{\lambda} = 0,678 (Gr Pr)^{1/4} \left(\frac{Pr}{0,952 + Pr} \right)^{1/4} \quad (6)$$

where Grashof number, and Prandtl number are:

$$Gr = \frac{g \beta (T_p - T_\infty) H^3}{\nu^2} = \frac{g \beta \Delta T H^3}{\nu^2}; Pr = \frac{\nu}{a} \quad (7)$$

The expression for the heat transfer coefficient follows:

$$\alpha = 0,678 \frac{\lambda}{H} \left(\frac{Pr}{0,952 + Pr} \right)^{1/4} \left(\frac{g \beta H^3}{\nu a} \right)^{1/4} \Delta T^{1/4} = C \Delta T^{1/4}$$

Dimensionless solution of unsteady temperature field (9) is obtained by integration eq. (5) after substituting the expression (8).

$$\frac{T_p(t) - T_\infty}{T_{p,0} - T_\infty} = \left[1 + \frac{C(T_p - T_\infty)^{1/4} t}{4c\rho l_0} \right]^{-4} \quad (9)$$

Equation (9) is valid for small values of Biot number.

$$0 < Bi = \alpha l_0 / \lambda < 0.1 \quad (10)$$

The three cases of a cooling of vertical plate of the height H and the thickness δ are considered: the height of the plate is: 0.2 m; 0.6 m and 1.0 m. According the equation (10), maximal value of the thickness δ is 10 mm.

The changes of current values of the temperature of the vertical plate and the heat transfer coefficient in time, for the three values of the height of the plate, obtained by the lumped capacitance method are shown in Figures 1 and 2.

Results of the calculations – comparison of the values of plate temperature obtained by the lumped capacitance method (dashed line) with the values of the volume averaged temperature obtained by numerical simulation (solid line designed by FVM) during time of 300 s are shown in Figures 3a and 3b.

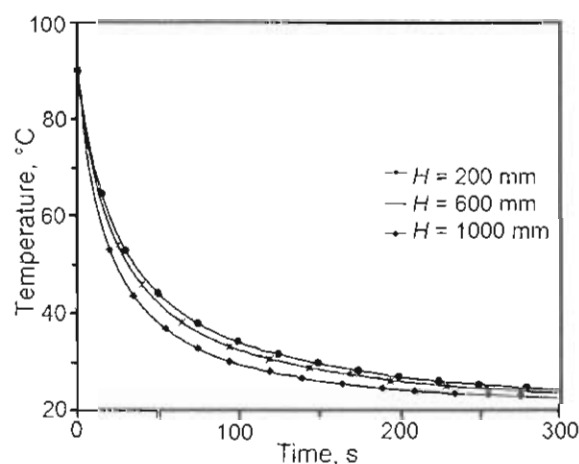


Figure 1. Dependence of the current value of the plate temperature (T_p) in time obtained by lumped capacitance method

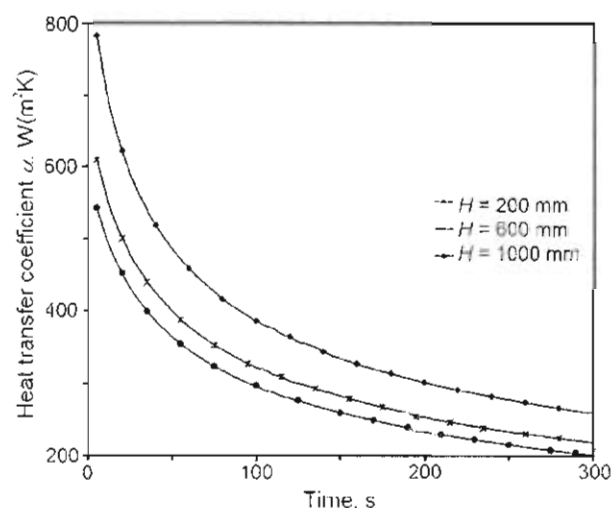


Figure 2. Dependence of the current value of the heat transfer coefficient in time obtained by lumped capacitance method

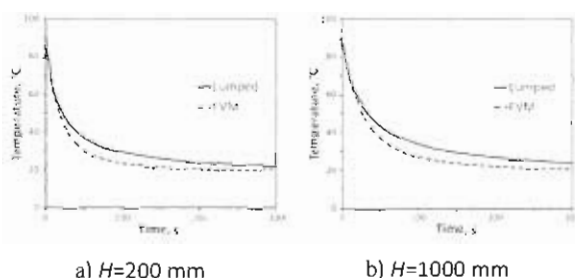


Figure 3. Comparison of the values of plate temperature (T_p) obtained by lumped capacitance method and by numerical simulation during time of 300 s

The discrepancy between results obtained by the lumped capacitance method and by AVL-FIRE is relatively small and could be explained by neglecting the transition of natural convection in lumped capacitance method (using correlation function for steady state condition during the whole integration time thus neglecting the inertia of the fluid flow).

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